5.5 - Factorials, Combinations, and Permutations

The factorial symbol \( ! \) denotes the product of decreasing positive whole numbers. For example, \( 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \).

By definition, \( 0! = 1 \).

5! is read "5 factorial".

The factorial symbol can be found on your calculator:
Press [MATH], select PRB, then select menu item 4 (which is the ! symbol).

ex. Find: \( 8! \)
\( (10 - 6)! \)
\( (12 - 12)! \)

The number of ways that we can select and arrange \( x \) items out of \( n \) different items is

\[
{n \choose x} = \frac{n!}{(n-x)!}
\]

\( nP_x \) is read as "the number of permutations (or arrangements) of selecting \( x \) elements from \( n \) elements." Note that we are selecting \( x \) items without replacement and that the order matters.

You can calculate permutations on your calculator:
First enter the value of \( n \). Then press [MATH] select PRB, then select menu item 2 (which is the \( nP_x \) symbol). Last enter the value of \( x \) (which the calculator calls \( r \)) and press [ENTER].

ex. Suppose our class starts a Statistics Club here at Cabrillo, and we want to select one student from the class to be the president, another to be the secretary, and a third to be the treasurer. If we randomly select these three people, out of the total 34 students, how many different arrangements of candidates are possible?

ex. An ice cream shop offers 30 flavors of ice-cream. Suppose you are going to buy a two scoop ice-cream cone, and you want two different flavors. If you do care how the ice-cream scoops are arranged, how many ways are there to arrange your 2 scoops?
The number of ways that we can select \( x \) items out of \( n \) different items is

\[
{n \choose x} = \frac{n!}{x!(n-x)!}
\]

\( n \choose x \) is read as "the number of combinations of \( n \) elements selected \( x \) at a time." Note that we are selecting \( x \) items \textit{without replacement} and that the \textit{order does not matter}.

You can calculate permutations on your calculator:
First enter the value of \( n \). Then press \( \text{MATH} \) select PRB, then select menu item 3 (which is the \( n \choose r \) symbol). Last enter the value of \( x \) (which the calculator calls \( r \)) and press \( \text{ENTER} \).

ex. Suppose our Statistics Club need to select a 3 person committee to investigate some peculiar probability problems. The three people are randomly selected out of the 34 club members. In how many ways can this committee be formed?

ex. You are now ready for another 2 scoop ice-cream, still with 30 flavors to choose from. You would like two different kinds of flavor, but you have decided that the order doesn't matter to you. How many different ice cream combinations do you have to choose from?

\textit{Some more fun counting and probability problems}

1. Your friend wants a 2 scoop ice-cream too. The order matters to her, but she doesn't necessarily need to have two different flavors. How many ice cream combinations does she have to choose from?

2. On a math test there are 10 multiple choice questions with 4 possible answers each, and 15 true-false questions. In how many possible ways can the 25 questions be answered? If you randomly guess on all problems, what is the probability that you get all of them right?
3. In California's Super Lotto Plus lottery game, winning the jackpot requires that you select the correct five numbers between 1 and 47 inclusive and, in a separate drawing, you must also select the correct single number between 1 and 27 inclusive. Find the probability of winning the jackpot.

4. A box contains 10 red marbles and 10 green marbles.

   (a) Sampling at random from the box five times with replacement, you have drawn a red marble all five times. What is the probability of drawing a red marble the sixth time?

   (b) Sampling at random from the box five times without replacement, you have drawn a red marble all five times. Without replacing any of the marbles, what is the probability of drawing a red marble the sixth time?

   (c) You have tossed a fair coin five times and have obtained heads all five times. A friend argues that according to the law of averages, a tail is due to occur and, hence, the probability of obtaining a head on the sixth toss is less than 0.50. Is he right? Is coin tossing mathematically equivalent to the procedure mentioned in part a or the procedure mentioned in part b? Explain.

5. A pizza parlor has 12 different toppings available for its pizzas, and 2 of these toppings are pepperoni and anchovies. If a customer picks 2 different toppings at random, find the probability that

   (a) neither topping is anchovies

   (b) pepperoni is one of the toppings
6. A trimotor plane has three engines - a central engine and an engine on each wing. The plane will crash only if the central engine fails and at least one of the two wing engines fails. The probability of failure during any given flight is 0.005 for the central engine and 0.008 for each of the wing engines. Assuming that the three engines operate independently, what is the probability that the plane will crash during a flight?

7. A screening test for a certain disease is prone to giving false positives or false negatives. If a patient being tested has the disease, the probability that the test indicates a (false) negative is 0.13. If the patient does not have the disease, the probability that the test indicates a (false) positive is 0.10. Assume that 3% of the patients being tested actually have the disease. Suppose that one patient is chosen at random and tested. Find the probability that

(a) this patient has the disease and tests positive

(b) this patient does not have the disease and tests positive

(c) this patient tests positive

(d) this patient has the disease given that he or she tests positive