LAB 8
RC Circuits

OBJECTIVES
1. Analyze the transient behavior of a series RC circuit.
2. Predict and measure the time constant $\tau$ of an RC circuit.
3. Predict and measure the response $V_C$ and $V_R$ to a square-wave input voltage.
4. Calculating the Capacitance of an Unknown Leaking Capacitor

EQUIPMENT
Circuits Kit, power supply, DMM, 1000 $\mu$F low-leakage capacitor, 22 k$\Omega$ resistor; DataStudio (Signal Generator & Voltage Sensors), unknown capacitor, decade resistor

THEORY
When a dc voltage source is connected across an uncharged capacitor, the rate at which the capacitor charges up decreases as time passes. At first, the capacitor is easy to charge because there is very little charge on the plates. But as charge accumulates on the plates, the voltage source must “do more work” to move additional charges onto the plates because the plates already have charge of the same sign on them. As a result, the capacitor charges exponentially, quickly at the beginning and more slowly as the capacitor becomes fully charged. The voltages of the capacitor and resistor during the charging phase is

$$V_C(t) = V_S(1 - e^{-t/\tau}) \quad \text{and} \quad V_R(t) = V_S e^{-t/\tau}$$

where $\tau = RC = \text{time constant}$ and $V_S$ is source voltage. Kirchhoff’s voltage law always applies such that the sum of the voltage drops across the capacitor and resistor must equal the voltage supplied by the power supply ($V_C + V_R = V_S$) as observed in the table below:

<table>
<thead>
<tr>
<th>$t/\tau$</th>
<th>$V_C = V_S(1 - e^{-t/\tau})$</th>
<th>$V_R = V_S e^{-t/\tau}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63%</td>
<td>37%</td>
</tr>
<tr>
<td>2</td>
<td>86%</td>
<td>14%</td>
</tr>
<tr>
<td>3</td>
<td>95%</td>
<td>5%</td>
</tr>
<tr>
<td>4</td>
<td>98%</td>
<td>2%</td>
</tr>
<tr>
<td>5</td>
<td>99%</td>
<td>1%</td>
</tr>
</tbody>
</table>

PROCEDURE
Part 1: Using a Stop Watch to Measure the Time Constant
a. Construct an RC circuit consisting of a 1000 $\mu$F low-leakage capacitor in series with a 22 k$\Omega$ resistor, and a power supply set to 10V. Make sure that the capacitor is attached to the ground and use a DMM to measure this voltage $V_C$.

b. Predict the time constant $\tau_{thy}$ of the RC circuit to two significant figures.

c. Energize the circuit and use a stopwatch to record the voltage $V_C$ at 5.0 s intervals until the capacitor is fully charged (after 6 time constants).

d. Plot $V_C$ vs. time and determine the time constant $\tau_{expt}$ when the capacitor’s voltage reaches 1$\tau$ (63% of its maximum voltage), 1$\tau$ (86%), and 1$\tau$ (95%).

e. Compare $\tau_{thy}$ and $\tau_{expt}$ using a percent difference. How do they compare?

Part 2: The Response $V_C$ and $V_R$ to a Square-Wave Input Voltage
A “positive-only” square wave imitates the action of charging and then discharging a capacitor by connecting and then disconnecting a dc voltage source.

a. Use DataStudio’s “positive-only” square voltage input (set to 5 V and 0.5 Hz) and two voltage sensors to measure the voltages across the resistor and capacitor. Replace the resistor in Part (1a) with a decade resistor box set at 100 $\Omega$ (using a DMM).
b. **Calculate**, in Excel, \((V_C)_{thy}\) and \((V_R)_{thy}\) for the charging phase of the cycle for \(t = 1\tau\), \(3\tau\) & \(5\tau\). Make sure to show sample calculations for \(V_C\) and \(V_R\).

c. Energize the circuit and **plot** \(V_C\) and \(V_R\) on the same time graph. Use the following technique to **measure** voltages \((V_C)_{expt}\) and \((V_R)_{expt}\):
   - Use the built-in analysis tools to find the first time constant at \(t = 1\tau\).
   - Now from this time, locate the value of \((V_C)_{expt}\) and \((V_R)_{expt}\) on the plot.
   - Repeat this process for \(t = 3\tau\) & \(5\tau\) and record your data in a table.

d. **Compare** \(V_{expt}\) and \(V_{thy}\) using a percent difference. How do they compare?

e. **Verify** that KVL holds for each time constant \(t = 1\tau\), \(3\tau\) & \(5\tau\). Is KVL satisfied at each time constant?

e. For each of the following modifications, first predict how the \(V_C(t)\) will change, then make the modification and compare the observed voltage waveforms with your predictions: (i) decrease the **amplitude** of the voltage source (do not exceed 10V), (ii) increase the **frequency** of the voltage source, and (iii) increase the **resistance** of the decade resistor.

**Part 3: Calculating the Capacitance of an Unknown Leaking Capacitor**

a. Construct a series RC circuit with DataStudio’s 5V-voltage source, an unknown capacitor \(C_X\) and a 22 k\(\Omega\) resistor. Use two voltage sensors to measure \(V_C\) and \(V_R\).

b. Energize the circuit and plot the voltages on the same graph. Continue to take data until the voltage across the capacitor and resistor has reached their steady-state voltages. Note that the capacitor will not charge up to 5V.

c. From the \(V_R\)-curve only, determine the time constant \(\tau\) and **predict** the unknown capacitance \(C_X\). Be careful not to include the “leaking voltage” of the resistor to get this time constant.