End of Chapter 4

4.4: A (possible) maximum profit $\pi$ occurs when either $\pi' = 0$ or, equivalently, $MR = MC$, that is, $R' = C'$.

Some suggested review problems: Ch. 4 Review: p. 222-225; 27a, b, 29a (these problems will NOT be collected; they are for your review only).

Chapter 5

5.1: If a rate of change of a quantity is known, areas of rectangles can be used to estimate the total change from a to b. Over (upper) and under (lower) estimates can be found using left and right sums (note: left or right estimates may give either over or under estimates!). Such estimates can be found using graphs or tables. Draw a picture to help out!

5.2: As the rectangles get narrower (equivalently as more rectangles are used), the estimates get closer to the actual value of the definite integral from a to b. Draw a picture to help out when estimating!

As $n \to \infty$ we get the exact value of the definite integral $\int_a^b f(x) \, dx$.

$\int_a^b |f(x)| \, dx$ is equal to the area between the x-axis and the curve.

5.3: The value of a general definite integral may be positive or negative, that is, it doesn't matter if a region is above or below the x-axis.

Areas are always all positive so take absolute value of any outputs that are negative. You may have to separate the interval into pieces and calculate the negative and positive parts separately and then add the results.

The area between two curves is found by deciding which curve is above the other and then integrating $\int_a^b (upper - lower) \, dx$.

5.4: This section repeats the idea that a definite integral finds the total change in a quantity (from a to b) from a rate of change. Sometimes this total change is added to an initial amount to answer a question. Pay attention to the units of the definite integral.

5.5: The Fundamental Theorem of Calculus will be used in Chapter 7.3. Basically, this Theorem ties together a function and its derivative.

$F'$ gives the moment-to-moment rate of change of $F$. Also, $\int_a^b F'(x) \, dx$ gives the total change in $F$ from a to b which is $F(b) - F(a)$. Thus,

$\int_a^b F'(x) \, dx = F(b) - F(a)$. This Theorem will be used from Ch. 7.3 on.

Some suggested review problems: Ch. 5 Review: p. 264-266: 1 – 19 (odds), 21, 29, 33 (these problems will NOT be collected; they are for your review only).

Chapter 7

7.1: Know the following rules for antiderivatives or indefinite integrals.

These rules come from the differentiation rules (see Ch. 3.1 - 3.3)

$C$ is a general constant.

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$  \hspace{1cm} \int x^{-1} \, dx = \frac{1}{x} \, dx = \ln|x| + C$, $x \neq 0$
\[ \int e^x \, dx = e^x + C \;; \quad \int e^{kx} \, dx = \frac{1}{k} e^{kx} + C \;; \quad \int k \, dx = kx + C \quad \text{(for a constant } k) \]

7.2: "Undoing" the Chain Rule requires using a substitution to get to a familiar formula (like one listed above). Look for a factor (part of a product) that might be the derivative of another factor (part of a product). After integrating substitute back to get the answer in terms of the original unknown. You may always "fix up" a constant factor but never a variable factor!

See Examples 1 - 5 in Chapter 7.2, pages 305-307.

7.3: The Fundamental Theorem of Calculus gives a method for finding exact numerical values of definite integrals. This method requires finding an antiderivative first.

\[ \int_a^b F'(x) \, dx = F(b) - F(a) \]

More specifically, find \( F(x) \), an antiderivative of \( F'(x) \), substitute the limits and subtract \( F(b) - F(a) \). This numerical value of the definite integral may be total change or may be area under a curve on a graph.

More integration practice is on p. 326: do any except those involving \( \sin x \) or \( \cos x \).

(these problems will NOT be collected; they are for your review only).