Chapter 9

9.1: Functions with 2 or more inputs are introduced. These functions may be given by:
1) words,
2) values in table form (see the example on page 350 - also done in class),
3) a formula or equation (see examples 1 & 2, pp. 350-351).

9.2: Read the section to know what a contour diagram is.

9.3: Review of the idea of rate of change and the units of rates of change.
For functions of two variables, say \( z = f(x, y) \), there are two rates of change,
namely one is the rate of change in the x-direction (hold y constant) and the other is in the direction of y
(hold x constant).

Using \( z = f(x, y) \), the notation is:
\[
\frac{\partial z}{\partial x} = f_x, \text{ the partial of } z \text{ or } f \text{ with respect to } x \text{ (hold } y \text{ constant)}
\]
\[
\frac{\partial z}{\partial y} = f_y, \text{ the partial of } z \text{ or } f \text{ with respect to } y \text{ (hold } x \text{ constant)}
\]

As done previously (Ch. 2 on) rates of change can be estimated using a table or a formula/equation. (See the
summary/review for Ch. 2.1 - 2.3.
Pay attention to the units!! Units of a derivative are still \( \frac{\text{output units}}{\text{input units}} \)!!!
(See p. 102)

9.4: Review of differentiation formulas using partial derivatives. Here you hold one variable constant and
differentiate with respect to the other.
There are 4 second order partial derivatives, namely
\[
f_{xx} = \frac{\partial^2 z}{\partial x^2}, \quad f_{yy} = \frac{\partial^2 z}{\partial y^2}, \quad f_{xy} = \frac{\partial^2 z}{\partial x \partial y}, \quad f_{yx} = \frac{\partial^2 z}{\partial y \partial x}
\]
The last 2 are called mixed partials and for our purposes are equal.

9.5: Review of critical points and local maximum and local minimum
Recall that critical points are only possibilities for local max or min!
The box on page 377 and the 2nd Derivative Test for functions of 2 variables (the box on p. 379) will be
given to you to use during the last exam.

9.6: The Method of Lagrange multipliers gives a method for finding max and min when there is a constraint.
The box on page 383 containing the Lagrange multipliers method will be given to you to use during the
last exam. Additionally, a copy of the example worked out in class will also be given to you to refer to
during the exam.

No suggested additional problems for Chapter 9.
After completing the Chapter 9 homework, go back and be sure you can do the problems again and do them
correctly without looking up anything in the book or in your notes or having to ask someone for help!
**THE BIG PICTURE**

Organize your thoughts, look at the previous summary/review sheets, the 3 exams, quizzes, the book, your notes, your homework. Ask about anything you don't understand. Try doing an outline or list of the important topics and skills. Below is a partial such list.

The idea of rate of change is basic to almost everything we've studied.

Using a table, a graph or a formula, some changes we've measured are:
- total change in y
- average rate of change between 2 points
- (approximate) instantaneous rate of change at a point.

Eventually, we called the (approximate) instantaneous rate of change the derivative at a specific point. Graphically, the derivative measures the slope of the tangent line drawn at that point. Numerically, the derivative gives the rate of change of the output when the input changes slightly. Applications include marginal quantities (MC, MR, MP) as well as other rates of change. Correct, appropriate units are important!! Finally, we learned formulas for finding derivatives directly from a formula.

If a rate of change is known, the total change is the value of the definite integral over an interval. These definite integrals are also areas under a curve that were first approximated by adding up areas of rectangles (Riemann sums). The Fundamental Theorem of Calculus gives a way to use antiderivatives to find exact values of definite integrals over an interval. Formulas for finding antiderivatives (indefinite integrals) together with the Fundamental Theorem of Calculus avoid the tedium of adding up more and more rectangles

Skills: Be sure you can differentiate and integrate using the formulas.

**Applications:** Some are: revenue, cost, profit; marginal revenue, cost and profit; max profit or max revenue; local max/min and global max/min for functions of one variable; local max/min for functions of 2 variables; Lagrange multipliers for constrained optimization.

Be sure you can interpret results using correct units!!!!

**Tables:** Be able to:
- use tables to estimate rates of change (as mentioned above) & definite integrals (areas);
- create a table from a function;
- use a table to determine if a rate of change is positive or negative, etc.

**Graphs:** Be able to use a graph to estimate rates of change, areas, critical points, inflection points, concavity, increasing/decreasing, local or global max/min.

Know the connection between derivatives and critical points, increasing/decreasing, concavity, local max/mins.

**Basics:**
- **linear functions:** rate of change (slope) is constant; slope m = \( \frac{\Delta y}{\Delta x} \) & y = mx + b
- **exponential functions:** ratios (division) of outputs are equal for equally spaced inputs;
  \( y = Ca^x \) where \( a = 1 + r \) and \( r \) is the rate of change per unit of time (see p. 41)
  Also, \( y = De^{kx} \) where \( k \) is the continuous rate of change  (see page 49)

**Word descriptions:** Be able to briefly answer the following, using correct English sentences!!

- What is a derivative?
- What is a definite integral versus an indefinite integral?

(continued next page)
What information can derivatives give that help to sketch the graph of a function?

How is a graph of a function used to estimate the value of a definite integral over an interval?

What is the Fundamental Theorem of Calculus and how does it relate a function with its derivative? How can it be used to find an exact value of a definite integral over an interval?

What is a partial derivative?

What is the First Derivative Test and how does it distinguish between local maximum and local minimum (output) values?

What is the Second Derivative Test and how does it distinguish between local maximum and local minimum (output) values?

Why does max profit occur when MR = MC or, graphically, where the tangents to the R and C curves are parallel.