Math 18 Solutions to Quiz #5

1. \( q \approx 400 \) because \( R > C \) and \( MR = MC \), that is, the tangent lines to revenue \( R \) and cost \( C \) are parallel.

OR

because \( R > C \) and the vertical distance between \( R \) and \( C \) is the greatest.

2. A point is \((2, f(2))\) where \( f(2) = \frac{1}{2} \) so a point on the line is \((2, \frac{1}{2})\).

Slope \( m = f'(2) = -\frac{1}{2^2} = -\frac{1}{4} \).

Using \( y = mx + b \Rightarrow y = -\frac{1}{4}x + b \). Use the point \((2, \frac{1}{2})\) to find \( b \):

\[
\frac{1}{2} = -\frac{1}{4}(2) + b \Rightarrow \frac{1}{2} = -\frac{1}{2} + b \Rightarrow 1 = b.
\]

So, the equation of the line is: \( y = -\frac{1}{4}x + 1 \).

OR

Using \( y \cdot y_1 = m(x - x_1) \Rightarrow y - \frac{1}{2} = -\frac{1}{4}(x - 2) \Rightarrow y - \frac{1}{2} = -\frac{1}{4}x - \frac{1}{4}(-2) \)

\[
\Rightarrow y - \frac{1}{2} = -\frac{1}{4}x + \frac{1}{2} \Rightarrow y = -\frac{1}{4}x + 1
\]

3. a) total pollution \( \approx f(0) \cdot 10 + f(10) \cdot 10 + f(20) \cdot 10 + f(30) \cdot 10 \)

\[
= 5 \cdot 10 + 7 \cdot 10 + 10 \cdot 10 + 9 \cdot 10 = (5 + 7 + 10 + 9)10 = 310 \text{ tons}
\]

b) total pollution \( \approx f(10) \cdot 10 + f(20) \cdot 10 + f(30) \cdot 10 + f(40) \cdot 10 \)

\[
= 7 \cdot 10 + 10 \cdot 10 + 9 \cdot 10 + 8 \cdot 10 = (7 + 10 + 9 + 8)10 = 340 \text{ tons}
\]

c) The estimate would be better if: 1) more rectangles were used or \( n \to \infty \)

OR 2) smaller intervals were used or \( \Delta t \to 0 \)

OR 3) the two estimates were averaged.