1. Write the equation for the nuclear reaction described in each of the following processes:

   a. Fluorine-18 (\(^{18}\)F) undergoes positron emission (one of the radionuclides used in PET scans)
      \[ {^{18}_9}\text{F} \rightarrow {^{18}_8}\text{O} + {^0_{+1}}\text{e} \quad t_{1/2} = 110. \text{ minutes} \]

   b. Technetium-99m (\(^{99m}\)Tc) undergoes gamma decay to form \(^{99}\)Tc (a diagnostic radioactive tracer used to locate tumors, the “m” indicates a metastable excited nuclear state)
      \[ {^{99m}_{43}}\text{Tc} \rightarrow {^{99}_{43}}\text{Tc} + {^0_0}\gamma \quad t_{1/2} = 6.01 \text{ hours} \]

   c. Chromium-51 (\(^{51}\)Cr) undergoes electron capture (a diagnostic radioactive tracer used to study blood)
      \[ {^{51}_{24}}\text{Cr} + {^0_{-1}}\text{e} \rightarrow {^{51}_{23}}\text{V} \quad t_{1/2} = 27.7 \text{ days} \]

2. Would you expect the three radionuclides described in Problem 1 to have short half-lives (minutes to days) or long half-lives (years)? Explain why.

   Radionuclides used in medicine for imaging and as diagnostic tracers have **short half-lives**. Decaying quickly (a short burst of radiation) allows less radioactive material to be used, while still emitting enough particles to be detectable from outside of the body. This also allows the patient to remain radioactive for a shorter period of time.

3. The decay series of Americium-241 (used in smoke detectors) eventually forms bismuth-209. Determine how many alpha decays and how many beta decays occur in the series, and write the overall nuclear reaction. Explain your reasoning.

   \[ {^{241}_{95}}\text{Am} \rightarrow {^{209}_{83}}\text{Bi} + 8 {^4_2}\text{He} + 4 {^0_{-1}}\text{e} \]

   All of the difference between the mass numbers of americium-241 and bismuth-209 must come from alpha decay, with each alpha particle emitted decreasing the mass number by 4 nucleons (beta decay does not change the mass number).

   **Number of nucleons (mass number):**
   
   241 nucleons – 209 nucleons = 32 nucleons (lost)
   
   32 nucleons \(\left[\frac{1\alpha-\text{particle}}{4\text{ nucleons}}\right]\) = 8 \(\alpha\)-particles (emitted)

   The remaining difference in the charge numbers must be from beta decay, with each beta particle emitted balancing the charge of 1 proton created in the nucleus \(\left(\frac{1}{0}n \rightarrow \frac{1}{1}p + \frac{0}{-1}\text{e}\right)\).

   **Number of protons (charge number):**
   
   95 protons = 83 protons + 8 \(\alpha\)-particles \(\left[\frac{2\text{ protons}}{1\alpha-\text{particle}}\right]\) + \(X\) \(\left[\frac{-1\text{ proton}}{1\beta-\text{particle}}\right]\)

   \(X = 4 \beta\)-particles (emitted)
4. Rocks can be dated using the beta decay of $^{40}$K into $^{40}$Ar. Assuming that there was no gaseous argon in the molten rock when it formed, and that all of the argon produced since solidification has remained trapped in the rock until it was crushed for analysis, use the following information to calculate the age of the rock in years BP (before present, 1950): Year analyzed: 2011, $^{40}$Ar/$^{40}$K ratio in the rock = 1.15, $t_{1/2}$ of $^{40}$K = $1.28 \times 10^9$ years.

**Finding the original amount of $^{40}$K:**

\[
\frac{^{40}\text{Ar}_p}{^{40}\text{K}_r} = 1.15 \quad \text{or} \quad ^{40}\text{Ar}_p = 1.15 \times ^{40}\text{K}_r
\]

\[
^{40}\text{K}_o = ^{40}\text{Ar}_p + ^{40}\text{K}_r = (1.15 \times ^{40}\text{K}_r) + ^{40}\text{K}_r = 2.15 ^{40}\text{K}_r
\]

From the 1st order integrated rate law:

\[
t_{1/2} = \frac{\ln (2)}{k} \quad \text{or} \quad k = \frac{\ln (2)}{t_{1/2}} = \frac{\ln (2)}{1.28 \times 10^9 \text{ years}} = 5.41521235 \times 10^{-10} \text{ years}^{-1}
\]

\[
\ln \left( \frac{[A]_t}{[A]_0} \right) = -kt
\]

\[
t = \frac{\ln \left( \frac{[A]_t}{[A]_0} \right)}{-k} = \frac{\ln \left( \frac{^{40}\text{K}_t}{^{40}\text{K}_o} \right)}{-k} = \frac{\ln \left( \frac{^{40}\text{K}_t}{2.15 \times ^{40}\text{K}_r} \right)}{-5.41521235 \times 10^{-10} \text{ years}^{-1}} = 1.413,550,925 \text{ years}
\]

Converting to years BP:

\[
1.413,550,925 \text{ years (before analysis in 2011)} - 61 \text{ years (between 2011 and 1950)} = 1.413,550,864 \text{ years (before 1950)}
\]

**Note:** Since $^{40}\text{Ar}_p > ^{40}\text{K}_r$, $t > 1 \times t_{1/2}$, so the answer is reasonable.

**Also note:** Simply multiplying the $t_{1/2}$ by the ratio does NOT give the correct answer!

5. The energy yield of nuclear weapons is measured in Mt (megatonnes of TNT). If the explosion of 1 tonne of TNT yields 4.2 GJ (gigajoules) of energy, what mass (in kg) of matter was converted to energy in the 57 Mt blast of the largest weapon ever tested (Tzar Bomba, USSR, 1961)?

\[
c = 2.998 \times 10^8 \text{ m/s}
\]

\[
E = mc^2
\]

\[
E = 57 \text{ Mt} \times \left( \frac{10^6 \text{ tonne}}{1 \text{ Mt}} \right) \times \left( \frac{4.2 \text{ GJ}}{1 \text{ tonne}} \right) \times \left( \frac{10^9 \text{ J}}{1 \text{ GJ}} \right) = 2.394 \times 10^{17} \text{ J}
\]

\[
m = \frac{E}{c^2} = \frac{2.394 \times 10^{17} \text{ J}}{(2.998 \times 10^8 \text{ m/s})^2} = 2.663550216 \text{ kg} = \boxed{2.7 \text{ kg}}
\]

**Note:** Only about 0.1% of the mass of each U atom is released as energy as daughter products and neutrons are formed. For visualization purposes only though, the total mass lost in this blast would only correspond to a cube of uranium 5.2 cm per side ($d_0 = 19.1 \text{ g/cm}^3$). That's a cube with faces the size of this text box.
6. What is the binding energy of $^{12}$C (in kJ/mol and MeV/nucleon)?

**Mass nucleons:**
\[
6 \text{ p (1.007276 u / p)} + 6 \text{ n (1.008665 u / n)} = 12.095646 \text{ u}
\]

**Mass $^{12}$C nucleus:**
\[
12 \text{ u} - 6 \text{ e}^- (5.485798 \times 10^{-4} \text{ u / e}^-) = 11.99670852 \text{ u}
\]

**Mass defect (per nucleus):**
\[
12.095646 \text{ u} - 11.99670852 \text{ u} = 0.098937488 \text{ u}
\]

**Binding energy (per nucleus):**
\[
E = mc^2 = \left( \frac{0.098937488 \text{ u}}{1 \text{ u}} \right) \left( 1.660539 \times 10^{-27} \text{ kg} \right) = 1.64289542 \times 10^{-28} \text{ kg}
\]

**Conversions:**
\[
\frac{1.47663506 \times 10^{-11} \text{ J}}{1 \text{ $^{12}$C nucleus}} \times \frac{6.022 \times 10^{23} \text{ $^{12}$C nuclei}}{1 \text{ mol $^{12}$C}} \times \frac{1 \text{ kJ}}{10^3 \text{ J}} = 8.892,296,341 \text{ kJ/mol} = \boxed{8.892 \times 10^9 \text{ kJ/mol}}
\]
\[
\frac{1.47663506 \times 10^{-11} \text{ J}}{1 \text{ $^{12}$C nucleus}} \times \frac{1 \text{ $^{12}$C nucleus}}{12 \text{ nucleons}} \times \frac{1 \text{ eV}}{1.602176 \times 10^{-19} \text{ J}} \times \frac{1 \text{ MeV}}{10^6 \text{ eV}} = 7.680362322 \text{ MeV/nucleon} = \boxed{7.680 \text{ MeV/nucleon}}
\]

7. How much energy (in kJ/mol) is released in the following fusion reaction occurring in the plasma phase? Hint: There are a total of 3 e$^-$ missing (compared to the neutral atoms given in the table) on both sides of the reaction, therefore their masses do not affect the change in mass.

\[
\frac{1}{2} \text{H}^+ + \frac{3}{2} \text{He}^{2+} \rightarrow \frac{4}{2} \text{He}^{2+} + \frac{1}{1} \text{H}^+
\]

**Mass products:**
\[
4.0026 \text{ u} + 1.0078 \text{ u} = 5.0104 \text{ u}
\]

**Mass reactants:**
\[
2.0141 \text{ u} + 3.0160 \text{ u} = 5.0301 \text{ u}
\]

**Δ Mass (per fusion event):**
\[
5.0104 \text{ u} - 5.0301 \text{ u} = -0.0197 \text{ u}
\]

**Energy released (per fusion event):**
\[
E = mc^2 = \left( \frac{-0.0197 \text{ u}}{1 \text{ u}} \right) \left( 1.660539 \times 10^{-27} \text{ kg} \right) = -3.2712618 \times 10^{-29} \text{ kg}
\]

**Conversion:**
\[
\left( \frac{2.94021144 \times 10^{-12} \text{ J}}{1 \text{ fusion event}} \right) \left( \frac{6.022 \times 10^{23} \text{ events}}{1 \text{ mol events}} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ J}} \right) = 1,770,595,330 \text{ kJ/mol} = \boxed{1.77 \times 10^9 \text{ kJ/mol}}
\]