Write your name only on this page. Show all work and answers on separate pages. Write on only one side of your pages and leave enough room between problems for my comments and grading. Each Problem Worth 10 Points

1. Verify that \( \int \frac{2(2x^2 + 1)}{\sqrt{x^2 + 1}} \, dx = 2x\left(x^2 + 1\right)^{\frac{1}{2}} + C \)

2. Evaluate the definite integral: \( \int_0^1 y\left(y^2 + 1\right)^5 \, dy \)

3. Evaluate the indefinite integral: \( \int \sin 3x \cos 3x \, dx \)

4. Explain the difference between a definite and an indefinite integral.

5. Find the area of the region enclosed by the curves \( y = x^2 \), and \( y = 4x - x^2 \).

6. Consider the solid formed by rotating the region bounded by \( y = x \) and \( y = \sqrt{x} \) about the line \( y = 1 \).
   a. Sketch the solid formed
   b. Show a typical slice taken perpendicular to the axis of symmetry (the axis of rotation) and state what the slice makes (a disk, washer, or shell).
   c. Give the expression that finds the volume of your typical slice. No need to integrate.

7. Set up (but do not evaluate) the integral necessary to find the volume of the solid obtained by rotating the region bounded by the curves \( x = 1 + y^2 \), \( x = 0 \), \( y = 1 \), and \( y = 2 \) about the \( x \)-axis.

8. Find the volume of the solid obtained by rotating the region bounded by the curves \( y = \sqrt{x} \) and \( y = \frac{1}{2}x \) about the \( x \)-axis.

9. A leaky 5 lb bucket is lifted from the bottom of a 20 ft well to the top of the well by pulling up a rope of negligible weight. The bucket starts with 16 lbs of water and leaks out at a constant rate. It finishes draining just as it reaches the top. How much work was spent lifting the bucket and the water?

10. Consider the function \( f(x) = \sqrt{x} \) on \([0,4]\).
    a. Sketch \( f \) over \([0,4]\) and find the average value.
    b. Find \( c \) such that \( f_{\text{avg}} = f(c) \).
    c. On your graph of \( f \) sketch a rectangle whose area is the same as the area under the graph of \( f \).