LAB 6
Work–Energy Theorem

OBJECTIVES
1. Observe mechanical energy as it transforms between the work done on an object and the kinetic energy.
2. Verify that the work done by a constant & varying force is equal to the change in kinetic energy according to the work-energy theorem $W_{\text{total}} = \Delta K$.

EQUIPMENT
Dynamics track, force & motion sensor, pulley, string, spring, supports, masses.

THEORY
For an object with mass $m$ that experiences a net force $F_{\text{net}}$ over a distance $d$ that is parallel to the net force, the work done is: $W = \int F_{\text{net}} \, dx$. If the work changes the object’s speed, the object’s kinetic energy changes according to the Work-Energy theorem:

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

PROCEDURE
Part 1: Work Done by a Constant Force
In this experiment, you will attach a hanging mass to a cart on a level track, release the cart from rest, and use DataStudio to measure (a) the tension $T$ in the connecting cord and (b) the position $x$ and velocity $v$ of the cart. The cart (with force sensor) has mass $M$ and the hanging mass has mass $m$.

a. Draw a force diagram and use Newton’s laws to obtain formulas in terms of $M$, $m$ and $g$ for (a) $T_{\text{rest}}$ before the cart is released and (a) $T_{\text{acc}}$ after the cart is released. Why should the tension change when you release the cart?

b. Apply the work-energy theorem to the cart to obtain a formula for $v$ in terms of $M$, $m$, $d$ and $g$, where $d$ is the distance from the release point.

c. Measure $M$, choose $m = 50$ g, and use your formulas to predict $T_{\text{rest}}$, $T_{\text{acc}}$ and $v$ when $d = 50$ cm.

d. Attach one end of the string to the hanging mass and other end to the force sensor’s hook. Connect the sensor to DataStudio, set the data sampling rate to 50 Hz.

e. Place the motion sensor at the end of the track so the cart moves away from it. Set the motion sensor sampling rate to 50 Hz and deselect the Acceleration measurement check box.

f. Make a force versus time graph. Add velocity to the same graph by dragging it onto the graph. Then change the horizontal axes from time to position. You should now have aligned force and velocity versus position graphs.

g. Before taking data, lift the hanging mass so no force is applied to the force sensor and push the tare button on the sensor to zero it. Start recording data and, after about 2 seconds, release the cart from rest. Stop recording and catch the cart just before it reaches the end stop.

h. Use DataStudio to select the recorded force data before the cart was released and determine $T_{\text{ave}}$ & $2\sigma_T$. Compare your result ($T_{\text{expt}}$ & $2\sigma_T$) with your predicted $T_{\text{rest}}$.

i. Repeat part (1h) for $T_{\text{acc}}$. 

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j. From the v-x graph, determine the velocity $v$ of the cart after it has travelled 50 cm.

k. To estimate the uncertainty in the measured velocity, repeat the experiment a total of 5 times and determine $v_{\text{exp}}$ & $2\sigma_v$.

l. Plot a distribution curve and state whether the theory is consistent or not with a $2\sigma$-confidence interval with your predicted $v_{\text{thy}}$.

m. Calculate the work done on the cart by the measured tension $T_{\text{acc}}$ over the 50 cm distance. Compare with the measured kinetic energy of the cart at the 50 cm point.

Part 2: Work Done by a Variable Force

Part 2A: Measure the Spring Constant $k$

Use the Force Sensor to measure the force that compresses the plunger spring in the Dynamics Cart. Measure the distance that the spring compresses and use EXCEL to plot the force vs. distance. The slope of the best-fit line of a graph of force versus distance is the spring constant $k$.

Part 2B: Setup and Data Taking

a. Connect a motion sensor to DataStudio and create two graph displays, force vs. time and velocity vs. position.

b. Put the cart on the Dynamics Track so the end of the plunger bar is pressed against the end stop, but the plunger spring is not compressed. Measure the length of the plunger relative to the end cap of the Dynamics Cart. Record the plunger length $x$.

c. Calculate the work done $W_{\text{thy}}$ by the spring based on $k$ and $x$.

d. Completely compress the plunger spring on the cart, and lock the spring in position. Put the plunger end of the cart against the end stop and begin data recording. Use the end of a pencil or ruler to tap down on the plunger release knob. The cart will be pushed away from the end stop by the plunger spring.

e. Repeat the data recording process a total of five times. For each trial, calculate the kinetic energy of the cart and then determine $K_{\text{ave}}$ and $2\sigma_K$.

f. Plot a distribution curve and state whether the work done by the spring $W_{\text{thy}}$ is consistent or not with a $K_{\text{ave}} \pm 2\sigma_K$-confidence interval?