LAB 8

Modeling a Spring-mass system with Excel

OBJECTIVES

1. To identify the difference between an analytical and numerical solutions.
2. To construct an Excel numerical model that recreates the motion of a mass-spring system from Lab 7.

EQUIPMENT

Excel and Tracker

THEORY

Numerical vs. Analytical Solutions

Suppose we wanted to determine the position of the thrown ball at certain times. One way to do this would be to start with fundamental ideas (the kinematics equations) and algebraically solve for y(t), this is called an **analytical solution**. What if instead we used simpler ideas (along with easier mathematics) and calculated (by hand) the position of the ball every hundredth of a second (0.01 s). In order to determine the position of the ball after 1 seconds, this method would require us to do 100 steps of calculations. This method is called **numerical solution** and it does not look inviting – UNLESS you get a computer to do all the tedious calculations. Analytical solutions are only possible for a small set of simplistic situations, but numerical solutions can be used in many more complex situations.

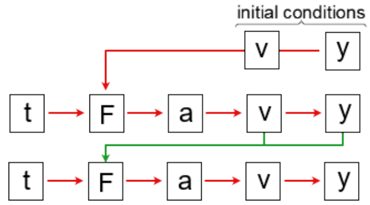
PROCEDURE

Part 1: Modeling the motion of a spring-mass system

We want to numerically model in Excel the position of a mass-spring that is displaced from its equilibrium position. To describe the motion of a mass, we determine the net force acting on the mass by performing numerous calculations over very, very short time frames or **steps** called **dt**. Since the spring force depends on the position of the mass, as the mass moves up and down, its position is constantly changing and there is a sequence of steps to determine this position and the net force. To know the position y, we need to know the velocity ($v = \Delta y/\Delta t$) at that point, but to know the velocity we need to know that acceleration ($a = \Delta v/\Delta t$) at that point as well. However, knowing the acceleration also determines the net force acting on the mass.

a. Define the **initial conditions** in Excel: spring constant k, mass m, acceleration due to gravity g, the initial position 0.20 m, and initial speed 0 m/s of the mass.
b. Break the motion into small pieces or steps of time. These steps time is a variable and called the **step size dt (**$=\Delta$**t)**. The best way to determine the appropriate step size is to start with, say, **dt = 0.1 s** and calculate the motion of the mass. Then decrease the time interval and see if your answer changes. If your answer does not change significantly with step size, then the value of the step size is a good value.
c. For each step, you are going to do the following: calculate the y_2 position using the average velocity definition $v = \Delta y/\Delta t$: $y_2 = y_1 + v_2dt$. To use this, we can calculate the current position (y_2) using the position it was before (y_1) and the velocity.
d. Now we need to update the velocity. From the definition of acceleration, write $v_2 = v_1 + a_2dt$. So similar to part (1C), calculate the current velocity (v_2) using the velocity it was before (v_1) and the acceleration.
e. With the updated position, update the net force acting on the mass.
f. The last thing we need to do is update the time $t_2 - t_1 = dt$.

g. The flow chart of these calculations looks like



h. Setup your variables, i.e., these are the things that will change each time step in Excel using the following.

t	F	a	v	x
0.0			0	0.20
0.1				
0.2				

i. Plot the position (y) vs. time (t) to see how your model looks.

Part 2: How to Model Damping

The mass-spring system will not oscillate forever. Therefore, we know there's some force causing mass to slow down and this force is always in the direction opposite the velocity.

a. Define the damping as a drag force that is velocity dependent and equal to $D = -bv$.

b. Modify Part 1 to include this drag force.

c. Plot the position (y) vs. time (t) to see how your model looks

What to turn in

1. Print the first two rows of the Excel sheet for dt = 0.1s and show the sample calculations of dt = 0.1s for F, a, v and y.
2. Print out two position plots (y vs. t) for the spring-mass system for (i) undamped and (ii) damped motion.