# Ballistic Pendulum and Conservation of Momentum

*Formal Report for Lab #8, Physics 4A*
*by Bryan Campbell*

## 1.0 Abstract

In this paper, we aim to validate one the most important and frequently used tools of physics: the law of conservation of momentum. We do so by comparing results from two experiments conducted with a single ballistic launcher/pendulum apparatus. In the first, we use kinematics methods alone to determine a mean initial velocity for a projectile. In the second, we add a ballistic pendulum and derive a projectile velocity using conservation of momentum and energy principles. Our results show momentum and kinematics methods yield highly accurate, nearly identical, results with sigma values within 1.1% and 1.7% of each mean velocity (respectively).

## 2.0 Introduction* /Theory

Education helps emancipate us from the misconceptions of today's media. As is usually the case, TV and movies are constantly reinforcing "bad" physics. A very common misconception is that if a bullet hits a person, they will immediately fly backwards and break any window they were standing in front of at that time. This lab helps us to understand via two key ideas that in these dramatic enactments on TV momentum is not conserved. The lab may be more similar to this analogy than first realized. Here the difference is we are talking of a human pendulum. Considering conservation of momentum begs the question, if the bullet has the ability to throw a person through a window, what would the recoil of the gun have done to the shooter? If the gun doesn't throw the shooter off his feet, the bullet can't throw the target off his feet. (Source: www.regentsprep.org/Regents/physics/phys01/miscons/default.htm)

Ballistic pendulum and kinematics have a plethora of applications in today's world. To appreciate their industriousness we shall explore the ideas of physics behind the scenes and explain the data analysis used to interpret the results.

The ballistic pendulum is a device where a ball is shot into and captured by a pendulum. The pendulum is initially at rest but acquires energy from the collision with the ball. Using conservation of energy it is possible to find the initial velocity of the ball. In this *ball-pendulum system* we cannot use the conservation of mechanical energy to relate the quantities because energy is transferred from mechanical to nonconservative forces.

### Kinematics

Strictly using kinematics methods, we can calculate an initial velocity for a projectile if we know its height of launch, launch angle, and launch distance. To simplify our system, we choose a horizontal launch angle. Given the launch height, we can discover the time the ball takes to fall due to gravity (its time of flight) as follows:

*Introduction by Andrew Schwartz

$$h = \tfrac{1}{2} g t^2$$
$$\Downarrow$$
$$t = \sqrt{\frac{2h}{g}}$$

Velocity is simply distance divided by time. This fact in combination with our expression for time above yields:

$$v_0 = \frac{d}{t} = d \Big/ \sqrt{\frac{2h}{g}}$$

We use this relation to determine a mean experimental velocity from 5 measured distances over five trials.

**Conservation of Momentum and Energy**

According to theory, an inelastic collision conserves momentum but not energy. This suggests we divide our analysis into two “views” where the conservation laws individually apply. By first adopting a momentum-view of the system, we can relate the states just before and after the collision as follows:

$$\begin{array}{c} \vec{P}_{b_0} + \vec{P}_{p_0} = \vec{P}_{(b+p)_f} \\ \Downarrow \\ \vec{P}_{b_0} = \vec{P}_{(b+p)_f} \end{array} \quad \Rightarrow \quad \begin{array}{c} m_b v_b = (m_b + m_p) v_p \\ \Downarrow \\ \left[ v_b = \left( \dfrac{m_b + m_p}{m_b} \right) v_p \right] \end{array}$$

With measurements of the pendulum and projectile masses, and their combined post-collision velocity, we can determine the initial velocity of the projectile. We don’t know the post-collision velocity, but we can find it by shifting to an energy-view. According to conservation of energy, the kinetic energy of a pendulum swinging at it’s lowest point is translated into potential energy at its highest:

$$\begin{array}{c} K_i + U_i = K_f + U_f \\ (U_i = 0, K_f = 0) \\ \Downarrow \\ K_i = U_f \end{array} \quad \Rightarrow \quad \begin{array}{c} \tfrac{1}{2} m_{b+p} v_p^2 = m_{b+p} g \Delta h \\ \Downarrow \\ \left[ v_p = \sqrt{2 g \Delta h} \right] \end{array}$$

Now, with the height of a pendulum swing we can derive its velocity at its lowest point, or the post-collision velocity. Plugging that value into the first equations yields the initial velocity of the ball.

## 3.0 Materials and Methods

The ballistic pendulum apparatus consists of a launcher, a projectile, and a pendulum. Attached to the swinging end of the pendulum is a bucket designed to catch the projectile and hold it, creating an inelastic collision. The bucket rests directly in front of the launcher, right at the exit point of the projectile (fig. 1).

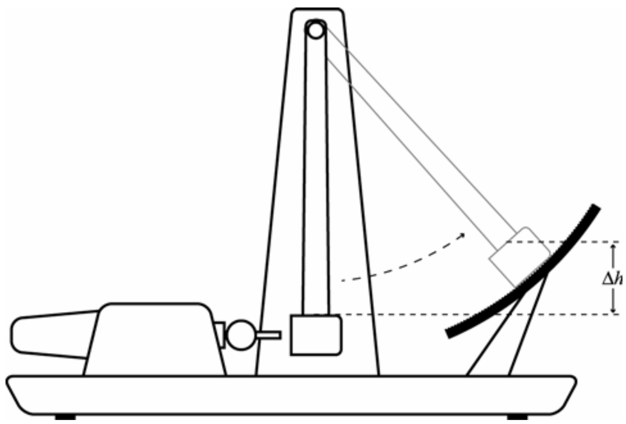



*figure 1*

Once the launch mechanism is released, the projectile enters the bucket and the arm swings up, moving past a grooved plastic strip. A ratchet mechanism on the bucket slides past each groove, allowing motion up but not back down. Once the pendulum stops (kinetic energy is fully transferred to potential), it stays locked in place recording the maximum height of the swing.

Our measurements consist of the initial height of the center of mass (CoM) of the pendulum, and a series of heights taken from several trial launches. This data is presented for analysis in the next section.

## 4.0 Results

Table 1 and Table 2 present data for the kinematics and momentum experiments respectively. Each show measured data in the first column (distance traveled in Table 1 and height of pendulum swing in Table 2) and a calculated velocity in the second (projectile velocity in Table 1 versus post-collision velocity in Table 2). Table 3 lists masses for the projectile and pendulum.

**Table 1**
Distance Measurements

| | x (m) | $v_0$ |
|---|---|---|
| 1 | 3.0903 | 6.867 |
| 2 | 3.1033 | 6.896 |
| 3 | 3.0383 | 6.752 |
| 4 | 3.0293 | 6.732 |
| 5 | 3.0863 | 6.858 |
| **mean** | **3.0695** | **6.82** |
| σ | **.033** | **.073** |

**Table 2**
Pendulum Height Measurements

| | Δh | $v_p$ |
|---|---|---|
| 1 | .120 | 1.53 |
| 2 | .118 | 1.52 |
| 3 | .124 | 1.56 |
| 4 | .118 | 1.52 |
| 5 | .124 | 1.56 |
| 6 | .122 | 1.55 |
| 7 | .126 | 1.57 |
| 8 | .120 | 1.53 |
| 9 | .112 | 1.48 |
| 10 | .120 | 1.53 |
| **mean** | **.12** | **1.54** |
| σ | **.004** | **.026** |

**Table 3**
Masses

| | |
|---|---|
| **Ball** | .0574 kg |
| **Pendulum** | .1966 kg |

Velocity and sigma values from Table 1 serve as our comparison values for numbers from the momentum experiment:

$$\bar{v}_0 = 6.82 \text{ m/s}$$
$$\sigma_{v_0} = .073 \text{ m/s}$$

The post-collision velocities of the pendulum in Table 2 are calculated from the second conservation equation derived in section 2:

$$v_p = \sqrt{2g\Delta h}$$

Plugging $\bar{v}_p$ and $\sigma_{v_p}$ into the first conservation equation from section 2 yields the initial velocity of the ball $\bar{v}_b$:

$$\bar{v}_b = \left(\frac{.0574\text{ kg} + .1966\text{ kg}}{.0574\text{ kg}}\right) \cdot 1.54\text{ m/s} \quad\Downarrow\quad \left[\bar{v}_b = 6.82\text{ m/s}\right]$$

$$\sigma_{v_b} = \left(\frac{.0574\text{ kg} + .1966\text{ kg}}{.0574\text{ kg}}\right) \cdot .026\text{ m/s} \quad\Downarrow\quad \left[\sigma_{v_b} = .115\text{ m/s}\right]$$

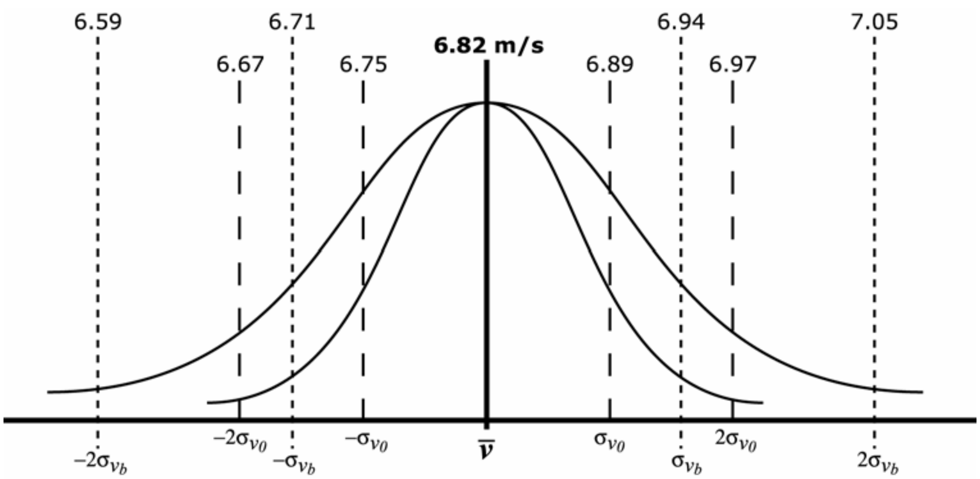
**4.1 Comparison of sigma values**

These results are surprisingly close, with both mean velocities matching up, and sigma values within .6% of each other (as a percentage of the mean velocity):

$$\frac{\sigma_{v_b}}{\bar{v}_b}\times 100\% = \frac{.115 \text{ m/s}}{6.82 \text{ m/s}}\times 100\% = [1.7\%]$$

$$\frac{\sigma_{v_0}}{\bar{v}_0}\times 100\% = \frac{.073 \text{ m/s}}{6.82 \text{ m/s}}\times 100\% = [1.1\%]$$

Because both velocities are the same, error distribution curves for both $\sigma_{v_b}$ , $\sigma_{v_0}$ and $2\sigma_{v_b}$ , $2\sigma_{v_0}$ can easily be compared on the same graph:




Here we see the values from the kinematics experiment are close to 1.5x more consistent compared to the momentum experiment. See discussion for an interpretation of these results.

**4.2 NULL Hypothesis**

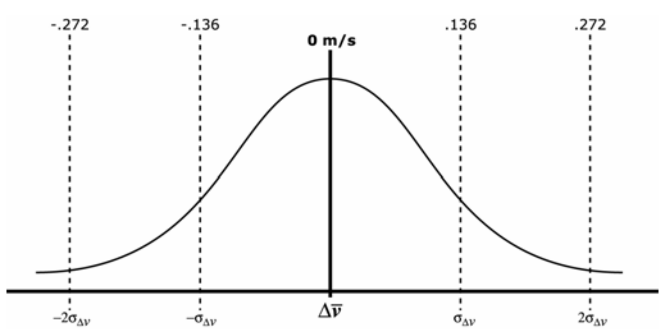
Using the NULL Hypothesis, we shift the comparison to the difference between experimental velocities. However, in this case the NULL hypothesis has limited usefulness because both velocities are the same. Graphing a combined sigma value against a zero axis yields the following:

$$\Delta v = 6.82 \text{ m/s} - 6.82 \text{ m/s} = 0$$

$$\sigma_{\Delta v} = \sqrt{\sigma_{v_b}^2 + \sigma_{v_0}^2}$$

$$\Downarrow$$

$$\sigma_{\Delta v} = \sqrt{.115^2 + .073^2} = \pm 0.136$$




**4.3 KE Differences**

A momentum specific view of kinetic energy dictates that the energy before and after the collision should equal the ratio of the initial mass (the ball) over the final (the ball and the pendulum). We attempt to confirm this against our traditional view of kinetic energy as follows:

$$K_0 = \frac{p^2}{2m_b}, K_f = \frac{p^2}{2(m_b + m_p)}$$

$$\Downarrow$$

$$\left(\frac{K_f}{K_0}\right)_{thy} = \frac{\left(\dfrac{p^2}{2(m_b + m_p)}\right)}{\left(\dfrac{p^2}{2m_b}\right)}$$

$$\Downarrow$$

$$\left(\frac{K_f}{K_0}\right)_{thy} = \frac{m_b}{m_b + m_p}$$

$$\Rightarrow$$

$$K_0 = \tfrac{1}{2}(.0574\text{ kg})(6.82\text{ m/s})^2 = 1.36\text{ J}$$

$$K_f = \tfrac{1}{2}(.0574\text{ kg} + .1966\text{ kg})(1.54\text{ m/s})^2 = .30\text{ J}$$

$$\Downarrow$$

$$\left(\frac{K_f}{K_0}\right)_{thy} = \frac{.30\text{ J}}{1.36\text{ J}} = .22$$

$$\frac{m_b}{m_b + m_p} = \frac{.0574\text{ kg}}{.0574\text{ kg} + .1966\text{ kg}} = .226$$

We can see that our ratio of kinetic energies is indeed very close, within 2.7%, to the ratio of the masses as predicted.

## 5.0 Discussion

We applied conservation principles to derive the velocity of a projectile after an inelastic collision with a ballistic pendulum. Comparing this analysis to one involving purely kinematics equations (Lab #2) gives us an opportunity to test the consistency of our system of analysis.

Surprisingly, our velocities for both experiments matched. At first glance, this seems to resoundingly confirm our expectations regarding conservation laws. And, looking at our sigma values for each as a percentage of the velocity, our accuracy is high (1.1% for kinematics vs. 1.7% for pendulum). However, considering the sigma values directly, we see a larger variance in measurement for the pendulum experiment (.073 m/s for the kinematics vs. .115 m/s for pendulum). This implies the kinematics experiment was roughly 1.5x more accurate than the pendulum experiment.

Friction also plays a greater role in the pendulum experiment, by the nature of the apparatus itself (ratcheting against plastic). This may explain why the ratio of initial and final kinetic energy differed from the initial and final mass ratio by 2.7%. It also suggests that our measured velocity in the pendulum experiment should be slightly lower.

Possible explanations for the difference in accuracy between experiments are straightforward to develop.

Perhaps the most significant is due to the configuration of the pendulum experiment, where the height of the pendulum is “stored” by a ratchet mechanism clicking against discrete grooves in a strip of plastic. This affects measurement in two ways: First, the graduation of the plastic serves to truncate or “round” height values not unlike rounding a decimal value to its closest integer; Second, if the pendulum swings just to the edge of a groove (to a state of unstable equilibrium), the chance it will fall on either side increases to 50/50 amplifying the apparatus' sensitivity to small chaotic factors such as vibration, nicks in the plastic, etc.

Another possibility involves the distances being measured. In the kinematics experiment, a projectile is launched several meters from its source. The point of impact is then measured from the launch point. The pendulum experiment however involves measuring the difference in height of the center of mass of a pendulum 30 to 40cm in length. The heights measured are only a fraction of that length, as the pendulum only swings through 30-60 degrees.

Taking these ideas into account, our measurements in the pendulum experiment are surprisingly accurate, perhaps due to the larger number of measurements taken (10 vs. 5 in the kinematics exp.).

I believe we can conclude that this experiment successfully validated conservation of momentum, and it's consistency with other forms of analysis.