Chapter 34
Macroscopic Reflection and Refraction

Macroscopic View of Reflections and Refractions
We’ll use the ray model of light to explore two of the most important aspects of light propagation: reflection and refraction. When a light wave strikes a smooth surface separating two transparent materials (such as air and glass or water and glass), the wave is in general partly reflected and partly refracted and partly transmitted into the second material. For example, when you look into a restaurant window from the street, you see a reflection of the street scene, but a person inside the restaurant can look out through the at the same scene as light reaches him by refraction.

**DEMO** Blackboard optics

We describe the directions of the incident, reflected, and refracted (transmitted) rays at a smooth interface between two optical materials in terms of the angles they make with the normal to the surface at the point of incidence.

Diffused and Specular Reflections
Light that is reflected in all directions is called diffuse reflection while light that is reflected in one direction is called specular reflection. Whether one gets diffused and specular reflections depends on the (i) surface and (ii) wavelength of light relative to the size of the surface particles.

(i) Surface Effects on Reflection
One gets diffused reflections when illuminating the road at night when driving on a dry night. Since the surface is rough, light is reflected in all directions. Due to the rough surface, lots of diffused light is reflected back to you, as well as light going forward (type does not help to see). However, on a rainy night a layer of water covers the rough road and smooth's it out. When the headlights reflect off the surface, the light is specular and very little reflected back to the driver. It is hard to see as a result!

(ii) Wavelength Effects on Reflection
As we already know, high frequency light has small wavelengths whereas low frequency light has long wavelengths. Therefore, small wavelengths are better for seeing small details than long wavelengths. So long wavelength light will reflect specularly even if the surface is rough.

Two examples of this specular reflection are (i) radio telescopes and (ii) Microwave ovens doors.

Our primary concern will be with specular reflection from a very smooth surface such as lightly polished glass, plastic, or metal. When referring to reflection we will always mean specular reflection.

**Laws of Reflection and Refraction**
Experimental studies of the directions of the incident, reflected, and refracted rays at a smooth surface between two materials lead to the following conclusions:

1. The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane. The plane of the three rays is perpendicular to the plane of the boundary surface between the two materials. We always draw ray diagrams so that the incident, reflected, and refracted rays are in the plane of the diagram.

2. The angle of reflection $\theta_r$ is equal to the angle of incidence $\theta_i$ for all wavelengths and for any pair of materials. That is,
   $$\theta_{\text{incidence}} = \theta_{\text{reflection}} \quad \text{(law of reflection)}$$

3. For monochromatic light and for a given pair of materials 1 and 2, on opposite sides of the interface, the ratio of the sine’s of the angles $\theta_1$ and $\theta_2$, where both angles are measured from the normal to the surface, is equal to the inverse ratio of the two indexes of refraction:
   $$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} \quad \Rightarrow \quad n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{(Law of refraction)}$$

**Interpretation**

Refraction occurs at a boundary between two medium (assume there is a change in the index of refraction). Consequently, this change in medium requires a change in the wave speed, which results a bending of the light ray.

**Analogy:** Lawnmower and sidewalk

**WEB** HWR site for reflection and refraction

When a light ray passes from one material into another, if $n_1 = n_2$ the beam does not bend because there is no change in wave speed and the refracted light is undeflected direction. However, if $n_1 \neq n_2$, there is a change in wave speed and therefore, the light ray is bent.

- If $n_1 = n_2$ (no change in medium), then the light ray is not bent and follows a straight line path.
- If $n_1 < n_2$ ($n_2$ is the slower speed medium), then the light ray is bent towards the normal.
- If $n_1 > n_2$ ($n_1$ is the slower speed medium), then the light ray is bent away from the normal.

**DEMO** oil with Pyrex containers with no refraction

We can only see objects because of the difference in the index of refraction. If there is no difference in the indexes of refraction, light rays are not bent and therefore, an object’s appearance becomes “invisible.”

**Metamaterials (don’t exist)**

**Web** Invisible Mercedes

The *refractive index* ($\equiv n$) of an optical material plays a central role in ray (or geometric) optics. The refractive index is a measure of the electromagnetic characteristics of a material (i.e., permittivity and permeability). However, for transparent materials in the optical frequency range, one can ignore it magnetic properties since the...
permeability does really change much. Therefore, the refractive index is then a measure of “electrical-ness” (or more accurate “dipole-ness”) characteristics of a material. The way the refractive index is computed is the ratio of its electrical-ness (assuming that the medium’s magnetic properties can be ignored) and we rate these to the ratio of the speed of light \( c \) in vacuum to the speed \( v_n \) in the material:

\[
\text{Index} \approx \sqrt{\frac{c_n}{c_0}} = \sqrt{\frac{\text{medium electricalness}}{\text{vacuum electricalness}}} \frac{c}{v_n}
\]

Units: \([n] = 1\) (unitless)

Light appears to travel more slowly in a refractive index relative to the vacuum value since \( n \geq 1 \). From the above equation, we see that the apparent speed of light in a refractive index \( (v_n) \) is inversely proportional to the index of refraction \( n \):

\[
v_n \propto \frac{1}{n}
\]

\[
\begin{align*}
\text{larger } n & \rightarrow \text{ lower apparent speed} \\
\text{smaller } n & \rightarrow \text{ higher apparent speed}
\end{align*}
\]

Indexes of refraction for several mediums are given in the table to the right for the particular wavelength of yellow light (589 nm).

The index of refraction depends not only on the medium but also on the frequency of light. The dependence on frequency is called dispersion and is characterized by \( n(\omega) \); we will consider this later. For example, by shining different frequencies (or wavelength) of light through water, the table on the right shows how the refractive index changes. This is what is meant by dispersion.

The index of refraction of air at standard temperature and press is about 1.0003, and we will take this at exactly unity. The index of refraction of a gas increases as its density increases.

### Index of Refraction and the Wave Aspects of Light

It is important to see what happens to the wave characteristics of the light when a light ray passes from one material to another material of different indexes of refraction. There two important points to be made:

1. **The frequency \( f \) of the wave does not change when passing from one material to another.** That is, the number of wave cycles arriving per unit time must equal the number leaving per unit time; this is a statement that the boundary surface cannot create or destroy waves.

\[
\text{number of waves entering medium} = \text{number of waves exiting a medium}
\]

Let’s take a glass-air interface and pass light through it. In one second, the frequency of crests pass through the interface. Now, a crest cannot be destroyed except via interference, so that many crests must exit. Remember, a crest is a zone of maximum amplitude. Since amplitude is related to energy, when there is maximum amplitude going in, there is maximum amplitude going out, though the two maxima need not have the same amplitude. Furthermore, we can directly say that to conserve energy (which depends solely on frequency), the frequency must remain constant.

\[
E \propto f = \frac{v_n}{\lambda}
\]

<table>
<thead>
<tr>
<th>Medium</th>
<th>Refractive index</th>
<th>( v_n = \frac{c}{n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.000293</td>
<td>0.9987</td>
</tr>
<tr>
<td>Water</td>
<td>1.333</td>
<td>0.7502</td>
</tr>
<tr>
<td>Ethanol</td>
<td>1.36</td>
<td>0.7353</td>
</tr>
<tr>
<td>Sugar solution (25%)</td>
<td>1.3723</td>
<td>0.7287</td>
</tr>
<tr>
<td>Sugar solution (50%)</td>
<td>1.42</td>
<td>0.7042</td>
</tr>
<tr>
<td>Sugar solution (75%)</td>
<td>1.4774</td>
<td>0.8769</td>
</tr>
<tr>
<td>Acrylic glass</td>
<td>1.490-1.492</td>
<td>0.8707</td>
</tr>
<tr>
<td>Human Eye</td>
<td>1.373-1.406</td>
<td>0.73-0.71</td>
</tr>
<tr>
<td>Glass</td>
<td>1.5-1.925</td>
<td>0.67-0.52</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
<td>0.413</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \lambda (\text{nm}) )</th>
<th>( n(\lambda) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>172</td>
<td>1.568</td>
</tr>
<tr>
<td>185</td>
<td>1.549</td>
</tr>
<tr>
<td>200</td>
<td>1.543</td>
</tr>
<tr>
<td>215</td>
<td>1.513</td>
</tr>
<tr>
<td>280</td>
<td>1.492</td>
</tr>
<tr>
<td>305</td>
<td>1.475</td>
</tr>
<tr>
<td>450</td>
<td>1.344</td>
</tr>
<tr>
<td>550</td>
<td>1.336</td>
</tr>
<tr>
<td>650</td>
<td>1.331</td>
</tr>
</tbody>
</table>
DEMO wave machine

The only variable thing left is the wavelength and it has to vary as the speed of light varies, which is due to the EM characteristics of the material.

2. The wavelength $\lambda$ of the wave is different for different optical mediums. Using the previous fact about frequency, we find

$$f = \text{constant} \Rightarrow f = \frac{c}{\lambda_0} = \frac{v_n}{\lambda} = \frac{c}{n} \Rightarrow \lambda = \lambda_0 \cdot \frac{c}{v_n} \Rightarrow \lambda = \frac{\lambda_0}{n}$$

In other words, when light enters a medium with index of refraction $n$, its speed and wavelength change, but not its frequency:

- higher index $n$ \( \Rightarrow \lambda = \frac{\lambda_0}{n} \Rightarrow \) slower wave speed, shorter wavelength
- lower index $n$ \( \Rightarrow \lambda = \frac{\lambda_0}{n} \Rightarrow \) faster wave speed, longer wavelength

Physically this makes sense: the waves get "squeezed" (the wavelength gets shorter) if the wave speed decreases since the wave is covering a shorter distance per unit time; on the other hand, the wave gets "stretched" (the wavelength gets longer) if the wave speed increases since its now covering more distance per unit time.

Example 34.1

Light is incident at angle $\theta_1 = 40.1^\circ$ on a boundary between two transparent materials. Some of the light travels down through the next three layers of transparent materials, while some of it reflects upward and then escapes into the air. (a) Physically interpret the refractions for this diagram. (b) Determine the values of $\theta_5$ and $\theta_4$?

Solution

a. Let's physically interpret the graph. Let's follow the ray as it goes through the four mediums and compare the index of refraction between each medium. In going from medium 1 to 2, note that the angle of refraction is bent towards the normal so $n_2 (1.40) > n_1 (1.30)$; from medium 2 to 3, its bent away from the normal so $n_2 (1.40) > n_3 (1.32)$; from medium 3 to 4 its bent towards the normal so $n_4 (1.45) > n_3 (1.32)$. These all agree with the numbers given.

b. Using basic ideas about angles and triangles, the incident angle from medium $n_1$ into air is the same as $\theta_1$. Applying Snell's law at the boundary, we have

$$n_1 \sin \theta_1 = n_5 \sin \theta_5 \Rightarrow \theta_5 = \sin^{-1} \left( \frac{n_5 \sin \theta_1}{n_5} \right) = \sin^{-1} \left( \frac{1.30 \sin 40.1^\circ}{n_5} \right) = 56.9^\circ - \theta_5$$

If we follow the ray from medium 1 all the way to medium 4, we are able to connect mediums 1 and 4 using Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3 = n_4 \sin \theta_4$$

34.4
so that
\[ \theta_4 = \sin^{-1} \left( \frac{n_1 \sin \theta_1}{n_4} \right) \]
\[ n_1 = 1.30, \quad n_4 = 1.45, \quad \theta_1 = 40.1^\circ \Rightarrow \theta_4 = 35.3^\circ = \theta_A \]

**Example 34.2**
The prism has a refractive index of 1.66, and the angles \( \theta \) are 25.0°. Two light rays \( R_1 \) and \( R_2 \) are parallel as they enter the prism. What is the angle between them after they emerge?

**Solution**
Apply Snell’s law to the refraction of each ray as it emerges from the glass. The paths of the two rays are sketched below.

The angle between rays #1 and #2 after they emerge is the angle \( 2\beta \), therefore, our job is to determine the angle \( \beta \) from the red triangle shown above. That is, to determine the angle \( \beta \), we must solve the sum of angles inside the red triangle where
\[ 180^\circ = 90^\circ + ? + \alpha + \beta \]

The issue is getting the correct angle values around the point of refraction using geometry. Focus first on the most top angle of 25° and “work” around the triangles by remembering that the sum of angles in a triangle sum to 180°. Or the two unknown angles sum to 90. The top most triangle has 90° = 25° + ?, which of course, tells us that the first mystery angle is \( \theta_1 = 25^\circ \). Apply this same type of thinking around the point of refraction, the angles are summarized in the diagram below where the incident angle between glass and air gives angle \( \theta_1 = A = 25.0^\circ \).

The equation that determines \( \beta \) now reads (since we know one of the mystery angles is 25°)
The focus now is to determine the angle $\alpha$. To solve for this angle, we apply Snell's law at the point of refraction:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \rightarrow n_{\text{glass}} \sin 25 = n_{\text{air}} \sin \theta_2$$

Solving for $\theta_2$, we get

$$\theta_2 = \sin^{-1} \left( \frac{n_{\text{glass}} \sin 25}{n_{\text{air}}} \right) = 44.55^\circ$$

To determine angle $\alpha$, note that the sum of $\theta_2$ and $\alpha$ sum to $90^\circ$, and so we can solve for angle $\alpha$:

$$\theta_2 + \alpha = 90^\circ \rightarrow \alpha = 90^\circ - 44.55^\circ = 45.45^\circ$$

Finally, the angle $\beta$ is

$$\beta = 65^\circ - \alpha = 65^\circ - 45.45^\circ = 19.55^\circ \rightarrow \text{angular spread} = 2\beta = 39.1^\circ$$

Question: The light is incident normally on the front face of the prism so the light is not bent as it enters the prism. Why?

**Total Internal Reflection**

We have described how light is partially reflected and partially transmitted at an interface between two materials with different indexes of refraction. Under certain circumstances, however, all of the light can be reflected back from the interface, with none of it being transmitted, even though the second material is transparent. Look at this image of a turtle in the ocean.

The figure shows how this can occur.

Several rays are radiating from a point source in water with index of refraction $n_1$. The rays strike the surface of a second material with index $n_2$ (of air), where $n_1 > n_2$. Using Snell's law,

$$\sin \theta_1 = \left( \frac{n_2}{n_1} \right) \sin \theta_2 \rightarrow \sin \theta_2 > \sin \theta_1 \rightarrow \begin{cases} \text{ray-2 is bent away} \\ \text{from the normal} \end{cases}$$
At some angle, however, there must be some value of $\theta_1$ less than 90° for which Snell’s law gives $\sin \theta_2 = 1 \rightarrow \theta_2 = 90°$. This is shown in the diagram which emerges just grazing the surface at an angle of refraction 90°. This angle of incidence for which the refracted ray emerges tangent to the surface is called the **critical angle**, denoted by $\theta_C$. (A more detailed analysis using Maxwell’s equations shows that as the incident angle approaches the critical angle, the transmitted intensity approaches zero.) **If the angle of incidence is greater than $\theta_C$, the sine of the angle of refraction, as computed by Snell’s law, would have to be greater than unity, which impossible:**

\[
\frac{n_2}{n_1} \sin 90° \textgreater 1 \quad \text{equal to one} \quad \sin (\theta_1 = \theta_C) \textgreater 1 \quad \text{impossible}
\]

**Beyond the critical angle, the ray cannot pass into the upper material; it is trapped in the lower material and is completely reflected at the boundary surface.** This situation, called **total internal reflection**:

![Jell-O transmitting a red light beam through total internal reflection.](image)

**Remarks**

1. Total internal reflection occurs only when a ray is incident on the interface with a second material whose index of refraction is smaller than that of the material in which the ray is traveling.

2. We can find the critical angle by setting $\theta_2 = 90°$ in Snell’s law:

\[
\sin \theta_C = \frac{n_2}{n_1} \sin 90° \quad \Rightarrow \quad \sin \theta_C = \frac{n_2}{n_1} \quad \text{(critical angle for total internal reflection)}
\]

3. Important value is for the glass-air surface with $n = 1.52$ for the glass. The critical angle is

\[
\sin \theta_C = \frac{n_{\text{air}}}{n_{\text{glass}}} = \frac{1}{1.52} = 0.658 \quad \Rightarrow \quad \theta_C(\text{glass-air}) = 41.1°
\]

Light propagating within this glass will be totally reflected if it strikes the glass-air surface at an angle of 41.1° or greater. Because the critical angle is slightly less than 45°, it is possible to use a prism with angles of 45°-45°-90° as a total reflecting surface. As reflectors, totally reflecting prisms have some advantages over metallic surfaces such as ordinary coated-glass mirrors. While no metallic surface reflects 100% of the light incident on it, light can be total reflected by a prism. The reflection properties of a prism have the additional advantages of being permanent and unaffected by tarnishing. Examples are binoculars and fiber optic cables.

**DEMO** fiber object, light tubes and fiber optic communication setup

**Example 34.3**
The ray is incident at the critical angle on the interface between materials 2 and 3. Find (a) index of refraction $n_3$ and angle $\theta$. (b) If $\theta$ is decreased, does light refract into material 3?

**Solution**

a. In the notation of this problem, Snell’s law becomes

$$\theta_c = \sin^{-1} \frac{n_2}{n_3} \rightarrow 60^\circ = \theta_c = \phi$$

b. Applying Eq. 33-44 law to the interface between material 1 and material 2, we have

$$n_2 \sin 30^\circ = n_1 \sin \theta \rightarrow \theta = \sin^{-1} \left( \frac{n_2 \sin 30^\circ}{n_1} \right) = 28.1^\circ = \theta$$

c. Decreasing $\theta$ will increase $\phi$ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than $\theta_c$. Therefore, no transmission of light into material 3 can occur.

**MICROSCOPIC VIEWPOINT OF REFLECTIONS AND REFRACTIONS**

**TRANSPARENT MATERIALS –Transmitted and Absorbed Light**

Light in Bulk Matter

Maxwell’s equations cannot describe $n(\omega)$ because it doesn’t “see” the atoms in the material. So we have to use a different model to describe this process.

**Question:** What is the physical basis for frequency dependence of the refractive index?

To answer this question, I will first need to remind you of resonance.

Analogy: harmonic oscillator

The beloved harmonic oscillator is a well-known example of resonance. From the 2nd law, we know that the amplitude is time dependent:

$$F_{\text{net}} = F \cos(\omega t) - kx = m \frac{d^2x}{dt^2} \rightarrow x(t) = x_m(t) \cos(\omega_t) \rightarrow x_m(t) = \frac{A}{\omega_d^2 - \omega^2} F(t)$$

If one includes damping, then the solution looks like
\[ x_m(t) = \frac{A}{\omega_0^2 - \omega_0^2 + B} \cdot F(t) \]

One can model the electrons in bulk matter in a similar manner:
\[ F_E \cos(\omega_d t) - kx = m \frac{d^2 x}{dt^2} \rightarrow x_m(t) = \frac{A}{\omega_0^2 - \omega_0^2 + B} E(t) \]

The refractive index \( n(\omega) \) is related to the number of electric dipoles in a medium or created by the incident EM wave, and this quantity is called the polarization:
\[
P = \frac{N}{\text{Number of dipoles}} \cdot \frac{q_0(t)}{\text{dipoles}} \propto \frac{\epsilon}{\text{electrical characteristics of a medium}} \propto \frac{n^2}{\text{index of refraction}}
\]

That is,
\[ n^2(\omega) \propto x_m(t) = \frac{A}{\omega_0^2 - \omega_0^2 + B} E(t) \]

Complex systems like glass are complicated because they have multiple resonate frequencies, not one! So the reason why the index of refraction is frequency dependent (= dispersion) is that complex systems have multiple resonate frequencies. Therefore, we have to modify the refractive index to show this:
\[ n^2(\omega) = \sum \frac{A_j}{\omega_0^2 - \omega_0^2 + B_j} E(t) \]

The index of refraction is therefore a measure of a medium’s “dipole-ness.” Because there are more dipoles in glass \((n = 1.5)\) than water \((1.33)\), so light interacts more electrically with glass atoms than it does with water atoms.

The interaction of an incident EM wave with an array of atoms constituting a dielectric medium causes the atoms to react to the incoming light’s E-field. This E-field applies an electric force \( F_E \) on the electrons that forces these charges to oscillate with the E-field frequency. That is, the oscillating E-field changes the directions of the dipoles (positive nucleus and negative electrons) at a rate of \( 10^{14} \) Hz.

So the electrons will then be shook or accelerated by the alternating E-field and consequently, these electrons will radiate EM waves. A single electron will become excited and spontaneously emit EM waves in about \( 10^8 \) seconds. This means that each electron emits \( 10^8 = 100 \) million “photons per second.” Here is the key point: the rate of emission of EM waves is the same for ALL electrons, regardless if these electrons are in acrylic, glass, water or whatever.

There are two extreme behaviors, (i) Nonresonate (incident light \( \neq \) resonate frequency of electrons) and (ii) resonate absorption (incident light \( = \) resonate frequency of electrons).

1. **Nonresonate Scattering.** If the incident frequency of light does not match the resonate frequency of the material, then resonance is not established.
incident frequency ≠ natural frequency of material \[ \omega_{\text{driving}} \neq \omega_{\text{natural}} \]

So the electrons will then be shook or accelerated by the alternating E-field and consequently, these electrons will radiate EM waves.

Picture-wise, the incident light gets absorbed, oscillates the electron, and re-emits the new light in all directions.

i. **ABSORBS**; incident light’s energy is completely absorbed by one electron.

ii. **OSCILLATES**; the excited electron gains energy and oscillates with the same frequency of the incident light.

iii. **RE-EMITS**; an oscillating electron produces an EM wave and re-emits the light.

The way light is transmitted through glass is atom by atom.

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### Analogy: A series of tuning forks resonating with each other and the Newton’s Cradle

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2. **Resonate Absorption**. Since glass is a complex system, it has three resonate frequencies: UV, IR and X-ray (think of lead glass) frequency range.

![Graph showing refractive index of glass](image)

The plot shows that the stronger resonance is at the IR range.

\[
\begin{align*}
n^2(\omega) \propto & \sum A_j \frac{\omega_d^2 - \omega_j^2 + B_j}{\omega_d^2 - \omega_j^2 + B_j} E(t) \\
\approx & \left( \frac{A_{\text{IR}}}{\omega_d^2 - \omega_{\text{IR}}^2 + B_{\text{IR}}} + \frac{A_{\text{UV}}}{\omega_d^2 - \omega_{\text{UV}}^2 + B_{\text{UV}}} + \frac{A_{\text{Xray}}}{\omega_d^2 - \omega_{\text{Xray}}^2 + B_{\text{Xray}}} \right) E(t)
\end{align*}
\]

If the incident light’s frequency is matched to one of the natural frequencies of the glass, then

\[
\begin{align*}
\text{incident freq} = \text{natural freq} \quad \rightarrow \quad & \text{large amplitudes} \\
\text{resonant condition} \quad \rightarrow \quad & \text{collisions heat up the atoms} \\
\text{absorption of light energy} \quad \rightarrow \quad & \text{no light transmission}
\end{align*}
\]
From this argument, the reason why visible light goes through glass but not UV or IR light is that
- Incident UV resonates electrons in the glass and is absorbed. In other words, light will only penetrate to within a few atoms thick before it is completely absorbed. So shining UV through a thin microscope glass slide will absorb it.
- Incident IR resonates whole atoms in the glass and is absorbed
- Incident Visible does not resonates either and passes through the glass

Interesting points
1. Since UV are the reason why one gets tan, tanning booths do not glass UV bulbs but quartz.
2. Lead glass is used for radiation shielding in medical treatments or imaging so that one can see the patient.
3. Suppose you put a potato into a microwave to cook. If the microwave oven was tuned to resonate water molecules, the microwaves would be absorbed at the surface of the potato and leave the inside uncooked. So microwave ovens are purposely set to not resonate water.