Chapter 3: Resistive Circuits

KIRCHHOFF’S LAWS
Kirchhoff’s laws are the governing laws of electric circuits. All techniques are derived from Kirchhoff’s laws (VDR, CDR, Node and Mesh equations) and Ohm’s law. I collectively called these “Simple Circuit Techniques” (SCT). This chapter is on identifying series and parallel on steroids.

Definition
A node (equipotential) is a junction of 2 or more elements.
When a circuit is limited to the absolute minimum of connections, then only nodes would be formed at junctions. For example, how many nodes are there in each circuit below?

Simple words of advice: until you become in expert circuit solver (which you will and fully expect), I strongly recommend that you use multiple colors to identify nodes.

Kirchhoff’s Current Law (KCL): the algebraic sum of currents entering any node is zero. Stated another way is the currents entering is equal to the currents coming out of the node.

\[ \sum_{n=1}^{N} i_n = 0 \Rightarrow i_{in} = i_{out} \]

Sign convention: currents into a node \( \equiv (+) \) while currents out of a node \( \equiv (-) \)

As an example, apply KCL to find currents \( i_3 \) and \( i_4 \).

Solution
Applying KCL to each of the 3 nodes:
Node x: \( i_6 - i_y = 0 \) \( \rightarrow \) \( i_6 = i_y \)
Node y: \( i_3 - i_6 + i_7 = 0 \) \( \rightarrow \) \( i_3 - i_6 + i_7 = i_3 = 0 \)
Node z: \( i_1 + i_2 - \frac{A}{\text{node}} + i_4 - i_5 = 0 \) \( \rightarrow \) \( i_4 = i_5 - i_1 - i_2 = 7 - (-1) - 5 = 3A = i_4 \)

Definition
A loop is a closed path formed within a circuit. “Loops” are closed potential paths so that the work done around a closed loop by a conservative force is always zero. How many possible loops are there within this circuit? There are 6 loops but only 3 independent ones.
Kirchhoff’s Voltage Law (KVL): the sum of voltage drops around any closed loop is zero.

\[ \sum_{n=1}^{N} v_n = 0 \]

To determine the sign of the voltage, one should use the Passive sign convention. However, it turns out that this sign convention is arbitrary. I am going to suggest that you use the following convention, which is different from the book convention. I find that this is much easier overall.

**Example 3.1**

Determine the power absorbed by each resistor.

![Circuit Diagram](image)

**Solution**

Always look for elements that are in series or parallel with voltage and current sources. Immediately we see that the 4Ω-resistor is in series with the 2A-source and the 8Ω-resistor is parallel with the 8V source. So we know that

\[ i_{4Ω} = 2A \quad \rightarrow \quad P_{4Ω} = i_{4Ω}^2 R_{4Ω} = 2^2 \cdot 4 = 16W = P_{4Ω} \]

\[ v_{8Ω} = 8V \quad \rightarrow \quad P_{8Ω} = \frac{v_{8Ω}^2}{R_{8Ω}} = \frac{8^2}{8} = 8W = P_{8Ω} \]

As for the 6Ω-resistor, if I apply a KVL loop (need to assume the sign of the 6Ω voltage drop):

\[ v_6 - 12 - 8 + 8 = 0 \quad \rightarrow \quad v_6 = 12V \quad \rightarrow \quad P_6 = \frac{v_6^2}{R_6} = \frac{12^2}{6} = 24W = P_{6Ω} \]

**SERIES & PARALLEL EQUIVALENTS**

The circuits we now encounter are much more complex, and until you learn to see these, at times, you will find that they can be overwhelming. This chapter will require that you see series and parallel at a higher level and to do this, you will need to learn how to see circuits as subcircuits or the series or parallel equivalents. Analogy: it’s like solving algebra problems for the first time. This is one of the most important skills you will learn in this course. That is, being able to look at a circuit and find the easiest way to reduce it to manageable parts so that it is solvable with SCT.

Another analogy comes to mind – one does not just eat a whole sandwich in one bite, you break it down by biting smaller pieces so that you can chew and swallow. In complex circuits, we will isolate a portion of a circuit (subcircuit) into a series or parallel equivalent. And then repeat the process to reveal its weakness to getting the desired current or voltage.

Warning: your ability to see series and parallel is of utmost importance and you should spend a lot of time learning "how to see." When the course is most challenging in Chapter 9 (deriving 2nd order equations) you will do so using these techniques.
extensively. If you do not see series and parallel by then, you should drop the course because it will be too late to pass the second and final exam. Do I have to convince you any more than this?

Here is the main approach to this chapter: is the primary behavior series or parallel? As soon as you identify the primary behavior of the circuit, mentally you will have a procedure that will take complicated circuits and solve them very quickly (analogy: solving algebra equations). However, after almost two decades of teaching circuits it is very clear now that without recognizing such behaviors, you will only look at a circuit with “blinded eyes” and no solution will come to you no matter how much time you spend on it. You MUST learn to see “series and parallel” to move to the next level.

**Series Equivalents**

As soon as one identifies the primary behavior of the circuit to be SERIES then these are the characteristics of the branch in question:

1. **Series Resistance add:** 
   
   \[ R_{\text{Series}} = R_1 + R_2 + \cdots \]

2. **Same current through series elements:** 
   
   \[ i_1 = i_2 = \cdots = i_{\text{Branch}} \]

3. **Voltages are divided among series elements according to**
   
   (i) **KVL:** \[ v_{\text{Branch}} = v_1 + \cdots + v_n \]
   
   (ii) **VDR:** \[ v_i = \left( \frac{R_i}{R_{\text{Series}}} \right) v_{\text{Branch}} \]

4. **Series Voltage sources added and all series current sources must be the same.**

Elements are in series because each adjacent element has only one common node. Since charge does not collect at any point in a wire carrying a steady current, the series resistors must carry the same current \( i \). Using KVL,

\[ v_s = v_1 + v_2 + v_3 = R_1 i + R_2 i + R_3 i = (R_1 + R_2 + R_3) i \]

What the battery sees is an effective resistance of \( R_{\text{Series}} = R_1 + R_2 + R_3 \). Generalizing this to \( n \) resistors,

\[ R_{\text{Series}} = \sum R_n \]

The equivalent series subcircuit can be drawn as

![Series Circuit Diagram]

From the series equivalent circuit, KVL and Ohm’s law, we can derive VDR (one of the most useful equations for solving for voltages):

\[
    i = \frac{v_1}{R_1} = \frac{v_2}{R_2} = \cdots = \frac{v_n}{R_n} = \frac{v_{\text{eq}}}{R_{\text{eq}}} \quad \text{VDR}
\]

\[ v_n = \frac{R_n}{R_{\text{Series}}} v_{\text{Branch}} \quad \text{(VDR)} \]

VDR shows how the voltages are divided up in direct proportion to their resistance:

\[
    \frac{R_n}{R_{eq}} \equiv \begin{cases} \text{percent of voltage drop} \\ \text{across series resistors} \end{cases} \quad \begin{cases} \text{larger } R_n \Rightarrow \text{larger voltage drop} \\ \text{smaller } R_n \Rightarrow \text{smaller voltage drop} \end{cases}
\]

**Voltage Sources in Series**

There are \( N \) voltage sources in a series branch, applying KVL around the “closed loop” to determine \( v_{oc} \), we get

\[
    -v_{oc} - (v_1 + v_2 + \cdots) - (v_3 + \cdots + v_n) = 0
\]

\[ v_{oc} = (\text{sum of active}) - (\text{sum of passive}) = v_{\text{net}} \]
That is, series voltage sources act as a single equivalent voltage source. For example, a branch of a circuit has series characteristics, which means that we can reduce that branch down to a single resistance and voltage source as

### Parallel Equivalents

As soon as one identifies the primary behavior of the circuit to be PARALLEL then these are the characteristics of the branches in question:

1. **Parallel Resistance add as inverses:**
   
   \[ \frac{1}{R_{\text{Parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \]
   
   **Trick:** \( \frac{1}{2} R_2 \parallel R_2 = \frac{1}{3} R_2, \quad \frac{1}{3} R_2 \parallel R_2 = \frac{1}{6} R_2, \quad \frac{1}{6} R_2 \parallel R_2 = \frac{1}{12} R_2, \quad \cdots \)

2. **Same voltages across parallel elements:**
   
   \( v_1 = v_2 = \cdots = v_{\text{Branch}} \)

3. **Currents are divided according to (i) KCL and (ii) CDR**
   
   i. **KCL:** \( i_{\text{Branch}} = i_1 + \cdots + i_n \)
   
   ii. **CDR:** \( i_n = \left( \frac{R_{\text{Parallel}}}{R_n} \right) i_{\text{Branch}} \)
   
   For two resistors: \( i_1 = \frac{R_2}{(R_1 + R_2)} i_{\text{Branch}} \) and \( i_2 = \frac{R_1}{(R_1 + R_2)} i_{\text{Branch}} \)

4. **Parallel current sources added and all parallel voltage sources must be the same.**

Elements are in parallel because each element has two common nodes. Since each node represents an equipotential surface, all parallel elements between these two nodes have the same voltage. Using KCL,

\[
i_s = i_1 + i_2 + i_3 = \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) v_s \quad \rightarrow \quad v_s = \left( \frac{i_s}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \right)
\]

What the battery sees is an effective resistance value of

\[
\frac{1}{R_{\text{Parallel}}} \equiv \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{or} \quad R_p = \sum \left( \frac{1}{R_n} \right)^{-1}
\]

The equivalent series subcircuit can be drawn as

For two parallel resistors,

\[
\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad \rightarrow \quad R_p = \frac{R_1 R_2}{R_1 + R_2}
\]

For purposes of checking results,

\( R_p \) is less than the smallest resistor value

**Summary:**

- When we add resistors in series, we increase \( R_{eq} \)
- When we add resistors in parallel, we decrease \( R_{eq} \)

When dealing with sums, we are able to interchange resistors at will to make our calculations easier. For example, resistors can be reduced quickly by moving resistors around to make the calculations easier:
From the fact that all parallel elements have the same voltage and Ohm’s law, we can solve for the individual currents $i_1$ and $i_2$:

$$v_S = R_p i_5 \quad \Rightarrow \quad i_1 = \frac{v_S}{R_1} = \frac{R_p}{R_1} i_5 \quad \text{and} \quad i_2 = \frac{v_S}{R_2} = \frac{R_p}{R_2} i_5$$

So the individual currents obey the **Current Divider Rule (CDR)**:

$$i_n = \frac{R_p}{R_n} i_S$$

Similar to VDR, CDR tells us the percentage of the total current which will flow through $R_1$ and $R_2$ is given by $R_p/R_n$. A very handy relation to remember is that for two resistors in parallel:

$$R_p = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

The currents flowing through resistor 1 & 2 are

$$i_1 = \frac{R_{eq}}{R_1} \cdot i_S = \frac{R_1 R_2}{R_1 (R_1 + R_2)} \cdot i_S = \frac{R_2}{R_1 + R_2} \cdot i_S \quad \text{\% of current flowing through } R_1$$

$$i_2 = \frac{R_{eq}}{R_2} \cdot i_S = \frac{R_1 R_2}{R_2 (R_1 + R_2)} \cdot i_S = \frac{R_1}{R_1 + R_2} \cdot i_S \quad \text{\% of current flowing through } R_2$$

If $R_2 \gg R_1$, the current $i_2 \gg i_1$. **The current follows the path of least resistance.**

**Example** Parallel circuits tend to cause most of the problems is SCT. Here are some tricks. Determine $R_p$ and the current through each resistor.

![Parallel Circuit Diagram](image)

Whenever there are more than two resistors in parallel, here is an important short cut for dealing with these situations.

- if one resistor is half as larger as another then

$$R_1 || R_2 = \frac{1}{\frac{1}{2}} R_2 \quad || \quad R_2 = \frac{\frac{1}{2} R_2 \cdot R_2}{\frac{1}{2} R_2 + R_2} = \frac{\frac{1}{2} R_2}{\frac{3}{2}} = \frac{1}{3} R_2$$

  **Example**: $6\Omega || 12\Omega = \frac{1}{3} \cdot 12\Omega = 4\Omega$

- if one resistor is one-third as larger as another then

$$R_1 || R_2 = \frac{1}{\frac{3}{4}} R_2 \quad || \quad R_2 = \frac{\frac{1}{3} R_2 \cdot R_2}{\frac{1}{3} R_2 + R_2} = \frac{\frac{1}{3} R_2}{\frac{4}{3}} = \frac{1}{4} R_2$$

  **Example**: $4\Omega || 12\Omega = \frac{1}{4} \cdot 12\Omega = 3\Omega$

So the equivalent resistance is one-fourth the value of the largest resistor value.

- if one resistor is one-fourth as larger as another then
\[ R_1 \parallel R_2 = \frac{1}{2} R_2 \parallel R_2 = \frac{\frac{1}{4} R_2 \cdot R_2}{\frac{1}{4} R_2 + R_2} = \frac{\frac{1}{2} R_2}{\frac{5}{4}} = \frac{3}{5} R_2 \]

Example: \( 3\Omega \parallel 12\Omega = \frac{1}{3} \cdot 12\Omega = \frac{12}{5} \Omega \)

So the equivalent resistance is one-fifth the value of the largest resistor value.

Using these ideas, look at how quick I can derive the parallel resistance for the circuit.

I demand that you be able to reduced parallel resistors this quickly in your head, of course, after practicing it a lot.

**Current Sources in Parallel**

There are \( N \) current sources in parallel branches, applying KCL at one of the nodes, we get

\[
I_{eq} = \left( I_1 + I_2 + \cdots \right) - \left( I_1 + \cdots + I_N \right)
\]

That is, parallel current sources act as a single equivalent current source.

Aside: voltage sources cannot be in parallel and have different voltage values

There are many ways to determine the currents of the three resistors.

(i) Physically approach is to realize that we start off with two \( 4\Omega \) resistors that equally split the current: each gets 3 A. The left-handed \( 4\Omega \) resistor is a \( 6\Omega \) and \( 12\Omega \) resistors splitting this 3A. Since the \( 12\Omega \) is twice as large, it gets half the current so I can immediately write \( i_{6\Omega} = 2A \) and \( i_{12\Omega} = 1A \).

(ii) Can use CDR:

\[
i_{6\Omega} = \frac{12}{6+12} \cdot 3A = 2A \quad \text{and} \quad i_{12\Omega} = \frac{6}{6+12} \cdot 3A = \frac{1A}{i_{12\Omega}}
\]

KCL: \( i_5 = i_{6\Omega} + i_{12\Omega} + i_{4\Omega} = 2 + 1 + 3 = 6A \)

**Current Sources in Series/Parallel**

Consider a series of \( N \) current sources. KCL tells us that the current in is equal to the current out. The current sources \( i_1 \) and \( i_2 \) at the center node must be the same, otherwise it violates KCL. Therefore, series current sources must have the same current value, otherwise, the current sources “discharge.” However, current sources add in parallel

**Summary**

**APPLICATIONS OF SIMPLE CIRCUIT TECHNIQUES (SCT)**

We will now start to look at circuits that have both series and parallel behaviors. However, the question still stands – **is the primary behavior series or parallel?** There are a zillion ways to solve circuits and no one way is more correct than the other. Nonetheless, different solving processes take longer than others and this may lead to trouble on quizzes and exams. Therefore, I will try to teach faster techniques that immediately lead to the heart of the problem.
**Example 3.2**

Find \( i \) and \( R_{ab} \) for the circuit.

**Solution**

How many nodes are there? 5! Use color pens to help you identify them in the beginning. Note that the current \( i \) is opposite of the voltage source, so keep in mind that there will be a negative sign to the current. There are two ways of solving this problem; in terms of series or parallel thinking.

a. Redrawing the circuit immediately leads to

\[
R_{eq} = 24\parallel 12 = \frac{24 \cdot 12}{24 + 12} = 8 \Omega = R_{eq}
\]

b. Solving this circuit both ways, we arrive at the following.

- **Series Thinking.** Using VDR, the voltage across the equivalent 4\( \Omega \) resistor is

\[
v = \frac{4}{20 + 4} \cdot 40V = 6.7V \quad \text{Ohm's law across 8\( \Omega \)} \quad i = \frac{6.67}{8} = 0.84A
\]

The two parallel resistors have the same voltage of 6.7V. However, each branch has two series resistors (3\( \Omega \) and 5\( \Omega \)) and therefore, have the same current. Using Ohm's law to determine this current across the 8\( \Omega \), we get

\[
i_{5\Omega} = i_{3\Omega} = i = \frac{6.67}{8} = 0.84A
\]

- **Parallel Thinking.** To use CDR, we need to determine the source current. From the equivalent resistance and Ohm's law, we get

\[
i_{S} = \frac{v_{S}}{R_{ab}} = \frac{40}{8} = 5A
\]

Solving for the current \( i_x \) using CDR:

\[
i_x = \frac{12}{12 + 24} \cdot 5A = \frac{5}{3} A
\]

However, the currents are divided across both 8\( \Omega \) resistors, so the current through the 5\( \Omega \) resistor is

\[
i_{8\Omega} = -\frac{1}{2} i_x = -\frac{1}{2} \cdot \frac{5}{3} A = -\frac{5}{6} A = i
\]

**Example 3.3**

Use Simple Circuit Methods (KVL, KCL, CDR, VDR, and Ohm’s law) only to determine \( i \) and \( v \) when \( v_{ab} = 12V \).

**Solution**

Remember that nodes are equipotential surfaces and therefore, only potential differences (difference in node voltages) produces a current.
How many nodes are there? 3!

I typically look at a circuit and immediately I start to find series and parallel equivalents to simplify the circuits as much as possible. I see two sets of parallel resistors that I can reduce to one. First of all, note that the top node has no potential difference across resistors 6Ω and 30Ω. Therefore, the “short” kills-off the 6Ω/30Ω resistors since there is no current flowing through those resistors. We say that the 6Ω/30Ω resistors are shorted out and removed from the circuit since they are only there for decorations.

This short makes it easier to find the equivalent resistance $R_{eq}$ for the circuit:

$$R_{eq} = 36 \|(10 + 72) | 9) = 36 \|(18) = 12Ω = R_{eq}$$

The current $i$ is through the 18Ω-resistor, so using Ohm’s law we get

$$i = \frac{V_s}{18Ω} = \frac{12V}{18} = \frac{2}{3}A$$

To solve this voltage $v$ is to realize that the 36Ω and the right branch are in parallel, therefore, they each have the same voltage of 12V. This immediately tells us that we can drop the 36Ω resistor from the circuit. Using series techniques to get voltage $v$, note (i) 9Ω is 1/8 of 72Ω, so 72Ω/9 = 8Ω and (ii) apply VDR across resistor $v = V_{8Ω}$:

$$v = \frac{72 \| 9}{72 \| 9 + 10} v_s = \frac{8}{18} 12V = \frac{16}{3}V = V$$

**Example 3.4**
Find $I_0$ in the circuit.

**Solution**
How many nodes are there? 4! Which resistors are in parallel or series?

- The 6kΩ and 12kΩ are in parallel between nodes A and B, so replace
  
  $$6k \| 12k = 4kΩ$$

- The 4kΩ and 12kΩ are in parallel between nodes B and C, so replace
  
  $$4k \| 12k = 3kΩ$$

However, note that if I do combine the $4kΩ \| 12kΩ = 3kΩ$, I immediately loose the information on $I_0$. 

3.8
Once again, there are multiple ways to solve for the current \( I_0 \). As soon as I combined the resistors \( 4\,k\Omega \parallel 12\,k\Omega = 3\,k\Omega \), I am forced to focus on the voltage \( v = v_{3k\Omega} \). I have the tendency to use VDR since there are still no currents in the circuit (although I could calculate the current easy enough).

To use VDR we need to setup the circuit to be in series resistors. The first step then is to identify this combination; combined \((9\,k\Omega + 3\,k\Omega)\parallel 4\,k\Omega = 3\,k\Omega\). Applying VDR to these two resistors in series gives

\[
v_{AB} = \frac{3k}{3k + 3k} \cdot 12V = 6V
\]

We now expand the resistor into the parallel resistors and reapply VDR:

\[
v_{3k\Omega} = v_{CB} = \frac{3k}{3k + 9k} \cdot v_{AB} = \frac{3k}{3k + 9k} \cdot 6V = \frac{3}{2} V = v_{4k\Omega} = v_{12k\Omega}
\]

Since the \( 4k\Omega \) and \( 12k\Omega \) are parallel to each other, they have the same voltages. We apply Ohm’s law and get the current through the \( 4k\Omega \) resistor:

\[
I_0 = \frac{v_{4k\Omega}}{4k\Omega} = \frac{\frac{3}{2} V}{4k\Omega} = \frac{3}{8} mA = I_0
\]

**Example 3.5**

Find \( I_0 \) in the circuit.

**Solution**

How many nodes are there in the circuit? 3! Therefore, these many parallel elements will go through a massive reduction to only a few resistors. Redraw the circuit will especially be key here in solving for the current \( I_0 \).

**Nodes 1 & 2:** the four left \( 12\Omega \)-resistors and the right most \( 3\Omega \)-resistor all share the same two nodes – they are all in parallel. Get their equivalent resistances.

\[
\frac{1}{R_p} = 12 \parallel 12 \parallel 12 \parallel 12 \parallel 3 = 4 \frac{12}{12} + \frac{1}{3} = \frac{2}{3} \rightarrow R_p = 1.5\Omega
\]

**Nodes 1 & 3:** there are 3 parallel elements: the \( 6mA, 12\Omega, 6\Omega \). One should not reduce the \( 12\Omega \) and \( 6\Omega \) resistors because you lose the details about the \( 6\Omega \) current \( I_0 \).

**Nodes 2 & 3:** there is only one resistor in series – \( 3\Omega \).

Redraw the circuit results in
As you can see this circuit is “trivial,” once you’ve seen it. So now we have a circuit that has 3 resistors in parallel and should use CDR to solve for the current \( i_0 \) through the 6Ω resistor. Reducing the 4.5Ω and 12Ω resistors into one resistance, CDR gives us \( i_0 \):

\[
R = \frac{4.5 \cdot 12}{4.5 + 12} = 3.3Ω \quad \Rightarrow \quad i_0 = \frac{3.3}{3.3 + 6} \cdot 6mA = 2.1 A = i_0
\]

**Practical Application: Not the Earth Ground from Geology (see page 65)**

Up to now, we have been drawing circuit schematics in a fashion

**Dependent Sources in Simple Circuits**

When one encounters a dependent source in a circuit, a good rule of thumb is to think in steps until you “see” how to solve dependent sources:

1. Apply simple circuit ideas, treating the dependent source like any other source.
2. Apply the **DEPENDENT SOURCE CONDITION** (DSC or Dep Condition):
   i. Find the controlling element in the circuit and see how it is defined. The most common way the controlling element is defined is the current through or voltage across a resistor. However, the controlling variable could be defined as the current at a node or a voltage around a loop.
   
   ![Resistor Diagram]

   ![Node Diagram]

   ![Loop Diagram]

   ii. Determine an additional equation that defines the controlling variable (current/voltage). If controlling variable is the voltage or current through a resistor, use Ohm’s law: \( v_c = R_i_c \). If the controlling variable is defined by a current at a node, apply KCL or CDR. If defined by a voltage around some loop, apply KVL or VDR.

3. **Warning:** do not place the controlling element into the equivalent resistances for the circuit. Why? All the information about the controlling variable is lost.

**Example 3.6**

Find \( v_x \) and the power delivered to the 10Ω resistor.

**Solution**

The dependent source is a VCVS. We are interested in the voltage \( v_x \) across the 5Ω resistor and the power \( P_{10Ω} \). Note that the dependent source is not defined in terms of an open circuit or a short circuit – it is defined by a resistor.

**Step 1: Simple Circuit Methods**

Since this is a series circuit, reduce all resistors (except the controlling resistor), and voltage sources into equivalents. However, because the controlling voltage is across the 5W-resistor, that resistor is left untouched. The equivalents are

\[
R_s = 10 + 15 = 25Ω \quad \text{and} \quad v_{net} = 30 - 3 + 3v_x = 27 + 3v_x
\]

The result is

![Equivalent Circuit Diagram]

Applying KVL around the loop leads us to

\[
3v_x + 27 + 25i + v_x = 0 \quad \text{reorganizing the equation} \quad 4v_x + 25i = -27
\]

This equation is unsolvable because it involves one equation with two unknowns. The second step in this process is applying the DC.
Step 2: Dependent Condition (DC)
Go to where the controlling voltage is defined and use Ohm’s law to get the second equation:

\[ v_x = 5i \quad \rightarrow \quad v_x - 5i = 0 \]

Now we have the two equations and two unknowns:

\[
\begin{align*}
25i + 4v_x &= -27 \\
-5i + 5v_x &= 0
\end{align*}
\]

\[
\text{multiply lower eq by 5 and add} \quad 9v_x = -27 \quad \rightarrow \quad v_x = -3V
\]

b. Using \( v_x \) to find the current \( i \) and power in the circuit, we get

\[
\begin{align*}
i &= \frac{v_x}{5} \\
&= \frac{-3}{5} \text{ A}
\end{align*}
\]

\[
\text{Power} = i^2 R = \left( -\frac{3}{5} \right)^2 \times 10 = 3.6 \text{ W} = P_{10\Omega}
\]

**Example 3.7**
Determine \( i_{3\Omega} \).

**Solution**
Since this is parallel, I immediately reduce resistors and sources accordingly. However, I need to be careful not to include the 2Ω resistor since it is the controlling current (I would lose all of the information on the controlling current \( i_x \)). The only parallel equivalents are the current sources: \( i_{\text{Parallel}} = 3i_x - 8 + 4 = 3i_x - 4 \). The new equivalent circuit is:

Step 1. Treat the circuit using SCT. That is, how do we solve for any current when there is a current source in the circuit with parallel resistors? We use CDR! So using CDR to calculate \( i_{3\Omega} \), we get

\[
i_{3\Omega} = \frac{2}{2 + 3} \cdot (3i_x - 4) = \frac{2}{5} \cdot (3i_x - 4) = i_{3\Omega}
\]

2 variables and 1 equation (not solvable)

Step 2. The DC tells me that I have to look at the circuit and relate the current \( i_{3\Omega} \) with \( i_x \) by any means possible. I can relate these two using KCL on the top node:

\[
\text{KCL: } 3i_x - 4 = i_{3\Omega} + i_x \quad \rightarrow \quad i_{3\Omega} = 2i_x - 4
\]

Now I take these two equations and solve for \( i_{3\Omega} \):

\[
\begin{align*}
\frac{5}{2} i_{3\Omega} &= 3i_x - 4 \\
i_{3\Omega} &= 2i_x - 4
\end{align*}
\]

Wrap-up of this chapter
Clearly, there are many ways to solve circuits. A key to solving circuits is to realize that each circuit has its own characteristics and one has to learn how to identify those characteristics.

- If a portion of the circuit is in series, then think along those ideas → combine series voltage sources, series resistances, and apply KVL & VDR. If a portion of the circuit is in parallel then think along those ideas → combine parallel current sources, parallel resistances, and apply KCL & CDR.
- If the circuit is not “simple,” then use the two-step process in solving dependent source problems.