Math 3 – Final Review Sheet

1) Find the radian measure of $300^\circ$.

2) Find the degree measure of $\frac{5\pi}{6}$.

3) Find the exact measure of the remaining sides of a 45-45-90 right triangle if the hypotenuse is 7cm.

4) Find the exact value of $\tan \theta$ if $\sin \theta = \frac{-3}{5}$ and $\cos \theta < 0$.

5) Find the exact value of the following:
   a) $\cos 225^\circ$
   b) $\tan \frac{3\pi}{2}$

6-7. Fill in the missing sides and angles of the following triangles.
6) side a = 1, side c = 2, angle C = 50 degrees
7) side a = 3, side b = 6, angle C = 30 degrees

Note: on 6-7, side a is opposite angle A, side b is opposite angle B, side c is opposite angle C. These are NOT (necessarily) right triangles.

8) A rocket is fired at sea level and climbs at a constant angle of $75^\circ$ for a distance of 10,000 feet. Approximate its altitude to the nearest foot.

9) Verify the following identity: $\frac{\tan x}{1 + \sec x} + \frac{1 + \sec x}{\tan x} = 2 \csc x$.

10-11. Find all of the solutions of the following equations.
10) $\cot x = -\frac{1}{\sqrt{3}}$
11) $\cos(4x - \frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

12) Find all of the solutions of $2\sin^3 x + \sin^2 x - 2\sin x - 1 = 0$ on the interval $[0, 2\pi]$.

13) If $x$ and $y$ are third-quadrant angles such that $\cos x = -2/5$ and $\cos y = -3/5$, find the exact value of the following:
   a) $\cos(x - y)$
   b) $\sin(x - y)$
   c) the quadrant containing $x - y$

14) Find the exact values of $\sin 2x$, $\cos 2x$, and $\tan 2x$ given that $\sin x = -\frac{4}{5}$, $270^\circ < x < 360^\circ$. 
15) Find the exact values of \( \sin(x/2) \), \( \cos(x/2) \), and \( \tan(x/2) \) given that 
\( \csc x = -\frac{5}{3}, -90^\circ < x < 0^\circ \)

16) Find all of the solutions of the equation \( \cos t - \sin 2t = 0 \).

17) Find the exact value of the following:
   a) \( \arcsin(\sin \frac{5\pi}{4}) \)
   b) \( \arccos(\cos \frac{5\pi}{4}) \)
   c) \( \arctan(\tan \frac{7\pi}{4}) \)

18) Find the exact value of the following:
   a) \( \sin(\tan^{-1} \sqrt{3}) \)
   b) \( \cos(\sin^{-1} 1) \)

19) Write the expression as an algebraic expression in \( x \) for \( x > 0 \) (i.e. get rid of the trig functions)
   \( \sin(\tan^{-1} x) \)

20) Solve \( \sin^2 x - \sin x - 1 = 0 \).

21) Graph \( y = -4 \cos(x + \frac{\pi}{6}) \). State the amplitude, the period, and the phase shift.

22) Graph \( y = \tan(\pi x - \frac{\pi}{2}) \). State the period and the phase shift.

23) The following points are given in polar coordinates. Find their rectangular coordinates.
   a) \((3, \frac{3\pi}{4})\)
   b) \((5, \frac{5\pi}{3})\)

24) If \((2, -1)\) are the rectangular coordinates of a point, find the polar coordinates.

25) Given the vectors \( \vec{v} = \langle 2, -3 \rangle \), \( \vec{w} = \langle -1, -4 \rangle \), find the following:
   a) \( \vec{v} + \vec{w} \)
   b) \( \vec{v} - \vec{w} \)
   c) \( 2\vec{v} - 3\vec{w} \)
   d) \( \|\vec{v}\| \)

26) Write the vector \( \vec{v} \) in the form \( a\hat{i} + b\hat{j} \), given that it has magnitude \( \|\vec{v}\| = 3 \) and it makes an angle of 120 degrees with the positive x-axis.

27) If \( \cos x = 8/17 \) where \( x \) is an acute angle, find \( \sin x \) and \( \tan x \).

28) Find the exact value of the following, if they are defined:
\[ a) \sin\left(\frac{3\pi}{2}\right) \quad b) \cos(-45^\circ) \quad c) \tan\left(\frac{3\pi}{4}\right) \]
\[ d) \sec\left(\frac{\pi}{6}\right) \quad e) \csc(300^\circ) \quad f) \cot(\pi) \]

29-31. Graph the following over 1 period. State the x-and y-intercepts, and any asymptotes.
29) \[ y = 2\sin(3x + \frac{\pi}{6}) + 1 \]
30) \[ y = -\cos(\pi x + \frac{\pi}{4}) \]
31) \[ y = \tan(2x + \frac{\pi}{3}) \]

32) Solve \[ \tan(2x - \frac{\pi}{4}) = \sqrt{3} \] where \( 0 \leq x \leq 2\pi \)

33) Find all of the solutions of \( \sin^2 u - 5\sin u + 4 = 0 \)

34) Find exact values of the following, if they are defined:
\[ a) \sin^{-1}(\sin\left(\frac{2\pi}{3}\right)) \quad b) \cos^{-1}(\cos\left(\frac{4\pi}{3}\right)) \quad c) \tan^{-1}(\tan\left(\frac{7\pi}{6}\right)) \]
\[ d) \cot(\sin^{-1}\left(-\frac{2}{5}\right)) \quad e) \sec(\tan^{-1}\left(-\frac{7}{4}\right)) \quad f) \csc(\cos^{-1}\left(-\frac{1}{5}\right)) \]

35) Rewrite the expression in terms of \( x \) (eliminate the trig functions):
\[ \tan(\arccos x) \]

36) Solve \( \tan^2 x + 2\tan x - 4 = 0 \) where \( x \) is in the interval \( (-\frac{\pi}{2}, \frac{\pi}{2}) \).

37-39. Solve the following triangles (fill in the missing sides and angles – here, angle A is opposite side a, angle B is opposite side b and angle C is opposite side c).
37) \( A = 32.32^\circ, c = 574.3, a = 263.6 \)
38) \( A = 42.17^\circ, a = 5.01, b = 6.12 \)
39) \( a = 10, b = 15, c = 12 \)

40) The following points are given in polar coordinates. Find their rectangular coordinates.
\[ a) (2, \frac{\pi}{4}) \quad b) (2, \frac{3\pi}{2}) \]

41) If \( (-1, -1) \) are the rectangular coordinates of a point, find the polar coordinates.

42) Given the vectors \( \vec{v} = <3, -1>, \vec{w} = <-5, 0> \), find the following:
\[ a) \vec{v} + \vec{w} \quad b) \vec{v} - \vec{w} \quad c) 2\vec{v} - 5\vec{w} \quad d) \|\vec{v}\| \]

43) Write the vector \( \vec{v} \) in the form \( a\hat{i} + b\hat{j} \), given that it has magnitude \( \|\vec{v}\| = 4 \) and it makes an angle of \( 150^\circ \) with the positive x-axis.
44) If \( \cot \theta = \frac{7}{24} \), where \( \theta \) is in quadrant III, find the following:
   a) \( \sin \theta \)                           b) \( \cos \theta \)
   c) \( \tan \theta \)

45) Find the exact value of the following:
   a) \( \sin 225^\circ \)                           b) \( \cos^{-1}(-1) \)
   c) \( \tan 120^\circ \)                                  d) \( \sec \left( \frac{3\pi}{4} \right) \)
   e) \( \csc \left( \frac{3\pi}{2} \right) \)                                    f) \( \cot(\tan^{-1} \sqrt{3}) \)

46) Graph \( y = -2 \sin(3x + \pi) - 1 \) over one period. State the amplitude, the period, the phase shift, and the x- and y-intercepts.

47) The tallest man-made structure in the world is a television transmitting tower in North Dakota. From a distance of 1 mile on level ground, the angle of elevation to the top of the tower is 21 degrees. Find the height of the tower to the nearest foot.

48) Find all of the solutions of \( \cos\left(\frac{1}{4} x\right) = -\frac{\sqrt{2}}{2} \)

49) Find all of the solutions of \( 2 \sin u \cos u - 2\sqrt{2} \sin u - \cos u + \sqrt{2} = 0 \)

50) Find the solutions of \( \cos^2 x + 2 \cos x - 1 = 0 \), where \( x \) is in the interval \( [0, 2\pi] \)

51) If \( \sin x = \frac{2}{3} \) and \( \cos y = -\frac{1}{5} \), where \( x \) and \( y \) are in quadrant II, find the following:
   a) \( \sin(x + y) \)                           b) \( \sin(2x) \)                           c) \( \cos\left(\frac{y}{2}\right) \)

52) Find all of the angles of a triangle with sides of length 6, 8, and 9.

53) Given the vectors \( \vec{v} = \langle -7, -1 \rangle \), \( \vec{w} = \langle -11, 2 \rangle \), find the following:
   a) \( \vec{v} + \vec{w} \)                           b) \( \vec{v} - \vec{w} \)                           c) \( 3\vec{v} + 2\vec{w} \)                           d) \( \|\vec{v}\| \)

54) Write the vector \( \vec{v} \) in the form \( a\hat{i} + b\hat{j} \), given that it has magnitude \( \|\vec{v}\| = 2 \) and it makes an angle of \( 210^\circ \) with the positive x-axis.