Math 5B Final Review

The actual exam will be structured the same, but with fewer questions…

1-6. Integrate.

1) \( \int \cos(\frac{\pi}{x}) \frac{1}{x^2} \, dx \)

2) \( \int_0^1 \frac{y}{e^{2y}} \, dy \)

3) \( \int_0^\frac{\pi}{2} \sin^2(2\theta) \, d\theta \)

4) \( \int_0^1 \sqrt{x^2 + 1} \, dx \)

5) \( \int_0^1 \frac{x-1}{x^2+3x+2} \, dx \)

6) \( \int_0^1 \frac{1}{4y-1} \, dy \)

7) Find the volume of the surface obtained by rotating the region bounded by \( y = x^3, y = 8, x = 0 \) about the x-axis.

8) A chain lying on the ground is 10m long and its mass is 80kg. How much work is required to raise one end of the chain to a height of 6m?

9) Given \( f(x) = \sqrt{x} \),
   a) Find the average value of the function on the interval \([0,4]\).
   b) Find \( c \) such that \( f_{\text{ave}} = f(c) \).

10) Find the centroid of the region bounded by \( y = \sqrt{x} \) and \( y = x \).

11) An aquarium 5 feet long, 2 feet wide, and 3 feet deep is full of water. Find the hydrostatic force on the short end of the aquarium.

12) Do the following series converge or diverge? Justify your answers.

   a) \( \sum_{n=1}^{\infty} \frac{n}{n^3 + 1} \)

   b) \( \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n}} \)

   c) \( \sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n} \)

   d) \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sqrt{n}}{n+1} \)

13) Find the radius of convergence and the interval of convergence of the series

   \( \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n} \)
14) Integrate \( \int \frac{e^x}{x} \, dx \) as an infinite series.

15) Find the arc length of the curve \( x = 3t - t^3, y = 3t^2 \) for \( 0 \leq t \leq 2 \).

16) Given the parametric equations \( x = 4 \sin^2 t \), \( y = 2 \cos(t) - 1 \):

   a) Sketch the curve from \( t = 0 \) to \( t = \pi \).
   b) Find the equation of the tangent line of this curve when it first crosses the x-axis.

17) Match the graphs with their equations:

   a) \( r = \sin \left( \frac{\theta}{2} \right) \)

   \( x = 3 \sin t \)
   \( y = 4 \cos t \)

   c) \( r = 1 + 4 \cos(5\theta) \)

Graphs for #17 to be given in class.