Chapter 6 - Discrete Probability Distributions

6.1 Random Variables

Introduction

If we roll a fair die, the possible outcomes are the numbers 1, 2, 3, 4, 5, and 6, and each of these numbers has probability 1/6. Rolling a die is a probability experiment whose outcomes are numbers. The outcome of such an experiment is called a random variable. Thus, rolling a die produces a random variable whose possible values are the numbers 1 through 6, each having probability 1/6.

Mathematicians and statisticians like to use letters to represent numbers. Uppercase letters are often used to represent random variables. Thus, a statistician might say,

“Let $X$ be the number that comes up on the next roll of the die.”

Random variable:

Discrete random variables:

Continuous random variables:
Examples: Which is discrete and which is continuous?

- Weights of passengers on a flight
- Number of passengers on a flight

**Probability Distribution**

A probability distribution for a discrete random variable specifies the probability for each possible value of the random variable.

**Properties:**

1. 

2. 

**Example**

Decide if each of the following is a probability distribution. Justify your answer:

a. 

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>0.659</td>
<td>0.287</td>
<td>0.050</td>
<td>0.004</td>
<td>0.001</td>
<td>0+</td>
</tr>
</tbody>
</table>
Example

When conducting research on color blindness in males, researchers form random groups of 5 males. Let $X$ be a random variable that represents the number of males in each group that exhibit symptoms of color blindness.

Find: (a) Find $P(2\ or\ 3)$  (b) Find $P(\text{More than } 1)$  (c) Find $P(\text{At least one})$
Example

In a class with 40 students, 12 have 0 siblings, 18 have 1 sibling, 6 have 2 siblings, 3 have 3 siblings, and 1 has 4 siblings. A student is randomly chosen from the class. Let $X$ represent the number of siblings the selected student has.

a) Find the probability distribution of $X$.

Solution

b) Find the probability of having 3 or more siblings.

c) Find the probability of having less than 3 siblings.

d) Would either of the situations in parts b) or c) be unusual?
**Probability Histograms**

Constructing a probability histogram from a probability distribution is just like constructing a relative frequency histogram from a relative frequency distribution for discrete data. We draw a rectangle for each possible value of the random variable, whose height is equal to the probability of that value.

**Example – Probability Histogram**

Create a probability histogram for the probability distribution of students’ siblings we just created.

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**Mean of a Random Variable**

The mean of a random variable provides a measure of center for the probability distribution of a random variable.

To find the mean of a discrete random variable,

\[ \mu_X = \]

Expected Value:
Mean of a Population

Law of Large Numbers for Means

Variance and Standard Deviation of a Random Variable

The variance of a discrete random variable $X$ is given by

$$\sigma_x^2 = \sum \left( x_i - \mu \right)^2 p(x_i)$$

or

$$\sigma_x^2 = \sum x_i^2 p(x_i) - \mu^2$$
The standard deviation of $X$ is the square root of the variance:

$$\sigma_x = \sqrt{\text{variance}}$$

**Example**

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Find the mean, variance, and standard deviation of the number of males that exhibit symptoms of color blindness.

**Example**

In the New York State Numbers Lottery, you pay $1 and can bet that the sum of the numbers that come up is 13. The probability of winning is 0.075, and if you win, you win $6.50 (which is a profit of $5.50). If you lose, you lose $1. What is the expected value of your profit?
Do You Know

- The difference between discrete and continuous random variables?
- How to determine a probability distribution for a discrete random variable?
- How to construct a probability distribution for a population?
- How to construct a probability histogram?
- How to compute the mean of a discrete random variable?
- How to compute the variance and standard deviation of a discrete random variable?

6.2 The Binomial Distribution

Introduction

Let’s say you have to take a multiple choice quiz, and you unfortunately have not studied at all. There are 10 questions on the quiz, each with 5 possible answers. What is the probability of getting 7 correct? What is the probability distribution for this quiz?

Binomial Distribution

Trials:

Successes:

Failures:
A random variable that represents the number of successes in a series of trials has a probability distribution called the binomial distribution. The conditions for a binomial distribution are:

- 
- 
- 
- 

**Notation**

In a binomial experiment, the following notation is generally used:

- 
- 
- 
- 
-
Example
Which of the following is a binomial experiment?

a) 10 babies are born in one night at Dominican Hospital. Let X be the number of boys born out of the 10 births.

b) A die is rolled twice. Let X be the sum of the two numbers obtained.

The Binomial Probability Distribution
For a binomial random variable X that represents the number of successes in \( n \) trials with success probability \( p \), the probability of obtaining \( x \) successes is

\[
P(x) = \]

The possible values of \( X \) are 0, 1, \( \ldots \) \( n \).

Example
Let X be the number of boys born out of 10 babies total.

a) Find the probability of 4 boys being born.

b) Find the probability of less than 2 boys being born.
c) Find the probability of at least 1 boy being born.

Note that we would rather not do this too often.

Table and Technology

The binomial probability distribution can require tedious calculations. For more involved problems it is better to use a table or technology.

Example

We roll a die 100 times. Find the probability that we roll a 1:

a) 10 times

b) more than 10 times

c) less than 20 times
d) Would it be unusual to roll a 1 80 or more times?

**Mean, Variance, and Standard Deviation**

Let $X$ be a binomial random variable with $n$ trials and success probability $p$.

The **mean** of $X$ is

$$
\mu_X = \text{ }
$$
The variance of $X$ is

$$\sigma_x^2 =$$

The standard deviation of $X$ is

$$\sigma_x =$$

Example

The probability of winning Powerball is $1/195,249,054$. If you buy one ticket a week for 50 years, you would have played Powerball 2600 times.

a) Find the mean and standard deviation for the number of wins for people who buy one ticket each week for 50 years.

b) Would it be unusual for someone to win who plays weekly for 50 years? Use the Empirical Rule to answer this question.
Do You Know

- How to determine whether a random variable is binomial?
- The notation for a binomial experiment?
- How to determine the probability distribution of a binomial random variable?
- How to compute binomial probabilities?
- How to compute the mean and variance of a binomial random variable?