Real Numbers

Throughout the semester we will be using certain words to describe numbers. This section teaches us about the types of numbers we will be working with.

First some terminology.
Set: A list of elements listed within braces. For example, \( \{2, a, 6, -5\} \)

Elements: The “things” inside of a set.
Roster notation: Set where all the elements are listed. Example: \( \{10, 20, 30\} \)
Set-builder notation: Set where we can’t list all the elements; written as
\[
\{ \text{element} \mid \text{rules element must satisfy to be in set} \}
\]
Example: \( \{x \mid x \text{ is greater than } 5\} \)

\( \in \): The symbol \( \in \) is used to indicate that a number is in a particular set.
\( \notin \): The symbol \( \notin \) is used to indicate that a number is not in a set.

Empty set (null set): A set that contains no elements. It can be denoted by \( \{ \} \) or \( \emptyset \).

Types of Numbers
Natural numbers: Numbers that you use to count. This is why the natural numbers are sometimes called the “counting numbers”. \( \{1, 2, 3, 4, 5, \ldots\} \).

Whole numbers: All of the natural numbers plus 0. \( \{0, 1, 2, 3, 4, 5, \ldots\} \).

Integers: All of the natural numbers plus their opposites and 0.
\( \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \)

Positive integers: The set of positive integers is the same as the set of natural or “counting” numbers. Notice that 0 is not included in this set. \( \{1, 2, 3, 4, 5, \ldots\} \).

Negative integers: The set of negative integers is \( \{\ldots, -5, -4, -3, -2, -1\} \). Notice that 0 is not included in this set.

The real number line: A “line” that illustrates the relationship (value comparison) between all real numbers.

Rational numbers: Any number that can be expressed as a quotient of integers where with the denominator is not 0. In other words, rational numbers are numbers that can be written as fractions where the numerator and denominator are integers and the denominator is not 0. Rational numbers include whole numbers, integers, fractions (of integers), and decimals that stop or repeat.

Here are a few. \( \left\{0, -1, 2 \frac{1}{4}, -13, 9, 15, 3.5, 0.\overline{3}, -\frac{5}{3}, \frac{1}{14}, \sqrt{4}\right\} \)

We can’t list them all rational numbers so we use set-builder notation:
\[
\left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers; } q \neq 0 \right\}
\]
Irrational numbers: Any number that can be represented on the real number line that is not rational. Irrational numbers are decimals that never stop and never repeat. There is no way to list all irrational numbers, so here are a few.

\[
\{ \pi, \sqrt{2}, \sqrt{8} \}
\]

Real numbers: If we put together all the rational and irrational numbers we get the real numbers. We can say real numbers are all of the numbers that can be represented on a real number line. Real numbers include the natural numbers, whole numbers, integers, rational numbers and irrational numbers.

There are other types of numbers, called complex numbers, which you will learn later on in this class. We see will where these numbers come from when we learn about square roots.

Use the roster method to list the elements in each set.

1. \( \{ x \mid x \text{ is a natural number less than 8} \} \)
2. \( \{ x \mid x \text{ is an integer between -5 and 4} \} \)
3. \( \{ x \mid x \text{ is an even whole number less than 8} \} \)
4. \( \{ x \mid x \text{ is an odd whole number less than 9} \} \)
5. \( \{ x \mid x \text{ is a natural number greater than 4} \} \)

Determine whether each statement is true or false.

1. \(-1 \notin \{ x \mid x \text{ is a negative number} \} \)
2. \(\sqrt{3} \in \{ x \mid x \text{ is a real number} \} \)
3. \(\frac{3}{5} \in \{ x \mid x \text{ is an integer} \} \)
4. \(0 \notin \{ x \mid x \text{ is a natural number} \} \)
5. \(9.2 \in \{ x \mid x \text{ is a rational number} \} \)

Write out the meaning of each inequality. Then determine whether the inequality is true or false.

1. \(-7 > -10 \) 
2. \(0 \geq -8 \) 
3. \(-3 \leq 1 \) 
4. \(4 > 4 \)
5. \(4 \geq 4 \)
6. \(4 \leq 4 \)