Ratios and Proportions

I. Ratios
   A ratio is a quotient of two quantities. The fraction bar, the symbol “:” and the word “to” are used to denote ratios.

   Examples: \[
   \frac{3 \text{ miles}}{10 \text{ minutes}} \quad \text{3 miles : 10 minutes} \quad \text{3 miles to 10 minutes}
   \]

   Example: The US population in 1990 was about 249 million. In 2006 it was about 299 million.

   a) What is the ratio of the US population in 2006 to the US population in 1990?

   b) Write the ratio as some quantity to 1. If necessary, round to the nearest hundredth.

   Example: In 2003, 78% of Americans drove alone to work, 10% carpooled, 5% used public transportation, 2% walked, 4% worked at home, and 1% found another means of transportation. Determine the ratio of workers who drove alone to work to those who walked to work.

II. Proportions
   A. A proportion is an equation showing two ratios are equal. \[
   \frac{a}{b} = \frac{c}{d}
   \]

   B. One way to solve a proportion is to cross-multiply.

      \[
      \text{If } \frac{a}{b} = \frac{c}{d}, \quad \text{then } ad = bc.
      \]

   C. It is okay to solve using other methods.

   Example: Solve.

   Using method from 2.3:

   \[
   \frac{x}{4} = \frac{12}{24} \quad \text{Cross-multiplying:} \quad \frac{x}{4} = \frac{12}{24}
   \]

   \[
   4 \cdot \frac{x}{4} = 4 \cdot \frac{12}{24} \quad 24 \cdot x = 12 \cdot 4
   \]
Examples:
1) \( \frac{9}{12} = \frac{x}{8} \)  
2) \( \frac{3}{12} = \frac{-1.4}{x} \)  
3) \( \frac{w}{6} = \frac{8}{-3} \)

III. Applications
A. Geometry
Two figures are similar if the measures of the corresponding angles are equal and the corresponding sides are proportional. In other words, they have the same shape but are not necessarily the same size.

Example: Find the length of the side indicated by \( x \).

1) \[
\begin{align*}
12 \text{ ft} & \quad x \\
9 \text{ ft} & \quad 13.5 \text{ ft}
\end{align*}
\]

2) \[
\begin{align*}
2 \text{ in} & \quad 1.8 \text{ in} \\
0.8 \text{ in} & \quad x
\end{align*}
\]

B. Word Problems
Write a proportion that can be used to solve the problem. Then solve the proportion and answer the question.

When you write the proportion make sure the units in the numerator match and the units in the denominator match.

Example: If a 40-lb bag of fertilizer covers 5000 square feet, how many pounds of fertilizer are needed to cover an area of 26,000 square feet?

\[
\frac{40 \text{ lb}}{5000 \text{ sq. ft.}} = \]
Example: A telephone cable crew is laying cable at a rate of 42 feet per hour. How long will it take them to lay 252 feet of cable?

Example: The property tax in the city of Junctionville is $8.325 per $1000 of assessed value. If the Jackson’s house is assessed at $325,000, how much property tax will they owe? Round to the nearest cent.

Example: Jon is currently reading a 462-page novel. If he reads 54 pages in 1.5 hours, how long will it take him to finish the novel? Round to the nearest tenth.