Solving Systems of Equations by Substitution

Main idea: substitute one equation into the other to turn a 2-variable equation into a 1-variable equation.

Steps to solving systems using the substitution method:
1. If necessary, solve one of the equations for one of the variables. (Isolate one variable in one of the equations.)
2. Substitute the expression in step 1 into the OTHER equation. This will result in one equation with one variable.
3. Solve the resulting equation. This will give you the value for one of the variables.*
4. Plug the value found in step 3 into one of the original equations and solve for the other variable.
5. Check the proposed solution in both original equations.

*If both variables cancel out, look at the equation you are left with:
   If it is a true statement, the system has an infinite number of solutions. Ex: 10 = 10
   If it is a false statement, the system has no solution. Ex: 3 = 5

Examples: Solve by substitution.

1) \[
\begin{aligned}
  x + y &= -5 \\
  y &= 2x + 7 \\
\end{aligned}
\]

2) \[
\begin{aligned}
  2x + 3y &= 7 \\
  6x - y &= 1 \\
\end{aligned}
\]

3) \[
\begin{aligned}
  x - 2y &= 4 \\
  2x - 4y &= 8 \\
\end{aligned}
\]

4) \[
\begin{aligned}
  3x + y &= 3 \\
  y + 5 &= -3x \\
\end{aligned}
\]

5) \[
\begin{aligned}
  x + 2y &= 6 \\
  2x - 3y &= 5 \\
\end{aligned}
\]

6) \[
\begin{aligned}
  5x - 2y &= -7 \\
  16 &= 2y - 8x \\
\end{aligned}
\]

7) \[
\begin{aligned}
  2x - y &= 14 \\
  y &= 2x - 14 \\
\end{aligned}
\]