Addition and Subtraction of Rational Expressions with a Common Denominator

I. Adding and Subtracting Rational Expressions with the SAME Denominator

\[
\frac{A}{C} + \frac{B}{C} = \frac{A + B}{C} = \frac{A, B, \text{ & } C \text{ poly’s; } C \neq 0}{C}
\]

Examples: Perform the indicated operation. Simplify if possible.

1) \[
\frac{x^2 + 7x}{x^2 - 5x} + \frac{x^2 - 4x}{x^2 - 5x}
\]

2) \[
\frac{x^2 + 6x + 2}{x^2 + x - 6} - \frac{2x - 1}{x^2 + x - 6}
\]

3) \[
\frac{x^2 - 2}{x^2 + 6x - 7} - \frac{-4x + 19}{x^2 + 6x - 7}
\]

4) \[
\frac{x^2 - 5x - 7}{2x^2 - 7x - 4} + \frac{4x - 5}{2x^2 - 7x - 4}
\]

II. The Least Common Denominator

The Least Common Denominator (LCD) is the same as the Least Common Multiple (LCM).

To find the LCD: 1) Factor completely each denominator.

2) Include in the LCD each factor the greatest number of times that it occurs in any one prime factorization.

The prime factorization of a number is the number written as the product of prime numbers. This is equivalent to factoring a polynomial completely.

Examples: Find the LCD.

1) \[
\frac{x}{12} + \frac{5}{18}
\]

2) \[
\frac{7}{15x^2} - \frac{5}{12x^3}
\]

3) \[
\frac{x}{x - 3} + \frac{5}{x + 4}
\]

4) \[
\frac{9}{x^2 - 4} + \frac{x - 5}{x^2 + 4x + 4}
\]

5) \[
\frac{x + 2}{(x - 3)^2} - \frac{x + 1}{x^2 + 2x - 15}
\]

6) \[
\frac{x - 7}{x^2 + 11x + 18} + \frac{x^2 + 1}{x^2 - 3x - 10}
\]