Solving Rational Equations

A. Definition: A **rational equation** is an equation containing at least one rational expression.

B. You saw rational equations in chapter 2 but we didn’t call them that. They had numbers in the denominators but not variables. \[ \frac{3x}{5} = \frac{2x}{3} + 1 \]

C. Steps:
1. Find the LCD.
2. **Get rid of denominators** by multiplying both sides of the equation by the LCD. When you are done multiplying the equation will no longer contain denominators.
3. Solve the resulting equation.
   a. **linear**: recognize: no exponent on variable; solve: get variables on one side, constants on the other
   b. **quadratic**: recognize: exponent of 2 on variable; solve: write as poly= 0, factor poly, set each factor = 0
4. REQUIRED CHECK: See if any of the value(s) found make a denominator equal to 0. If so, reject it as a solution. (These values are known as extraneous solutions.)
   USUAL CHECK: Make sure you didn’t make mistakes along the way and do the usual check, that is, plug the value(s) it into the original equation to see if it satisfies the equation.

Examples: Solve. If an equation has no solution, so state.

1) \[ \frac{x}{2} - \frac{12}{x} = 1 \]
2) \[ \frac{x^2}{x + 6} = \frac{36}{x + 6} \]

3) \[ \frac{r + 2}{r - 5} - \frac{3}{4} = \frac{6}{r - 5} \]
4) \[ \frac{2n^2 - 15}{n^2 + n - 6} - \frac{n - 3}{n - 2} = \frac{n + 1}{n + 3} \]
CAUTION: Don’t confuse adding & subtracting rational expressions with solving rational equations. BOTH types of problems use the LCD BUT for different reasons.

**EXPRESSION:**
- Two EXPRESSIONS can be added or subtracted.
- No = in the problem.
- Use the LCD to build up your rational expressions so they have the same denominator.
- When done you have a RATIONAL EXPRESSION, that is, a FRACTION as your answer (or possibly 1 if everything cancels).

Add: \[ \frac{1}{r} + \frac{2r}{r+15} \]

**EQUATION:**
- An EQUATION is SOLVED.
- An EQUATION has an = in it.
- Multiply both sides of the equation by the LCD to get rid of the denominators.
- When done you have a SOLUTION to the equation, that is, \( x = \# \) (or possibly no solution).

Solve: \[ \frac{1}{r} = \frac{2r}{r+15} \]