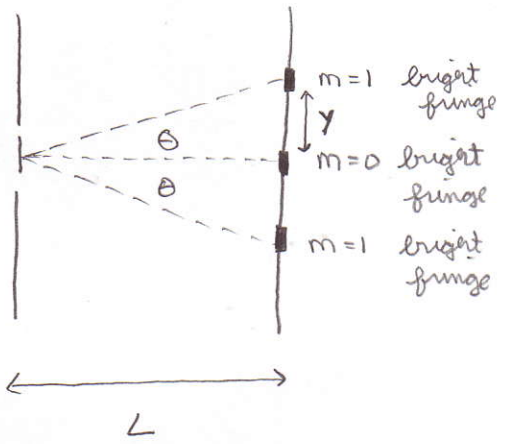
Interference and Wave Nature of Light

Problem 1

Light with a wavelength of 646 nm passes through two slits and forms an interference pattern on a screen 8.75 m away. The distance between the central bright fringe and the first-order (m = 1) bright fringe is 5.16 cm. (a) What is the separation between the slits? (b) What will be the distance between the central bright fringe and the second-order (m = 2) *minimum*?



Note: the distance between the central bright fringe and a point on the screen is given by

$$y = L \tan\theta$$

$$y = L\tan\theta \longrightarrow \theta = \tan^{-1}\left(y/L\right)$$

$\lambda = 646\text{ nm}$

$L = 8.75\text{ m}$

$y_1 = 5.16\text{ cm} = 5.16\times10^{-2}\text{ m}$

$$\theta = \tan^{-1}\left(\frac{5.16\times10^{-2}\text{ m}}{8.75\text{ m}}\right) \longrightarrow \theta = 0.338^\circ$$

Note: this is the angle of the m = 1 bright fringe

$$\sin\theta = m\lambda/d \longrightarrow d = m\lambda/\sin\theta$$

$$d = \frac{(1)(646\text{ nm})}{\sin(0.338^\circ)} \longrightarrow \boxed{d = 1.1\times10^{-4}\text{ m}}$$

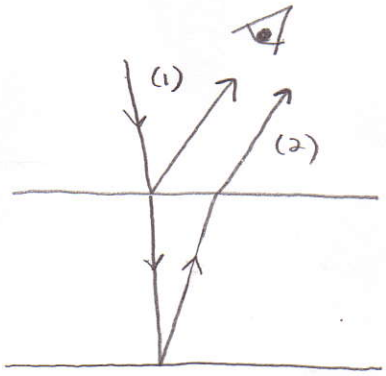
(b) for dark fringes: $\sin\theta = (m+1/2)\lambda/d$

$$m = 2 \longrightarrow \sin\theta = \frac{(2+1/2)(646\times10^{-9}\text{ m})}{(1.1\times10^{-4}\text{ m})} \longrightarrow \theta = 0.845^\circ$$

$$y = L\tan\theta \longrightarrow y = (8.75\text{ m})\tan(0.845^\circ) = \boxed{0.13\text{ m}}$$

Problem 2

Light reflected from a thin film of oil (n = 1.40) floating on water (n = 1.33) constructively interferes at a wavelength of $\lambda = 550$ nm. (a) Sketch the situation and indicate which of the reflected rays undergo a phase shift. (b) Find the minimum thickness of the film that could produce this constructive interference.



Note: light undergoes a $\frac{1}{2}\lambda$ phase shift when it reflects off a material with a higher index of refraction

(a) Reflected ray 1 undergoes a phase shift because

$n_{oil} > n_{air}$

(b) Note: when only one of the light rays undergoes a $\frac{1}{2}\lambda$ phase shift, the conditions for constructive & destructive interference are

$2t = (m + \frac{1}{2})\lambda/n$ $\quad m = 0, 1, 2, \ldots$ constructive

$2t = m\lambda/n$ $\quad m = 0, 1, 2, \ldots$ destructive

} $n = n_{film}$

constructive $\Rightarrow$ $2t = (m + \frac{1}{2})\lambda/n$

Note: for minimum thickness choose $m = 0$

$2t = (\frac{1}{2})\lambda/n$

$t = \lambda/4n = \frac{550\text{nm}}{4(1.40)} \rightarrow$

$t = 98.2\text{nm}$
$= 98.2 \times 10^{-9}\text{m}$