The Nature of the Atom

Problem 1
In the line spectrum of atomic hydrogen there is also a group of lines known as the Pfund series. These lines are produced when electrons, excited to high energy levels, make transitions to the \( n = 5 \) level. Determine (a) the longest wavelengths and (b) the shortest wavelengths in this series.

Note: the Pfund series is given by:

\[
\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{n^2} \right) \quad n = 6, 7, 8, \ldots
\]

The longest wavelength occurs when \( n = 6 \) and the shortest wavelength occurs when \( n = \infty \).

a) Longest wavelength (\( n = 6 \))

\[
\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{6^2} \right)
\]

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{5^2} - \frac{1}{6^2} \right) = 1.34 \times 10^5 \text{ m}^{-1}
\]

\[
\lambda = 7.46 \times 10^{-6} \text{ m}
\]

b) Shortest wavelength (\( n = \infty \))

\[
\frac{1}{\lambda} = R \left( \frac{1}{5^2} - \frac{1}{\infty} \right)
\]

\[
\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1}) \left( \frac{1}{5^2} - 0 \right) = 4.39 \times 10^3 \text{ m}^{-1}
\]

\[
\lambda = 2.28 \times 10^{-6} \text{ m}
\]
Problem 2
(a) The electron in a hydrogen atom is in the first excited state, when the electron acquires an additional 2.86 eV of energy. What is the quantum number \( n \) of the state into which the electron moves?

\[
E_n = -\frac{13.6\text{ eV}}{n^2}.
\]

The first excited state corresponds to \( n = 2 \).

\[
n = 2 \rightarrow E_a = -\frac{13.6\text{ eV}}{2^2} = -3.4\text{ eV}
\]

\[
\Rightarrow \text{ after the electron acquires an additional 2.86 eV of energy, the new energy is } E = -3.4\text{ eV} + 2.86\text{ eV} = -0.54\text{ eV}
\]

\[
E = \frac{-13.6\text{ eV}}{n^2} \rightarrow n = \sqrt{\frac{-13.6\text{ eV}}{E}} = \sqrt{\frac{-13.6\text{ eV}}{-0.54\text{ eV}}} = \sqrt{25}
\]

\[
n = 5
\]

(b) A laser is used in eye surgery to weld a detached retina back into place. The wavelength of the laser beam is 514 nm, and the power is 1.5 W. During surgery, the laser beam is turned on for 0.050 s. During this time, how many photons are emitted by the laser?

\[
\lambda = 514\text{ nm} \quad P = \frac{E}{t}
\]

\[
P = 1.5\text{ W} \quad t = 0.050\text{ s}
\]

\[
n = \frac{P}{\frac{nhf}{t}}
\]

\[
\text{Note: the energy emitted by the laser is given by } E = nhf
\]

\[
f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{514 \times 10^{-9} \text{ m}} = 5.84 \times 10^{14} \text{ Hz}
\]

\[
n = \frac{Pt}{hf} = \frac{(1.5\text{ W})(0.050\text{ s})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(5.84 \times 10^{14} \text{ Hz})}
\]

\[
n = 1.9 \times 10^{17}
\]