Kinematics in Two Dimensions

Displacement: $\Delta \vec{r} = \vec{r} - \vec{r}_0$

Average velocity: $\vec{v}_{\text{ave}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r} - \vec{r}_0}{t - t_0}$

Instantaneous velocity: $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$

Average acceleration: $\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - t_0}$

Instantaneous acceleration: $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

Note: In two dimensional motion, each component (x and y) can be treated separately. The x-component is independent of the y-component and vice versa. The two components are connected by time $t$.

Equations of Constant Acceleration:

- **x-component of motion**
  
  $v_x = v_{0x} + a_x t$
  
  $x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$
  
  $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$
  
  $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

- **y-component of motion**
  
  $v_y = v_{0y} + a_y t$
  
  $y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$
  
  $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$
  
  $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

 Projectile Motion:

⇒ In projectile motion, we assume no air resistance so:

  $a_x = 0 \text{ m/s}^2$
  
  $a_y = -9.8 \text{ m/s}^2$ (assuming up is +)

⇒ Since $a_x = 0 \text{ m/s}^2$, the 4 equations for the horizontal component of projectile motion reduce down to two:

  $v_x = v_{0x}$
  
  $x - x_0 = v_{0x}t$

⇒ The equations for the vertical component of projectile motion are the same as the above equations for the y-component of motion with $a_y = -9.8 \text{ m/s}^2$