Kinematics in Two Dimensions

Displacement: \( \Delta \vec{r} = \vec{r} - \vec{r}_0 \)

Average velocity: \( \vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r} - \vec{r}_0}{t - t_0} \)

Instantaneous velocity: \( \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \)

Average acceleration: \( \vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - t_0} \)

Instantaneous acceleration: \( \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \)

Note: In two dimensional motion, each component (x and y) can be treated separately. The x-component is independent of the y-component and vice versa. The two components are connected by time \( t \).

Equations of Constant Acceleration:

x-component of motion

\[ v_x = v_{0x} + a_x t \]
\[ x - x_0 = \frac{1}{2}(v_{0x} + v_x) t \]
\[ x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 \]
\[ v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \]

y-component of motion

\[ v_y = v_{0y} + a_y t \]
\[ y - y_0 = \frac{1}{2}(v_{0y} + v_y) t \]
\[ y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 \]
\[ v_y^2 = v_{0y}^2 + 2a_y(y - y_0) \]

Projectile Motion:

⇒ In projectile motion, we assume no air resistance so:

\[ a_x = 0 \text{ m/s}^2 \]
\[ a_y = -9.8 \text{ m/s}^2 \text{ (assuming up is +)} \]

⇒ Since \( a_x = 0 \text{ m/s}^2 \), the 4 equations for the horizontal component of projectile motion reduce down to two:

\[ v_x = v_{0x} \]
\[ x - x_0 = v_{0x} t \]

⇒ The equations for the vertical component of projectile motion are the same as the above equations for the y-component of motion with \( a_y = -9.8 \text{ m/s}^2 \)