Physics 11
Chapter 18: Ray Optics

"... Everything can be taken from a man but one thing; the last of the human freedoms — to choose one’s attitude in any given set of circumstances, to choose one’s own way."
Victor E. Frankl

"Your own mind is a sacred enclosure into which nothing harmful can enter except by your promotion." - Ralph Waldo Emerson

Reading: pages 574 – 602

Outline:
⇒ the ray model of light (PowerPoint)
⇒ reflection
  specular and diffuse reflection
  images from plane mirrors
⇒ refraction
  Snell’s law
  total internal reflection
⇒ image formation by refraction
⇒ thin lenses
  ray tracing (PowerPoint)
  focal point and focal length
  magnification
  real and virtual images
⇒ image formation with Spherical Mirrors
  concave and convex mirrors
  principle axis and radius of curvature
  ray tracing for mirrors (PowerPoint)
⇒ the thin-lens / mirror equation and the magnification equation
  sign conventions
  lots of examples

Problem Solving

Whenever a light ray hits the interface between two transparent materials, generally some of the light is reflected (the angle of reflection equals the angle of incidence: \( \theta_r = \theta_i \)) and some of the light is transmitted or refracted. The angle of reflection equals the angle of incidence: \( \theta_r = \theta_i \). The angle of the refracted ray is determined by Snell’s law: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) where \( n \) is the index of refraction of the material which is given by \( n = c/v \). Remember that in optics all angles are measured with respect to the normal.
When light hits the interface between a medium with a larger index of refraction and a medium with a smaller index of refraction (i.e. from water to air), it is possible for the light to be totally internally reflected. When this happens, no light at all escapes the medium with the larger index of refraction and 100% of the light is reflected. Total internal reflection will occur if the incident angle is greater than the critical angle, where \( \theta_c \) is given by \( \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \).

You should know how to find the orientation, height and position of the image from a thin lens (both converging and diverging) and from a spherical mirror (both concave and convex) by using ray tracing. Ray tracing always involves drawing three light rays.

To find the orientation, height, and position of the image using a more mathematical approach, the thin-lens / mirror and magnification equations can be used.

The thin lens / mirror equation is

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

where \( s \) is the object distance, \( s' \) is the image distance, and \( f \) is the focal length.

The magnification equation is

\[
m = \frac{h'}{h} = -\frac{s'}{s}
\]

where \( h_i \) and \( h_o \) are the image and object heights.

It is **very important** to know the sign conventions for thin lenses and spherical mirrors. They are listed below for your convenience.

- \( f \) is + for a concave mirror or converging lens
- \( f \) is - for a convex mirror or diverging lens
- \( s \) is + if the object is real
- \( s \) is - if the object is virtual (only possible with multiple lens/mirror systems)
- \( s' \) is + if the image is real
- \( s' \) is - if the image is virtual
- \( m \) is + for an image that is upright with respect to the object
- \( m \) is - for an image that is inverted with respect to the object
Reflection

Law of reflection: \( \theta_i = \theta_r \)

Reflection can be specular (mirror-like) or diffuse (from rough surfaces).

Plane mirrors: A virtual image is formed at with where \( s \) is the object distance and \( s' \) is the image distance.

\[
\text{ Incident ray} \quad \theta_i \\
\text{ Normal} \quad \theta_r \\
\text{ Refracted ray} \\
\]

Refraction

Snell’s law of refraction:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Index of refraction is \( n = c/v \). The ray is closer to the normal on the side with the larger index of refraction.

If total internal reflection (TIR) occurs when the angle of incidence \( \theta_i \) is greater than \( \theta_c = \sin^{-1}(n_2/n_1) \).

**Image formation**

If rays diverge from P and, after interacting with a lens or mirror, appear to diverge from P’ without actually passing through P’, then P’ is a virtual image of P.

If rays diverge from P and interact with a lens or mirror so that the refracted rays converge at P’, then P’ is a real image of P. Rays actually pass through a real image.

**The thin-lens equation**

For a lens or curved mirror, the object distance \( s \), the image distance \( s' \), and the focal length \( f \) are related by the thin-lens equation:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} 
\]

The magnification of a lens or mirror is \( m = -s'/s \).

**Sign conventions** for the thin-lens equation:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Positive when</th>
<th>Negative when</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>Always</td>
<td>Not treated here</td>
</tr>
<tr>
<td>( s' )</td>
<td>Real image; on opposite side of a lens from object, or in front of a mirror</td>
<td>Virtual image; on same side of a lens as object, or behind a mirror</td>
</tr>
<tr>
<td>( f )</td>
<td>Converging lens or concave mirror</td>
<td>Diverging lens or convex mirror</td>
</tr>
<tr>
<td>( m )</td>
<td>Image is upright.</td>
<td>Image is inverted.</td>
</tr>
</tbody>
</table>
Example Problems from Chapter 18

Problem 1
At high noon, the sun is almost directly above (about 2.0° from the vertical) and a tall redwood tree casts a shadow that 10.0 ft long. How tall is the redwood tree?

Problem 2
You shine your laser pointer through the flat glass side of a rectangular aquarium at an angle of incidence of 45°. The index of refraction of this type of glass is 1.55.

(a) At what angle from the normal does the beam from the laser pointer enter the water inside the aquarium? (b) Does your answer to part a depend on the index of refraction of the glass?

Problem 3
Two plane mirrors are separated by 120°, as the drawing illustrates. If a ray strikes mirror M₁ at a 65° angle of incidence, at what angle θ does it leave mirror M₂?
Problem 4
The glass core of an optical fiber has index of refraction 1.60. The index of refraction of the cladding is 1.48. What is the maximum angle between a light ray and the wall of the core if the ray is to remain inside the core?

Problem 5
An object is 15 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. What are the image characteristics: (i) real or virtual?, (ii) upright or inverted?, (iii) smaller, larger or same?, and (iv) the image location?

Problem 6
A 2.0-cm-high object is situated 15.0 cm in front of a concave mirror that has a radius of curvature of 10.0 cm. Using a ray diagram drawn to scale, measure (a) the location and (b) the height of the image. The mirror must be drawn to scale.
Problem 7
A 10.0-cm-high object is situated 25.0 cm in front of a convex mirror that has a radius of curvature of $1.00 \times 10^2$ cm. Using a ray diagram drawn to scale, measure (a) the location and (b) the height of the image. The mirror must be drawn to scale.

The ray diagram is shown below. (Note: $f = -50.0$ cm and $s = 25.0$ cm)

The ray diagram indicates that the image is 16.7 cm behind the mirror.

The ray diagram indicates that the image height is 6.67 cm.

Problem 8
A 2.0-cm tall object is 15 cm in front of a converging lens that has a 20 cm focal length. Where is the image located and what is the height of the image? What are the characteristics of the image (real or virtual, upright or inverted, enlarged or reduced)?
**Problem 9**
The focal length of a concave mirror is 17 cm. An object is located 38 cm in front of this mirror. Where is the image located?

**Problem 10**
The image behind a convex mirror (radius of curvature = 68 cm) is located 22 cm from the mirror. (a) Where is the object located and (b) what is the magnification of the mirror? Determine whether the image is (c) upright or inverted and (d) larger or smaller than the object.

**Problem 11**
A 1.0-cm tall object is 60 cm in front of a diverging lens that has a -30 cm focal length. Where is the image located and what is the height of the image? What are the characteristics of the image (real or virtual, upright or inverted, enlarged or reduced)?
Problem 12
Convex mirrors are being used to monitor the aisles in a store. The mirrors have a radius of curvature of 4.0 m. (a) What is the image distance if a customer is 15 m in front of the mirror? (b) Is the image real or virtual? (c) If a customer is 1.6 m tall, how tall is the image?

Problem 13
A clown is using a concave makeup mirror to get ready for a show and is 27 cm in front of the mirror. The image is 65 cm behind the mirror. Find (a) the focal length of the mirror and (b) the magnification.

Problem 14
A concave mirror \( f = 45 \) cm produces an image whose distance from the mirror is one-third the object distance. Determine (a) the object distance and (b) the (positive) image distance.