**SUMMARY**

The goals of Chapter 30 have been to understand the physics of the nucleus and some of the applications of nuclear physics.

**GENERAL PRINCIPLES**

**The Nucleus**

The nucleus is a small, dense, positive core at the center of an atom.

- Z protons, charge $+e$, spin $\frac{1}{2}$
- N neutrons, charge 0, spin $\frac{1}{2}$

The mass number is

$$A = Z + N$$

**Isotopes** of an element have the same value of $Z$ but different values of $N$.

The strong force holds nuclei together:

- It acts between any two nucleons.
- It is short range.

Adding neutrons to a nucleus allows the strong force to overcome the repulsive Coulomb force between protons.

The binding energy $B$ of a nucleus depends on the mass difference between an atom and its constituents:

$$B = (Zm_H +Nm_n - m_{atom}) \times (931.49 \text{ MeV/u})$$

**Nuclear Stability**

Most nuclei are not stable. Unstable nuclei undergo **radioactive decay**. Stable nuclei cluster along the **line of stability** in a plot of the isotopes.

Mechanisms by which unstable nuclei decay:

<table>
<thead>
<tr>
<th>Decay</th>
<th>Particle</th>
<th>Penetration</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>$^4\text{He}$ nucleus</td>
<td>low</td>
</tr>
<tr>
<td>beta-minus</td>
<td>$e^-$</td>
<td>medium</td>
</tr>
<tr>
<td>beta-plus</td>
<td>$e^+$</td>
<td>medium</td>
</tr>
<tr>
<td>gamma</td>
<td>photon</td>
<td>high</td>
</tr>
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Alpha and beta decays change the nucleus; the daughter nucleus is a different element.

**Alphas decay:**

$$\frac{1}{2}X \rightarrow \frac{1}{2}Y + \alpha + \text{energy}$$

**Betas decay:**

$$\frac{1}{2}X \rightarrow \frac{1}{2}Y + \beta + \text{energy}$$

**IMPORTANT CONCEPTS**

**Energy levels**

Nucleons fill nuclear energy levels, similar to filling electron energy levels in atoms. Nucleons can often jump to lower energy levels by emitting beta particles or gamma photons.

**The quark model**

Nucleons (and other particles) are made of quarks. Quarks and leptons are fundamental particles.

**APPLICATIONS**

**Radioactive decay**

The number of undecayed nuclei decreases exponentially with time $t$:

$$N = N_0 e^{-\lambda t}$$

$$N = N_0 \left( \frac{1}{2} \right)^{\ln 2}$$

The half-life

$$t_{1/2} = \frac{\ln 2}{\lambda} = 0.693 \tau$$

is the time in which half of any sample decays.

**Measuring radiation**

The **activity** of a radioactive sample is the number of decays per second. Activity is related to the half-life as

$$R = \frac{0.693 N}{t_{1/2}} = \frac{N}{\tau}$$

The radiation **dose** is measured in grays, where

$$1 \text{ Gy} = 1.00 \text{ J/kg of absorbed energy}$$

The **relative biological effectiveness** (RBE) is the biological effect of a dose relative to the biological effects of x rays. The **dose equivalent** is measured in sieverts, where

$$\text{dose equivalent in Sv} = \text{dose in Gy} \times \text{RBE}$$
Conceptual Questions

1. Atom A has a larger atomic mass than atom B. Does this mean that atom A also has a larger atomic number? Explain.
2. Given that \( m_H = 1.007825 \) u, is the mass of a hydrogen atom \(^1\)H greater than, less than, or equal to 1/12 the mass of a \(^{12}\)C atom? Explain.
3. a. Is there a stable \(^{3}\)Li nucleus? Explain how you made your determination.
   b. Is there a stable \(^{18}\)O nucleus? Explain how you made your determination.
4. Rounding slightly, the nucleus \(^3\)He has a binding energy of 2.5 MeV/nucleon and the nucleus \(^6\)Li has a binding energy of 5 MeV/nucleon.
   a. What is the binding energy of \(^3\)He?
   b. What is the binding energy of \(^6\)Li?
   c. Is it energetically possible for two \(^3\)He nuclei to join or fuse together into a \(^{6}\)Li nucleus? Explain.
   d. Is it energetically possible for a \(^{6}\)Li nucleus to split or fission into two \(^3\)He nuclei? Explain.
5. A sample contains a mix of isotopes of an element. Using a spectrometer to measure the spectrum of emitted light will not reveal the mix of isotopes; analyzing the sample with a mass spectrometer will. Explain.
6. For each nuclear energy-level diagram in Figure Q30.6, state whether it represents a nuclear ground state, an excited nuclear state, or an impossible nucleus.

7. Figure Q30.7 shows how the number of nuclei of one particular isotope varies with time. What is the half-life of the nucleus?

8. A radioactive sample has a half-life of 10 s. 10,000 nuclei are present at \( t = 20 \) s.
   a. How many nuclei were there at \( t = 0 \) s?
   b. How many nuclei will there be at \( t = 40 \) s?

9. Nucleus A decays into the stable nucleus B with a half-life of 10 s. At \( t = 0 \) s there are 1000 A nuclei and no B nuclei. At what time will there be 750 B nuclei?
10. A radioactive sample's half-life is 1.0 min, so each nucleus in the sample has a 50% chance of undergoing a decay sometime between \( t = 0 \) and \( t = 1 \) min. One particular nucleus has not decayed at \( t = 15 \) min. What is the probability this nucleus will decay between \( t = 15 \) and \( t = 16 \) min?
11. Four samples each contain a single radioactive isotope. Sample A has 1 mol of matter and an activity of 100 Bq. Sample B has 10 mol and 100 Bq, sample C has 100 mol and 100 Bq, and sample D has 100 mol and 1000 Bq. Rank in order, from largest to smallest, the half-lives of these four isotopes. Explain.
12. Oil and coal generally contain no measurable \(^{14}\)C. What does this tell us about how long they have been buried?
13. Radiocarbon dating assumes that the abundance of \(^{14}\)C in the environment has been constant. Suppose \(^{14}\)C was less abundant 10,000 years ago than it is today. Would this cause a lab using radiocarbon dating to overestimate or underestimate the age of a 10,000-year-old artifact? (In fact, the abundance of \(^{14}\)C in the environment does vary slightly with time. But the issue has been well studied, and the ages of artifacts are adjusted to compensate for this variation.)
14. Identify the unknown \( X \) in the following decays:
   a. \(^{226}\)Rn \( \rightarrow ^{222}\)Rn + \( X \)
   b. \(^{228}\)Ra \( \rightarrow ^{224}\)Ra + \( X \)
   c. \(^{140}\)Xe \( \rightarrow ^{140}\)Xe + \( X \)
   d. \(^{20}\)Ne \( \rightarrow ^{20}\)Ne + \( X \)
15. Are the following decays possible? If not, why not?
   a. \(^{233}\)U \( \rightarrow ^{233}\)U + \( \alpha \)
   b. \(^{238}\)U \( \rightarrow ^{238}\)U + \( \alpha \)
   c. \(^{35}\)P \( \rightarrow ^{35}\)S + e\(^-\)
16. What kind of decay, if any, would you expect for the nuclei with the energy-level diagrams shown in Figure Q30.16?

17. The nuclei of \(^{4}\)He and \(^{16}\)O are very stable and are often referred to as “doubly magic” nuclei. Use what you know about energy levels to explain what is special about these particular nuclei.
18. A and B are fresh apples. Apple A is strongly irradiated by nuclear radiation for 1 hour. Apple B is not irradiated. Afterward, in what ways are apples A and B different?
19. A patient’s tumor is irradiated with gamma rays from an external source. Afterward, is his body radioactive? Explain.
20. It’s possible that a bone tumor will not show up on an x-ray image but will show up in a gamma scan. Explain why this is so.

21. Four radiation doses are as follows: Dose A is 10 rad with an RBE of 1, dose B is 20 rad with an RBE of 1, dose C is 10 rad with an RBE of 2, and dose D is 20 rad with an RBE of 2.
   a. Rank in order, from largest to smallest, the amount of energy delivered by these four doses.
   b. Rank in order, from largest to smallest, the biological damage caused by these four doses.

22. Two different sources of radiation give the same dose equivalent in Sv. Does this mean that the radiation from each source has the same RBE? Explain.

23. Some types of MRI can produce images of resolution and detail similar to PET. Though the images are similar, MRI is generally preferred over PET for studies of brain function involving healthy subjects. Why?

24. Sulfur colloid particles tagged with 99mTc are taken up and retained by cells in the liver and spleen. A patient is suspected of having a liver tumor that would destroy these cells. Explain how a gamma camera scan could be used to confirm or rule out the existence of a tumor.

25. The first two letters in the acronym SPECT, which describes a nuclear imaging technique, stand for “single photon.” Is a SPECT done with a gamma emitter or a positron emitter?

The following two questions concern an uncommon nuclear decay mode known as electron capture. Certain nuclei that are proton-rich but energetically prohibited from undergoing beta-plus decay can capture an electron from the 1s shell, which then combines with a proton to make a neutron. The basic reaction is

\[ p + e^- \rightarrow n + \nu_e \]

26. Give a description of the electron capture process in terms of quarks.

27. Electron capture is usually followed by the emission of an x-ray. Why?

Multiple-Choice Questions

28. A significant fraction of the radiation dose you will receive during your life comes from radioactive materials in your body. The most important source of this radiation is the potassium isotope 40K, which decays to the stable calcium isotope 40Ca. What particle is emitted in the decay?
   A. A helium nucleus  
   B. A neutron  
   C. An electron  
   D. A positron

29. A certain watch’s luminous glow is due to zinc sulfide paint that is energized by beta particles given off by tritium, the radioactive hydrogen isotope 3H, which has a half-life of 12.3 years. This glow has about 1/10 of its initial brightness. How many years old is the watch?
   A. 20 yr  
   B. 30 yr  
   C. 40 yr  
   D. 50 yr

30. What is the unknown isotope in the following fission reaction: \( n + ^{235}\text{U} \rightarrow ^{141}\text{X} + ? + 3n \)
   A. \(^{88}\text{Rb}\)  
   B. \(^{102}\text{Rb}\)  
   C. \(^{89}\text{Y}\)  
   D. \(^{102}\text{Y}\)

31. The uranium in the earth’s crust is 0.7% \( ^{235}\text{U} \) and 99.3% \( ^{238}\text{U} \). Two billion years ago, \( ^{235}\text{U} \) comprised approximately 3% of the uranium in the earth’s crust. This tells you something about the relative half-lives of the two isotopes. Suppose you have a sample of \( ^{235}\text{U} \) and a sample of \( ^{238}\text{U} \), each with exactly the same number of atoms.
   A. The sample of \( ^{235}\text{U} \) has a higher activity.
   B. The sample of \( ^{238}\text{U} \) has a higher activity.
   C. The two samples have the same activity.

32. Suppose you have a 1 g sample of \( ^{226}\text{Ra} \), half-life 1600 years. How long will it be until only 0.1 g of radium is left?
   A. 1600 yr  
   B. 3200 yr  
   C. 5300 yr  
   D. 16,000 yr

33. A sample of \( ^{121}\text{I} \), half-life 8.0 days, is registering 100 counts per second on a Geiger counter. How long will it be before the sample registers only 1 count per second?
   A. 8 days  
   B. 53 days  
   C. 80 days  
   D. 800 days

34. The complete expression for the decay of the radioactive hydrogen isotope tritium may be written as \( ^3\text{H} \rightarrow ^3\text{He} + X + Y \). The symbols \( X \) and \( Y \) represent
   A. \( X = e^- \), \( Y = \bar{\nu}_e \)  
   B. \( X = e^- \), \( Y = \nu_e \)  
   C. \( X = e^+ \), \( Y = \bar{\nu}_e \)  
   D. \( X = e^+ \), \( Y = \nu_e \)

35. The quark composition of the proton and neutron are, respectively, uud and udd, where \( u \) is an up quark (charge \( +\frac{2}{3}e \)) and \( d \) is a down quark (charge \( -\frac{1}{3}e \)). There are also anti-up \( \bar{u} \) (charge \( -\frac{2}{3}e \)) and anti-down \( \bar{d} \) (charge \( +\frac{1}{3}e \)) quarks. The combination of a quark and an antiquark is called a meson. The mesons known as pions have the composition \( \pi^+ = u\bar{d} \) and \( \pi^- = d\bar{u} \). Suppose a proton collides with an antineutron. During such collisions, the various quarks and antiquarks annihilate whenever possible. When the remaining quarks combine to form a single particle, it is a
   A. Proton  
   B. Neutron  
   C. \( \pi^+ \)  
   D. \( \pi^- \)

Section 30.1 Nuclear Structure

1. How many protons and how many neutrons are in (a) \( ^3\text{H} \), (b) \( ^{40}\text{Ar} \), (c) \( ^{40}\text{Ca} \), and (d) \( ^{239}\text{Pu} \)?
2. How many protons and how many neutrons are in (a) \( ^3\text{He} \), (b) \( ^{20}\text{Ne} \), (c) \( ^{60}\text{Co} \), and (d) \( ^{226}\text{Ra} \)?
3. Use the data in Appendix D to calculate the chemical atomic mass of lithium, to two decimal places.
4. Use the data in Appendix D to calculate the chemical atomic mass of neon, to two decimal places.

Section 30.2 Nuclear Stability

5. Calculate (in MeV) the total binding energy and the binding energy per nucleon for (a) \( ^3\text{H} \) and (b) \( ^3\text{He} \).
6. Calculate (in MeV) the total binding energy and the binding energy per nucleon for (a) \( ^{40}\text{Ar} \) and (b) \( ^{40}\text{K} \).
7. Calculate (in MeV) the binding energy per nucleon for \( ^3\text{He} \) and \( ^3\text{He} \). Which is more tightly bound?
8. Calculate (in MeV) the binding energy per nucleon for \( ^{12}\text{C} \) and \( ^{13}\text{C} \). Which is more tightly bound?
9. Calculate (in MeV) the binding energy per nucleon for (a) $^{14}$N, (b) $^{56}$Fe, and (c) $^{207}$Pb.

10. When a nucleus of $^{235}$U undergoes fission, it breaks into two smaller, more tightly bound fragments. Calculate the binding energy per nucleon for $^{235}$U and for the fission product $^{137}$Cs.

11. When a nucleus of $^{239}$Pu undergoes fission, it breaks into two smaller, more tightly bound fragments. Calculate the binding energy per nucleon for $^{239}$Pu and for the fission product $^{135}$Xe.

Section 30.3 Forces and Energy in the Nucleus

12. Draw an energy-level diagram, similar to Figure 30.9 for the protons and neutrons in $^{11}$Be. Do you expect this nucleus to be stable?

13. Draw energy-level diagrams, similar to Figure 30.9, for all $A = 10$ nuclei listed in Appendix D. Show all the occupied neutron and proton levels. Which of these nuclei do you expect to be stable?

14. Draw energy-level diagrams, similar to Figure 30.9, for all $A = 14$ nuclei listed in Appendix D. Show all the occupied neutron and proton levels. Which of these nuclei do you expect to be stable?

15. You have seen that filled electron energy levels correspond to chemically stable nuclei. A similar principle holds for nuclear energy levels; nuclei with equally filled proton and neutron energy levels are especially stable. What are the three lightest isotopes whose proton and neutron energy levels are both filled, and filled equally?

Section 30.4 Radiation and Radioactivity

16. $^{15}$O and $^{131}$I are isotopes used in medical imaging. $^{15}$O is a beta-plus emitter, $^{131}$I a beta-minus emitter. What are the daughter nuclei of the two decays?

17. Spacecraft have been powered with energy from the alpha decay of $^{239}$Pu. What is the daughter nucleus?

18. Identify the unknown isotope $X$ in the following decays.
   a. $^{234}$U $\rightarrow X + \alpha$
   b. $^{32}$P $\rightarrow X + e^-$
   c. $X \rightarrow ^{30}$Si $+ e^-$
   d. $^{24}$Mg $\rightarrow X + \gamma$

19. Identify the unknown isotope $X$ in the following decays.
   a. $X \rightarrow ^{22}$Ra $+ \alpha$
   b. $X \rightarrow ^{207}$Pb $+ e^-$
   c. $^{7}$Be $+ e^-$ $\rightarrow X$
   d. $X \rightarrow ^{60}$Ni $+ \gamma$

20. What is the energy (in MeV) released in the alpha decay of $^{239}$Pu?

21. What is the energy (in MeV) released in the alpha decay of $^{220}$Th?

22. What is the total energy (in MeV) released in the beta decay of a neutron?

23. Medical gamma imaging is generally done with the technetium isotope $^{99}$Tc, which decays by emitting a gamma-ray photon with energy 140 keV. What is the mass loss of the nucleus, in u, upon emission of this gamma ray?

Section 30.5 Nuclear Decay and Half-Lives

24. The radioactive hydrogen isotope $^3$H is called tritium. It decays by beta-minus decay with a half-life of 12.3 years.
   a. What is the daughter nucleus of tritium?
   b. A watch uses the decay of tritium to energize its glowing dial. What fraction of the tritium remains 20 years after the watch was created?

25. The barium isotope $^{132}$Ba has a half-life of 10.5 years. A sample begins with $1.0 \times 10^{10}$ $^{132}$Ba atoms. How many are left after (a) 2 years, (b) 20 years, and (c) 200 years?

26. The cadmium isotope $^{109}$Cd has a half-life of 462 days. A sample begins with $1.0 \times 10^{12}$ $^{109}$Cd atoms. How many are left after (a) 50 days, (b) 500 days, and (c) 5000 days?

27. How many half-lives must elapse until (a) 90% and (b) 99% of a radioactive sample of atoms has decayed?

28. The Chernobyl reactor accident in what is now Ukraine was the worst nuclear disaster of all time. Fission products from the reactor core spread over a wide area. The primary radiation exposure to people in western Europe was due to the short-lived (half-life 8.0 days) isotope $^{131}$I, which fell across the landscape and was ingested by grazing cows that concentrated the isotope in their milk. Farmers couldn’t sell the contaminated milk, so many opted to use the milk to make cheese, aging it until the radioactivity decayed to acceptable levels. How much time must elapse for the activity of a block of cheese containing $^{131}$I to drop to 1.0% of its initial value?

29. What is the age in years of a bone in which the $^{14}$C/$^{12}$C ratio is measured to be $1.65 \times 10^{-13}$?

30. $^{85}$Sr is a short-lived (half-life 65 days) isotope used in bone scans. A typical patient receives a dose of $^{85}$Sr with an activity of 0.10 mCi. If all of the $^{85}$Sr is retained by the body, what will be its activity in the patient’s body after one year has passed?

31. What is the half-life in days of a radioactive sample with $5.0 \times 10^{12}$ atoms and an activity of $5.0 \times 10^7$ Bq?

32. What is the activity, in Bq and Ci, of 1.0 g of $^{228}$Ra? Marie Curie was the discoverer of radium; can you see where the unit of activity named after her came from?

33. Many medical PET scans use the isotope $^{18}$F, which has a half-life of 1.8 hr. A sample prepared at 10:00 a.m. has an activity of 20 mCi. What is the activity at 1:00 p.m., when the patient is injected?

34. An investigator collects a sample of a radioactive isotope with an activity of 370,000 Bq. 48 hours later, the activity is 120,000 Bq. What is the half-life of the sample?

Section 30.6 Medical Applications of Nuclear Physics

35. A 50 kg nuclear plant worker is exposed to 20 mCi of neutron radiation with an RBE of 10. What is the dose in mSv?

36. A gamma scan showing the active volume of a patient’s lungs can be created by having a patient breathe the radioactive isotope $^{131}$Xe, which undergoes beta-minus decay with a subsequent gamma emission from the daughter nucleus. A typical procedure gives a dose of 0.30 rem to the lungs. How much energy is deposited in the 1.2 kg mass of a patient’s lungs?

37. How many rad of gamma-ray photons cause the same biological damage as 30 rad of alpha radiation?

38. 150 rad of gamma radiation are directed into a 150 g tumor.

39. During the 1950s, nuclear bombs were tested on islands in the South Pacific. In one test, personnel on a nearby island received 10 mGy per hour of beta and gamma radiation. At this rate, how long would it take to receive a potentially lethal dose equivalent of 4.5 Sv?
40. 131I undergoes beta-minus decay with a subsequent gamma emission from the daughter nucleus. Iodine in the body is almost entirely taken up by the thyroid gland, so a gamma scan using this isotope will show a bright area corresponding to the thyroid gland with the surrounding tissue appearing dark. Because the isotope is concentrated in the gland, so is the radiation dose, most of which results from the beta emission. In a typical procedure, a patient receives 0.050 mCi of 131I. Assume that all of the iodine is absorbed by the 0.15 kg thyroid gland. Each 131I decay produces a 0.97 MeV beta particle. Assume that half the energy of each beta particle is deposited in the gland. What dose equivalent in Sv will the gland receive in the first hour?

41. The doctors planning a radiation therapy treatment have determined that a 100 g tumor needs to receive 0.20 J of gamma radiation. What is the dose in Gy?

42. 90Sr decays with the emission of a 2.8 MeV beta particle. Strontium is chemically similar to calcium and is taken up by bone. A 75 kg person exposed to waste from a nuclear accident absorbs 90Sr with an activity of 370,000 Bq. Assume that all of this 90Sr ends up in the skeleton. The skeleton forms 17% of the person's body mass. If 50% of the decay energy is absorbed by the skeleton, what dose equivalent in Sv will be received by the person's skeleton in the first month?

Section 30.7 The Ultimate Building Blocks of Matter

43. What are the minimum energies of the two oppositely directed gamma rays in a PET procedure?

44. Positive and negative pions, denoted π+ and π−, are antiparticles of each other. Each has a rest mass of 140 MeV/c². Suppose a collision between an electron and positron, each with kinetic energy K, produces a π+, π− pair. What is the smallest possible value for K?

45. In a particular beta-minus decay of a free neutron (that is, one not part of an atomic nucleus), the emitted electron has exactly the same kinetic energy as the emitted antineutrino. What is the value, in MeV, of that kinetic energy? Assume that the recoiling proton has negligible kinetic energy.

46. The masses of the neutrinos are still not precisely determined, but let us assume for the purpose of this problem that the mass of an electron neutrino is one millionth the mass of an electron. What is the kinetic energy, in eV, of an electron neutrino moving at 0.999c?

General Problems

47. The chemical atomic mass of hydrogen, with the two stable isotopes 1H and 2H (deuterium), is 1.00798 u. Use this value to determine the natural abundance of these two isotopes.

48. You learned in Chapter 29 that the binding energy of the electron in a hydrogen atom is 13.6 eV.
   a. By how much does the mass decrease when a hydrogen atom is formed from a proton and an electron? Give your answer both in atomic mass units and as a percentage of the mass of the hydrogen atom.
   b. By how much does the mass decrease when a helium nucleus is formed from two protons and two neutrons? Give your answer both in atomic mass units and as a percentage of the mass of the helium nucleus.

49. Use the graph of binding energy of Figure 30.6 to estimate the total energy released if a nucleus with mass number 240 fissions into two nuclei with mass number 120.

50. Could a 56Fe nucleus fission into two 28Al nuclei? Your answer, which should include some calculations, should be based on the curve of binding energy of Figure 30.6.

51. What are the isotopic symbols of all isotopes in Appendix D with A = 17?
   a. Which of these are stable nuclei?
   b. For those that are not stable, identify both the decay mode and the daughter nucleus.

52. What is the activity in Bq and in Ci of a 2.0 mg sample of 3H?

53. The activity of a sample of the cesium isotope 137Cs is 2.0 × 10⁸ Bq. Many years later, after the sample has fully decayed, how many beta particles will have been emitted?

54. A 115 mCi sample of a radioactive isotope is made in a reactor. When delivered to a hospital, its activity is 95 mCi. The lowest usable activity level is 10 mCi.
   a. What is the isotope's half-life?
   b. For how long after delivery is the sample usable?

55. You are assisting in an anthropology lab over the summer by carrying out 14C dating. A graduate student found a bone he believes to be 20,000 years old. You extract the carbon from the bone and prepare an equal-mass sample of carbon from modern organic material. To determine the activity of a sample with the accuracy your supervisor demands, you need to measure the time it takes for 10,000 decays to occur.
   a. The activity of the modern sample is 1.06 Bq. How long does that measurement take?
   b. It turns out that the graduate student’s estimate of the bone’s age was accurate. How long does it take to measure the activity of the ancient carbon?

56. A sample of wood from an archaeological excavation is dated by using a mass spectrometer to measure the fraction of 14C atoms. Suppose 100 atoms of 14C are found for every 1.0 × 10ⁱ⁵ atoms of 12C in the sample. What is the wood’s age?

57. A sample of 1.0 × 10⁹ atoms that decay by alpha emission has a half-life of 100 min. How many alpha particles are emitted between t = 50 min and t = 200 min?

58. A sample contains radioactive atoms of two types, A and B. Initially there are five times as many A atoms as there are B atoms. Two hours later, the numbers of the two atoms are equal. The half-life of A is 0.50 hours. What is the half-life of B?

59. The technique known as potassium-argon dating is used to date old lava flows and thus any fossilized skeletons found in them, like this 1.8-million-year old hominid skull. The potassium isotope 40K has a 1.28-billion-year half-life and is naturally present at very low levels. 40K decays by beta emission into the stable isotope 40Ar. Argon is a gas, and there is no argon in flowing lava because the gas escapes. Once the lava solidifies, any argon produced in the decay of 40K is trapped inside and cannot escape. What is the age of a piece of solidified lava with a 40Ar/40K ratio of 0.12?
60. III Corals take up certain elements from seawater, including uranium but not thorium. After the corals die, the uranium isotopes slowly decay into thorium isotopes. A measurement of the relative fraction of certain isotopes therefore provides a determination of the coral’s age. A complicating factor is that the thorium isotopes decay as well. One scheme uses the alpha decay of $^{238}\text{U}$ to $^{234}\text{Th}$. After a long time, the two species reach an equilibrium in which the number of $^{234}\text{U}$ decays per second (each producing an atom of $^{230}\text{Th}$) is exactly equal to the number of $^{230}\text{Th}$ decays per second. What is the relative concentration of the two isotopes—the ratio of $^{234}\text{U}$ to $^{230}\text{Th}$—when this equilibrium is reached?

61. II All the very heavy atoms found in the earth were created long ago by nuclear fusion reactions in a supernova, an exploding star. The debris spewed out by the supernova later coalesced to form the sun and the planets of our solar system. Nuclear physics suggests that the uranium isotopes $^{235}\text{U}$ ($t_{\frac{1}{2}} = 7.04 \times 10^9$ yr) and $^{238}\text{U}$ ($t_{\frac{1}{2}} = 4.47 \times 10^9$ yr) should have been created in roughly equal amounts. Today, 99.28% of uranium is $^{238}\text{U}$ and 0.72% is $^{235}\text{U}$. How long ago did the supernova occur?

62. II $^{235}\text{U}$ decays to $^{207}\text{Pb}$ via the decay series shown in Figure 30.16. The first decay in the chain, that of $^{235}\text{U}$, has a half-life of 7.0 $\times$ 10$^8$ years. The subsequent decays are much more rapid, so we can take this as the half-life for the decay of $^{235}\text{U}$ to $^{208}\text{Pb}$. $^{238}\text{U}$ decays, with a half-life of $4.5 \times 10^9$ years, via a similar decay series that ends in a different lead isotope, $^{206}\text{Pb}$. Again, the subsequent decays are much more rapid, so we can take this as the half-life for the decay of $^{238}\text{U}$ to $^{206}\text{Pb}$. The two uranium decay chains can be used to precisely determine the age of certain minerals that exclude lead from their crystal structure but easily incorporate uranium. When these minerals form, they contain both $^{238}\text{U}$ and $^{235}\text{U}$, but no lead. As time goes on, the isotopes of uranium decay, producing isotopes of lead. A measurement of the $^{238}\text{U}/^{206}\text{Pb}$ ratio allows a determination of the half-life—which can be checked using a measurement of the $^{235}\text{U}/^{207}\text{Pb}$ ratio. If 75% of the $^{235}\text{U}$ present in a particular rock has decayed to lead, what percent of the $^{238}\text{U}$ has decayed to lead?

63. II A 75 kg patient swallows a 30 $\mu\text{Ci}$ beta emitter with a half-life of 5.0 days. The beta particles are emitted with an average energy of 0.35 MeV. Ninety percent of the beta particles are absorbed within the patient’s body and 10% escape. What dose equivalent does the patient receive?

64. III About 12% of your body mass is carbon; some of this is radioactive $^{14}\text{C}$, a beta-emitter. If you absorb 100% of the 49 keV energy of each $^{14}\text{C}$ decay, what dose equivalent in Sv do you receive each year from the $^{14}\text{C}$ in your body?

65. III Ground beef may be irradiated with high-energy electrons from a linear accelerator to kill pathogens. In a standard treatment, 1.0 kg of beef receives 4.5 kGy of radiation in 40 s.
   a. How much energy is deposited in the beef?
   b. What is the average rate (in W) of energy deposition?
   c. Estimate the temperature increase of the beef due to this procedure. The specific heat of beef is approximately 3/4 of that of water.

66. II A 70 kg human body typically contains 140 g of potassium.

Potassium has a chemical atomic mass of 39.1 u and has three naturally occurring isotopes. One of those isotopes, $^{40}\text{K}$, is radioactive with a half-life of 1.3 billion years and a natural abundance of 0.012%. Each $^{40}\text{K}$ decay deposits, on average, 1.0 MeV of energy into the body. What yearly dose in Gy does the typical person receive from the decay of $^{40}\text{K}$ in the body?

67. III What dose in rads of gamma radiation must be absorbed by a block of ice at 0°C to transform the entire block to liquid water at 0°C?

68. III A chest x-ray uses 10 keV photons. A 60 kg person receives a 30 mrem dose from one x-ray that exposes 25% of the patient’s body. How many x-ray photons are absorbed in the patient’s body?

69. III The plutonium isotope $^{239}\text{Pu}$ has a half-life of 24,000 years and decays by the emission of a 5.2 MeV alpha particle. Plutonium is not especially dangerous if handled because the activity is low and the alpha radiation doesn’t penetrate the skin. But the tiniest speck of plutonium can cause problems if it is inhaled and lodges deep in the lungs. Let’s see why.
   a. Soot particles are roughly 1 $\mu\text{m}$ in diameter, and it is known that these particles can go deep into the lungs. How many $^{239}\text{Pu}$ atoms are in a 1.0-$\mu\text{m}-\text{diameter}$ particle of $^{239}\text{Pu}$? The density of plutonium is 19,800 kg/m$^3$.
   b. What is the activity, in Bq, of this 1.0-$\mu\text{m}-\text{diameter}$ particle?
   c. The activity of the particle is very small, but the penetrating power of alpha particles is also very small, so the damage is concentrated. The alpha particles deposit their energy in a 50-$\mu\text{m}$-diameter sphere around the plutonium particle. In one year, what is the dose equivalent in mSv to this small sphere of tissue in the lungs? Assume that the tissue density is that of water.
   d. How does the exposure to this tissue compare to the natural background exposure?

70. III Uranium is naturally present at low levels in many soils and rocks. The $^{232}\text{U}$ decay series includes the short-lived radon isotope $^{222}\text{Rn}$, with $t_{\frac{1}{2}} = 3.82$ days. Radon is a gas, and it can seep into basements. The Environmental Protection Agency recommends that homeowners take steps to remove radon if the radon activity exceeds 4 pCi per liter of air. The daughter nuclei from radon decay are of significant concern, but the radon itself does provide some exposure.
   a. How many $^{222}\text{Rn}$ atoms are there in 1.0 m$^3$ of air if the activity is 4.0 pCi/L?
   b. The range of alpha particles in air is 3 cm. Let’s model a person as a 180-cm-tall, 25-cm-diameter cylinder with a mass of 65 kg. Only decays within 3 cm of the cylinder cause exposure, and only 50% of the decays direct the alpha particle toward the person. What is the dose equivalent (in mSv) for a person who spends an entire year in a room where the activity is 4 pCi/L?
Passage Problems

Nuclear Fission

The uranium isotope $^{235}\text{U}$ can fission—break into two smaller-mass components and free neutrons—if it is struck by a free neutron. A typical reaction is

$$^{0}\text{n} + ^{235}_{92}\text{U} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{0}\text{n}$$

As you can see, the subscripts (the number of protons) and the superscripts (the number of nucleons) “balance” before and after the fission event; there is no change in the number of protons or neutrons. Significant energy is released in this reaction. If a fission event happens in a large chunk of $^{235}\text{U}$, the neutrons released may induce the fission of other $^{235}\text{U}$ atoms, resulting in a chain reaction. This is how a nuclear reactor works.

The number of neutrons required to create a stable nucleus increases with atomic number. When the heavy $^{235}\text{U}$ nucleus fissions, the lighter reaction products are thus neutron rich and are likely unstable. Many of the short-lived radioactive nuclei used in medicine are produced in fission reactions in nuclear reactors.

71. What statement can be made about the masses of atoms in the above reaction?
   A. $m(^{235}\text{U}) > m(^{141}\text{Ba}) + m(^{92}\text{Kr}) + 2m(^{0}\text{n})$
   B. $m(^{235}\text{U}) < m(^{141}\text{Ba}) + m(^{92}\text{Kr}) + 2m(^{0}\text{n})$
   C. $m(^{235}\text{U}) = m(^{141}\text{Ba}) + m(^{92}\text{Kr}) + 2m(^{0}\text{n})$
   D. $m(^{235}\text{U}) = m(^{141}\text{Ba}) + m(^{92}\text{Kr}) + 3m(^{0}\text{n})$

72. Because the decay products in the above fission reaction are neutron rich, they will likely decay by what process?
   A. Alpha decay
   B. Beta decay
   C. Gamma decay

73. $^{235}\text{U}$ is radioactive, with a long half-life of 704 million years. The decay products of a $^{235}\text{U}$ fission reaction typically have half-lives of a few minutes. This means that the decay products of a fission reaction have
   A. Much higher activity than the original uranium.
   B. Much lower activity than the original uranium.
   C. The same activity as the original uranium.

74. If a $^{238}\text{U}$ nucleus is struck by a neutron, it may absorb the neutron. The resulting nucleus then rapidly undergoes beta-minus decay. The daughter nucleus of that decay is
   A. $^{238}_{94}\text{Pu}$
   B. $^{234}_{92}\text{U}$
   C. $^{234}_{93}\text{Np}$
   D. $^{238}_{94}\text{Pu}$

Stop to Think 30.1: Three. Different isotopes of an element have different numbers of neutrons but the same number of protons. The number of electrons in a neutral atom matches the number of protons.

Stop to Think 30.2: Yes. $^{12}\text{C}$ has filled levels of protons and neutrons; the neutron we add to make $^{13}\text{C}$ will be in a higher energy level, but there is no “hole” in a lower level for it to move to, so we expect this nucleus to be stable.

Stop to Think 30.3: B. An increase of Z with no change in A occurs when a neutron changes to a proton and an electron, ejecting the electron.

Stop to Think 30.4: C. One-quarter of the atoms are left. This is one-half of one-half, or $(1/2)^2$, so two half-lives have elapsed.

Stop to Think 30.5: A. Dose equivalent is the product of dose in Gy (the same for each) and RBE (highest for alpha particles).