Chapter 17: Wave Optics
Solutions

Questions: 3, 7, 11, 15
Exercises & Problems: 4, 7, 8, 10, 12, 24, 44, 55

Q17.3: The wavelength of a light wave is 700 nm in air; this light appears red. If this wave enters a pool of water, its wavelength becomes $\lambda_{\text{air}} / n = 530$ nm. If you were swimming underwater, the light would still appear red. Given this, what property of a wave determines its color? Explain.

Q17.3. Reason: The wavelength changes ($\lambda_{\text{water}} = \lambda_{\text{air}} / n$) when the light goes from one medium to another, but the frequency stays the same. Since the color doesn’t change, then it must be associated with the frequency.
Assess: Energy of the light is directly related to the frequency as well.

Q17.7: The figure shows the viewing screen in a double-slit experiment with monochromatic light. Fringe C is the central maximum.

a. What will happen to the fringe spacing if the wavelength of the light is decreased?
b. What will happen to the fringe spacing if the spacing between the slits is decreased?
c. What will happen to the fringe spacing if the distance to the screen is decreased?
d. Suppose the wavelength of the light is 500 nm. How much farther is it from the dot on the screen in the center of fringe E to the left slit than it is from the dot to the right slit?

Q17.7. Reason: The fringe separation for the light intensity pattern of a double slit is determined by $\Delta y = \lambda L / d$. We can answer this question by inspecting this relationship.
(a) If $\lambda$ decreases, $\Delta y$ will decrease.
(b) If $d$ decreases, $\Delta y$ will increase.
(c) If $L$ decreases, $\Delta y$ will decrease.
(d) Since the dot is in the $m = 2$ bright fringe, the path length difference from the two slits is $2\lambda = 1000$ nm.
Assess: The ability to inspect a relationship and answer “what if” questions is a skill that physics will help you develop.

Q17.11: Why does light reflected from peacock feathers change color when you see the feathers at a different angle? Explain.

Q17.11. Reason: Peacock feathers consist of thin nearly parallel rods of melanin. Since they are thin, thin-film interference is a factor. Since they are very small and nearly parallel, they also act as a diffraction grating. As a result different wavelengths add up constructively at different locations and you will see different colors when viewing the feather from different angles.
Assess: This effect is common from biological structures whose size is similar to the wavelength of light.

Q17.15: Should the antireflection coating of a microscope objective lens designed for use with ultraviolet light be thinner, thicker, or the same thickness as the coating on a lens designed for visible light? Explain.

Q17.15. Reason: The wavelength of ultraviolet light is shorter than the wavelength of visible light. The formulas for the thickness of thin films, such as those used as antireflection coatings, show the thickness is proportional to the wavelength. Hence, if the wavelength is shorter, then the film needs to be thinner.

Assess: It is difficult to make antireflection coatings that work at a wide range of wavelengths, but it can be done somewhat with multiple layers of films of different indices of refraction.

P17.4: A light wave has a 670 nm wavelength in air. Its wavelength in a transparent solid is 420 nm.

a) What is the speed of light in this solid?

b) What is the light’s frequency in the solid?

P17.4. Prepare: Light rays travel in straight lines and light’s speed in a material is characterized by its refractive index, defined by Equation 17.1. Also, the frequency does not change as the wave moves from one medium to another.

Solve: (a) The refractive index is

\[
n = \frac{c}{v_{\text{solid}}} = \frac{\lambda_{\text{vac}}}{\lambda_{\text{solid}}} \Rightarrow v_{\text{solid}} = c \frac{\lambda_{\text{solid}}}{\lambda_{\text{vac}}} = (3.0 \times 10^8 \text{ m/s}) \frac{420 \text{ nm}}{670 \text{ nm}} = 1.88 \times 10^8 \text{ m/s}
\]

or \(1.9 \times 10^8 \text{ m/s}\) to two significant figures.

(b) The frequency is

\[
f = \frac{v_{\text{solid}}}{\lambda_{\text{solid}}} = \frac{1.88 \times 10^8 \text{ m/s}}{420 \text{ nm}} = 4.5 \times 10^4 \text{ Hz}
\]

Assess: We must not forget that the frequency of a wave does not change as the wave moves from one medium into another.

P17.7: Two narrow slits 50 μm apart are illuminated with light of wavelength 500 nm. What is the angle of the \(m = 2\) bright fringe in radians? In degrees?

P17.7. Prepare: Two closely spaced slits produce a double-slit interference pattern given by Equation 17.6. The interference pattern looks like the photograph of Figure 17.10. It is symmetrical with the \(m = \pm 2\) fringes on both sides of and equally distant from the central maximum.

Solve: The bright fringes occur at angles \(\theta_m\) such that

\[
d \sin \theta_m = m \lambda
\]

\(m = 0, 1, 2, 3, \ldots\)

\[
\Rightarrow \sin \theta_m = \frac{2(500 \times 10^{-9} \text{ m})}{(50 \times 10^{-6} \text{ m})} = 0.02 \Rightarrow \theta_m = 0.020 \text{ rad} = 0.020 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = 1.1^\circ
\]
Assess: We did expect the angle to be small because the wavelength of light is much smaller than the separation of the two slits.

P17.8: Light from a sodium lamp ($\lambda = 589$ nm) illuminates two narrow slits. The fringe spacing on a screen 150 cm behind the slits is 4.0 mm. What is the spacing (in mm) between the two slits?

P17.8. Prepare: Two closely spaced slits produce a double-slit interference pattern. The interference pattern looks like the photograph of Figure 17.10.

Solve: The fringe spacing is given by Equation 17.9 as follows:

$$\Delta y = \frac{\lambda L}{d} \Rightarrow d = \frac{\lambda L}{\Delta y} = \frac{(589 \times 10^{-9} \text{m})(150 \times 10^{-3} \text{m})}{4.0 \times 10^{-3} \text{m}} = 0.22 \text{ mm}$$

P17.10: A double-slit experiment is performed with light of wavelength 600 nm. The bright interference fringes are spaced 1.8 mm apart on the viewing screen. What will the fringe spacing be if the light is changed to a wavelength of 400 nm?

P17.10. Prepare: Two closely spaced slits produce a double-slit interference pattern. The interference pattern looks like the photograph of Figure 17.10.

Solve: The formula for fringe spacing is Equation 17.9.

$$\Delta y = \frac{\lambda L}{d} \Rightarrow 1.8 \times 10^{-3} \text{m} = (600 \times 10^{-9} \text{m}) \frac{L}{d} \Rightarrow \frac{L}{d} = 3000$$

The wavelength is now changed to 400 nm, and $L/d$, being a part of the experimental setup, stays the same. Applying the above equation once again,

$$\Delta y = \frac{\lambda L}{d} = (400 \times 10^{-9} \text{m})(3000) = 1.2 \text{ mm}$$

P17.12: Two narrow slits are 0.12 mm apart. Light of wavelength 550 nm illuminates the slits, causing an interference pattern on a screen 1.0 m away. Light from each slit travels to the m = 1 maximum on the right side of the central maximum. How much farther did the light from the left slit travel than the light from the right slit??

P17.12. Prepare: For a double slit, constructive interference occurs when $\Delta r = m\lambda$ where $m = 0, 1, 2, 3, \ldots$

Solve: At the first interference maximum ($m = 1$) the path difference from the two slits is one wavelength, which in this case is $\Delta r = m\lambda = (1)\lambda = 550 \text{ nm}$.

Assess: For a double slit, the first interference maximum occurs when the path difference from the two slits is one wavelength.

P17.24: Antireflection coatings can be used on the inner surfaces of eyeglasses to reduce the reflection of stray light into the eye, thus reducing eyestrain.

a) A 90-nm-thick coating is applied to the lens. What must be the coating’s index of refraction to be most effective at 480 nm? Assume that the coating’s index of refraction is less than that of the lens.
b) If the index of refracting of the coating is 1.38, what thickness should the coating be so as to be most effective at 480 nm? The thinnest possible coating is best.

**P17.24. Prepare:** First note that the coating is on the inside of the glass. As the incident light reflects off the air-coating interface there is no phase change. As the light that is transmitted through the coating reflects off the coating-air interface there is no phase change. Destructive interference for two reflective no-phase changes occurs when

\[ 2t = (m + 1/2)\frac{\lambda}{n}. \]

We will use \( m = 1 \) in order to obtain the thinnest possible coating.

**Solve:**

(a) The index of refraction for the coating is

\[ n = (m + 1/2)\frac{\lambda}{2t} = \frac{\lambda}{480\text{ nm}/(4(90\text{ nm}))} = 1.33. \]

(b) The thickness of the coating is

\[ t = \left( m + \frac{1}{2} \right) \frac{\lambda}{2n} = \frac{\lambda}{4n} = \frac{(480\text{ nm})}{(1.38)} = 87\text{ nm} \]

**Assess:** This is a reasonable value for the index of refraction thickness of the coating (see Problem 17.26).

**P17.44:** The two most prominent wavelengths in the light emitted by a hydrogen discharge lamp are 656 nm (red) and 486 nm (blue). Light from a hydrogen lamp illuminates a diffraction grating with 500 lines/mm, and the light is observed on a screen 1.50 m behind the grating. What is the distance between the first-order red and blue fringes?

**P17.44. Prepare:** A diffraction grating produces an interference pattern. The interference pattern looks like the diagram of Figure 17.12. The bright interference fringes are given by Equation 17.12 \( d \sin \theta = m\lambda; \) \( m = 0, 1, 2, 3, \ldots\) The slit spacing is \( d = 1\text{ mm}/500 = 2.00\times10^{-4}\text{ m} \) and \( m = 1 \).

**Solve:** For the red and blue light,

\[ \theta_{\text{red}} = \sin^{-1} \left( \frac{656\times10^{-9}\text{ m}}{2.00\times10^{-4}\text{ m}} \right) = 19.15^\circ \]
\[ \theta_{\text{blue}} = \sin^{-1} \left( \frac{486\times10^{-9}\text{ m}}{2.00\times10^{-4}\text{ m}} \right) = 14.06^\circ \]

The distance between the fringes, then, is \( \Delta y = y_{\text{red}} - y_{\text{blue}} \) where

\[ y_{\text{red}} = (1.5\text{ m})\tan19.15^\circ = 0.521\text{ m} \]
\[ y_{\text{blue}} = (1.5\text{ m})\tan14.06^\circ = 0.376\text{ m} \]

So, \( \Delta y = 0.145\text{ m} = 14.5\text{ cm} \).

**P17.55:** If sunlight shines straight onto a peacock feather, the feather appears bright blue when viewed from 15° on either side of the incident beam of sunlight. The blue color is due to diffraction from the from the melanin bands in the feather barbules. Blue light with a wavelength of 470 nm is diffracted at 15° by these bands (this is the first-order diffraction) while other wavelengths in the sunlight are diffracted at different angles. What is the spacing of the melanin bands in the feather?

**P17.55. Prepare:** The peacock feather is acting as a reflection grating, so we may use Equation 17.12: \( d \sin \theta = m\lambda \) with \( m = 1 \) (we are told it is first-order diffraction), \( \lambda = 470\text{ nm} \), and \( \theta = 15^\circ = 0.262\text{ rad} \).

**Solve:** Solve Equation 17.12 for \( d \).
\[ d = \frac{m\lambda}{\sin \theta_m} = \frac{(1)(470 \times 10^{-9} \text{ m})}{\sin(0.262 \text{ rad})} = 1.8 \mu\text{m} \]

**Assess:** The answer is small, but plausible for the bands on the barbules.