Center of Mass  The center of mass of a system of \( n \) particles is defined to be the point whose coordinates are given by
\[
\mathbf{x}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \mathbf{x}_i, \quad \mathbf{y}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \mathbf{y}_i, \quad \mathbf{z}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \mathbf{z}_i,
\]
(9-5)
or
\[
\mathbf{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \mathbf{r}_i,
\]
(9-8)
where \( M \) is the total mass of the system.

Newton's Second Law for a System of Particles  The motion of the center of mass of any system of particles is governed by Newton's second law for a system of particles, which is
\[
\mathbf{F}_{\text{net}} = M \mathbf{a}_{\text{com}}.
\]
(9-14)
Here \( \mathbf{F}_{\text{net}} \) is the net force of all the external forces acting on the system, \( M \) is the total mass of the system, and \( \mathbf{a}_{\text{com}} \) is the acceleration of the system's center of mass.

Linear Momentum and Newton's Second Law  For a single particle, we define a quantity \( \mathbf{p} \) called its linear momentum as
\[
\mathbf{p} = m \mathbf{v},
\]
(9-22)
and can write Newton's second law in terms of this momentum:
\[
\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt}.
\]
(9-23)
For a system of particles these relations become
\[
\mathbf{P} = M \mathbf{v}_{\text{com}} \quad \text{and} \quad \mathbf{F}_{\text{net}} = \frac{d\mathbf{P}}{dt}.
\]
(9-25, 9-27)

Collision and Impulse  Applying Newton's second law in momentum form to a particle-like body involved in a collision leads to the impulse–linear momentum theorem:
\[
\mathbf{p}_f - \mathbf{p}_i = \Delta \mathbf{p} = \mathbf{J},
\]
(9-31, 9-32)
where \( \mathbf{p}_f - \mathbf{p}_i = \Delta \mathbf{p} \) is the change in the body's linear momentum, and \( \mathbf{J} \) is the impulse due to the force \( \mathbf{F}(t) \) exerted on the body by the other body in the collision:
\[
\mathbf{J} = \int_{t_i}^{t_f} \mathbf{F}(t) \, dt.
\]
(9-30)
If \( F_{\text{avg}} \) is the average magnitude of \( \mathbf{F}(t) \) during the collision and \( \Delta t \) is the duration of the collision, then for one-dimensional motion
\[
J = F_{\text{avg}} \Delta t.
\]
(9-35)
When a steady stream of bodies, each with mass \( m \) and speed \( v \), collides with a body whose position is fixed, the average force on the fixed body is
\[
F_{\text{avg}} = -\frac{n}{\Delta t} \Delta p = -\frac{n}{\Delta t} m \Delta v,
\]
(9-37)
where \( n/\Delta t \) is the rate at which the bodies collide with the fixed body, and \( \Delta v \) is the change in velocity of each colliding body. This average force can also be written as
\[
F_{\text{avg}} = -\frac{\Delta m}{\Delta t} \Delta v,
\]
(9-40)
where \( \Delta m/\Delta t \) is the rate at which mass collides with the fixed body. In Eqs. 9-37 and 9-40, \( \Delta v = -v \) if the bodies stop upon impact and \( \Delta v = -2v \) if they bounce directly backward with no change in their speed.

Conservation of Linear Momentum  If a system is isolated so that no net external force acts on it, the linear momentum \( \mathbf{P} \) of the system remains constant:
\[
\mathbf{P} = \text{constant} \quad \text{(closed, isolated system)}.
\]
(9-42)
This can also be written as
\[
\mathbf{P}_i = \mathbf{P}_f \quad \text{(closed, isolated system)},
\]
(9-43)
where the subscripts refer to the values of \( \mathbf{P} \) at some initial time and at a later time. Equations 9-42 and 9-43 are equivalent statements of the law of conservation of linear momentum.

Inelastic Collision in One Dimension  In an inelastic collision of two bodies, the kinetic energy of the two-body system is not conserved. If the system is closed and isolated, the total linear momentum of the system must be conserved, which we can write in vector form as
\[
\mathbf{p}_{i1} + \mathbf{p}_{i2} = \mathbf{p}_{f1} + \mathbf{p}_{f2},
\]
(9-50)
where subscripts \( i \) and \( f \) refer to values just before and just after the collision, respectively.

If the motion of the bodies is along a single axis, the collision is one-dimensional and we can write Eq. 9-50 in terms of velocity components along that axis:
\[
m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}.
\]
(9-51)
If the bodies stick together, the collision is a completely inelastic collision and the bodies have the same final velocity \( V \) (because they are stuck together).

Motion of the Center of Mass  The center of mass of a closed, isolated system of two colliding bodies is not affected by a collision. In particular, the velocity \( \mathbf{v}_{\text{com}} \) of the center of mass cannot be changed by the collision.

Elastic Collisions in One Dimension  An elastic collision is a special type of collision in which the kinetic energy of a system of colliding bodies is conserved. If the system is closed and isolated, its linear momentum is also conserved. For a one-dimensional collision in which body 2 is a target and body 1 is an incoming projectile, conservation of kinetic energy and linear momentum yield the following expressions for the velocities immediately after the collision:
\[
v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i},
\]
(9-67)
and
\[
v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.
\]
(9-68)

Collisions in Two Dimensions  If two bodies collide and their motion is not along a single axis (the collision is not head-on), the collision is two-dimensional. If the two-body system is closed and isolated, the law of conservation of momentum applies to the
collision and can be written as
\[
\vec{P}_1 + \vec{P}_2 = \vec{P}_1' + \vec{P}_2'.
\] (9-77)
In component form, the law gives two equations that describe the collision (one equation for each of the two dimensions). If the collision is also elastic (a special case), the conservation of kinetic energy during the collision gives a third equation:
\[
K_1 + K_2 = K_{1'} + K_{2'}.
\] (9-78)

**Variable-Mass Systems**

In the absence of external forces a rocket accelerates at an instantaneous rate given by
\[
Rv_{rel} = Ma
\] (first rocket equation),
\[
\text{(9-87)}
\]

in which \(M\) is the rocket’s instantaneous mass (including unexpended fuel), \(R\) is the fuel consumption rate, and \(v_{rel}\) is the rocket’s exhaust speed relative to the rocket. The term \(Rv_{rel}\) is the thrust of the rocket engine. For a rocket with constant \(R\) and \(v_{rel}\), whose speed changes from \(v_i\) to \(v_f\) when its mass changes from \(M_i\) to \(M_f\),
\[
v_f - v_i = v_{rel} \ln \frac{M_i}{M_f}
\] (second rocket equation).
\[
\text{(9-88)}
\]

**Questions**

1. Figure 9-23 shows an overhead view of three particles on which external forces act. The magnitudes and directions of the forces on two of the particles are indicated. What are the magnitude and direction of the force acting on the third particle if the center of mass of the three-particle system is (a) stationary, (b) moving at a constant velocity rightward, and (c) accelerating rightward?

2. Figure 9-24 shows an overhead view of four particles of equal mass sliding over a frictionless surface at constant velocity. The directions of the velocities are indicated; their magnitudes are equal. Consider pairing the particles. Which pairs form a system with a center of mass that (a) is stationary, (b) is stationary and at the origin, and (c) passes through the origin?

3. Consider a box that explodes into two pieces while moving with a constant positive velocity along an \(x\) axis. If one piece, with mass \(m_1\), ends up with positive velocity \(\vec{v}_1\), then the second piece, with mass \(m_2\), could end up with (a) a positive velocity \(\vec{v}_2\) (Fig. 9-25a), (b) a negative velocity \(\vec{v}_2\) (Fig. 9-25b), or (c) zero velocity (Fig. 9-25c). Rank those three possible results for the second piece according to the corresponding magnitude of \(\vec{v}_2\), greatest first.

4. Figure 9-26 shows graphs of force magnitude versus time for a body involved in a collision. Rank the graphs according to the magnitude of the impulse on the body, greatest first.
7 A block slides along a frictionless floor and into a stationary second block with the same mass. Figure 9-29 shows four choices for a graph of the kinetic energies $K$ of the blocks. (a) Determine which represent physically impossible situations. Of the others, which best represents (b) an elastic collision and (c) an inelastic collision?

![Figure 9-29](image1.png)

**Fig. 9-29** Question 7.

8 Figure 9-30 shows a snapshot of block 1 as it slides along an $x$ axis on a frictionless floor, before it undergoes an elastic collision with stationary block 2. The figure also shows three possible positions of the center of mass (com) of the two-block system at the time of the snapshot. (Point $B$ is halfway between the centers of the two blocks.) Is block 1 stationary, moving forward, or moving backward after the collision if the com is located in the snapshot at (a) $A$, (b) $B$, and (c) $C$?

![Figure 9-30](image2.png)

**Fig. 9-30** Question 8.

9 Two bodies have undergone an elastic one-dimensional collision along an $x$ axis. Figure 9-31 is a graph of position versus time for those bodies and for their center of mass. (a) Were both bodies initially moving, or was one initially stationary? Which line segment corresponds to the motion of the center of mass (b) before the collision and (c) after the collision? (d) Is the mass of the body that was moving faster before the collision greater than, less than, or equal to that of the other body?

![Figure 9-31](image3.png)

**Fig. 9-31** Question 9.

10 Figure 9-32: A block on a horizontal floor is initially either stationary, sliding in the positive direction of an $x$ axis, or sliding in the negative direction of that axis. Then the block explodes into two pieces that slide along the $x$ axis. Assume the block and the two pieces form a closed, isolated system. Six choices for a graph of the momenta of the block and the pieces are given, all versus time $t$. Determine which choices represent physically impossible situations and explain why.

![Figure 9-32](image4.png)

**Fig. 9-32** Question 10.

11 Block 1 with mass $m_1$ slides along an $x$ axis across a frictionless floor and then undergoes an elastic collision with a stationary block 2 with mass $m_2$. Figure 9-33 shows a plot of position $x$ versus time $t$ of block 1 until the collision occurs at position $x_c$ and time $t_c$. In which of the lettered regions on the graph will the plot be continued (after the collision) if (a) $m_1 > m_2$ and (b) $m_1 < m_2$? (c) Along which of the numbered dashed lines will the plot be continued if $m_1 = m_2$?

![Figure 9-33](image5.png)

**Fig. 9-33** Question 11.

12 Figure 9-34 shows four graphs of position versus time for two bodies and their center of mass. The two bodies form a closed, isolated system and undergo a completely inelastic, one-dimensional collision on an $x$ axis. In graph 1, are (a) the two bodies and (b) the center of mass moving in the positive or negative direction of the $x$ axis? (c) Which graphs correspond to a physically impossible situation? Explain.

![Figure 9-34](image6.png)

**Fig. 9-34** Question 12.
CHAPTER 9 CENTER OF MASS AND LINEAR MOMENTUM

sec. 9-2 The Center of Mass

1 A 2.00 kg particle has the xy coordinates (−1.20 m, 0.500 m), and a 4.00 kg particle has the xy coordinates (0.600 m, −0.750 m). Both lie on a horizontal plane. At what (a) x and (b) y coordinates must you place a 3.00 kg particle such that the center of mass of the three-particle system has the coordinates (−0.500 m, −0.700 m)?

2 Figure 9-35 shows a three-particle system, with masses \(m_1 = 3.0\) kg, \(m_2 = 4.0\) kg, and \(m_3 = 8.0\) kg. The scales on the axes are set by \(x_s = 2.0\) m and \(y_s = 2.0\) m. What are (a) the x coordinate and (b) the y coordinate of the system’s center of mass? (c) If \(m_1\) is gradually increased, does the center of mass of the system shift toward or away from that particle, or does it remain stationary?

3 Figure 9-36 shows a slab with dimensions \(d_1 = 11.0\) cm, \(d_2 = 2.80\) cm, and \(d_3 = 13.0\) cm. Half the slab consists of aluminum (density = 2.70 g/cm\(^3\)) and half consists of iron (density = 7.85 g/cm\(^3\)). What are (a) the x coordinate, (b) the y coordinate, and (c) the z coordinate of the slab’s center of mass?

4 In Fig. 9-37, three uniform thin rods, each of length \(L = 22\) cm, form an inverted U. The vertical rods each have a mass of 14 g; the horizontal rod has a mass of 42 g. What are (a) the x coordinate and (b) the y coordinate of the system’s center of mass?

5 What are (a) the x coordinate and (b) the y coordinate of the center of mass for the uniform plate shown in Fig. 9-38 if \(L = 5.0\) cm?

6 Figure 9-39 shows a cubical box that has been constructed from uniform metal plate of negligible thickness. The box is open at the top and has edge length \(L = 40\) cm. Find (a) the x coordinate, (b) the y coordinate, and (c) the z coordinate of the center of mass of the box.

7 In the ammonia (NH\(_3\)) molecule of Fig. 9-40, three hydrogen (H) atoms form an equilateral triangle, with the center of the triangle at distance \(d = 9.40 \times 10^{-10}\) m from each hydrogen atom. The nitrogen (N) atom is at the apex of a pyramid, with the three hydrogen atoms forming the base. The nitrogen-to-hydrogen atomic mass ratio is 13.9, and the nitrogen-to-hydrogen distance is \(L = 10.14 \times 10^{-10}\) m. What are the (a) x and (b) y coordinates of the molecule’s center of mass?
A uniform soda can of mass 0.140 kg is 12.0 cm tall and filled with 0.354 kg of soda (Fig. 9-41). Then small holes are drilled in the top and bottom (with negligible loss of metal) to drain the soda. What is the height \( h \) of the com of the can and contents (a) initially and (b) after the can loses all the soda? (c) What happens to \( h \) as the soda drains out? (d) If \( x \) is the height of the remaining soda at any given instant, find \( x \) when the com reaches its lowest point.

**sec. 9-3 Newton’s Second Law for a System of Particles**

9. ILW A stone is dropped at \( t = 0 \). A second stone, with twice the mass of the first, is dropped from the same point at \( t = 100 \text{ ms} \). (a) How far below the release point is the center of mass of the two stones at \( t = 300 \text{ ms} \)? (Neither stone has yet reached the ground.) (b) How fast is the center of mass of the two-stone system moving at that time?

10. A 1000 kg automobile is at rest at a traffic signal. At the instant the light turns green, the automobile starts to move with a constant acceleration of 4.0 m/s\(^2\). At the same instant a 2000 kg truck traveling north at 41 km/h turns east and accelerates to 51 km/h. (a) What is the change in its linear momentum? (b) What is the velocity of the com as a function of time \( t \)? (c) Sketch the path taken by the com.

11. A big olive \((m = 0.50 \text{ kg})\) lies at the origin of an \( xy \) coordinate system, and a big Brazil nut \((M = 1.5 \text{ kg})\) lies at the point \((1.0, 2.0) \text{ m}\). At \( t = 0\), a force \( F_y = (2.0\hat{i} + 3.0\hat{j}) \text{ N}\) begins to act on the olive, and a force \( F_x = (-3.0\hat{i} - 2.0\hat{j}) \text{ N}\) begins to act on the nut. In unit-vector notation, what is the displacement of the center of mass of the olive–nut system at \( t = 4.0 \text{ s}\), with respect to its position at \( t = 0? \)

12. Two skaters, one with mass 65 kg and the other with mass 40 kg, stand on an ice rink holding a pole of length 10 m and negligible mass. Starting from the ends of the pole, the skaters pull themselves along the pole until they meet. How far does the 40 kg skater move?

13. SSM A shell is shot with an initial velocity \( V_0\) of 20 m/s, at an angle of \( \theta_0 = 60° \) with the horizontal. At the top of the trajectory, the shell explodes into two fragments of equal mass (Fig. 9-42). One fragment, whose speed immediately after the explosion is zero, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?

14. In Figure 9-43, two particles are launched from the origin of the coordinate system at time \( t = 0 \). Particle 1 of mass \( m_1 = 5.00 \text{ g}\) is shot directly along the \( x \) axis on a frictionless floor, with constant speed 10.0 m/s. Particle 2 of mass \( m_2 = 3.00 \text{ g}\) is shot with a velocity of magnitude 20.0 m/s, at an upward angle such that it always stays directly above particle 1. (a) What is the maximum height \( H_{\text{max}}\) reached by the com of the two-particle system? In unit-vector notation, what are the (b) velocity and (c) acceleration of the com when the com reaches \( H_{\text{max}}\)?

15. Figure 9-44 shows an arrangement with an air track, in which a cart is connected by a cord to a hanging block. The cart has mass \( m_1 = 0.600 \text{ kg}\), and its center is initially at \( xy \) coordinates \((-0.500 \text{ m}, 0 \text{ m})\); the block has mass \( m_2 = 0.400 \text{ kg}\), and its center is initially at \( \text{xy coordinates} (0, -0.100 \text{ m})\). The mass of the cord and pulley are negligible. The cart is released from rest, and both cart and block move until the cart hits the pulley. The friction between the cart and the air track and between the pulley and its axle is negligible. (a) In unit-vector notation, what is the acceleration of the center of mass of the cart–block system? (b) What is the velocity of the com as a function of time \( t \)? (c) Sketch the path taken by the com. (d) If the path is curved, determine whether it bulges upward to the right or downward to the left, and if it is straight, find the angle between it and the \( x \) axis.

16. Ricardo, of mass 80 kg, and Carmelita, who is lighter, are enjoying Lake Merced at dusk in a 30 kg canoe. When the canoe is at rest in the placid water, they exchange seats, which are 3.0 m apart and symmetrically located with respect to the canoe’s center. If the canoe moves 40 cm horizontally relative to a pier post, what is Carmelita’s mass?

17. In Fig. 9-45a, a 4.5 kg dog stands on an 18 kg flatboat at distance \( D = 6.1 \text{ m}\) from the shore. It walks 2.4 m along the boat toward shore and then stops. Assuming no friction between the boat and the water, find how far the dog is then from the shore. (Hint: See Fig. 9-45b.)

18. A 0.70 kg ball moving horizontally at 5.0 m/s strikes a vertical wall and rebounds with speed 2.0 m/s. What is the magnitude of the change in its linear momentum?

19. ILW A 2100 kg truck traveling north at 41 km/h turns east and accelerates to 51 km/h. (a) What is the change in the truck’s kinetic energy? What are the (b) magnitude and (c) direction of the change in its momentum?

20. At time \( t = 0 \), a ball is struck at ground level and sent over level ground. The momentum \( p \) versus \( t \) during the flight is
given by Fig. 9-46 ($p_0 = 6.0 \text{ kg} \cdot \text{m/s}$ and $p_1 = 4.0 \text{ kg} \cdot \text{m/s}$). At what initial angle is the ball launched? (Hint: find a solution that does not require you to read the time of the low point of the plot.)

-21 A 0.30 kg softball has a velocity of 15 m/s at an angle of 35° below the horizontal just before making contact with the bat. What is the magnitude of the change in momentum of the ball while in contact with the bat if the ball leaves with a velocity of (a) 20 m/s, vertically downward, and (b) 20 m/s, horizontally back toward the pitcher?

-22 Figure 9-47 gives an overhead view of the path taken by a 0.165 kg cue ball as it bounces from a rail of a pool table. The ball’s initial speed is 2.00 m/s, and the angle $\theta_1$ is 30.0°. The bounce reverses the $y$ component of the ball’s velocity but does not alter the $x$ component. What are (a) angle $\theta_2$, and (b) the change in the ball’s linear momentum in unit-vector notation? (The fact that the ball rolls is irrelevant to the problem.)

sec. 9-6 Collision and Impulse

-23 Until his seventies, Henri LaMothe (Fig. 9-48) excited audiences by belly-flopping from a height of 12 m into 30 cm of water. Assuming that he stops just as he reaches the bottom of the water and estimating his mass, find the magnitude of the impulse on him from the water.

-24 In February 1955, a paratrooper fell 370 m from an airplane without being able to open his chute but happened to land in snow, suffering only minor injuries. Assume that his speed at impact was 56 m/s (terminal speed), that his mass (including gear) was 85 kg, and that the magnitude of the force on him from the snow was at the survivable limit of $1.2 \times 10^5$ N. What are (a) the minimum depth of snow that would have stopped him safely and (b) the magnitude of the impulse on him from the snow?

-25 A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?

-26 In a common but dangerous prank, a chair is pulled away as a person is moving downward to sit on it, causing the victim to land hard on the floor. Suppose the victim falls by 0.50 m, the mass that moves downward is 70 kg, and the collision on the floor lasts 0.082 s. What are the magnitudes of the (a) impulse and (b) average force acting on the victim from the floor during the collision?

-27 SSM A force in the negative direction of an $x$ axis is applied for 27 ms to a 0.40 kg ball initially moving at 14 m/s in the positive direction of the axis. The force varies in magnitude, and the impulse has magnitude 32.4 N·s. What are the ball’s (a) speed and (b) direction of travel just after the force is applied? What are (c) the average magnitude of the force and (d) the direction of the impulse on the ball?

-28 In tae-kwon-do, a hand is slammed down onto a target at a speed of 13 m/s and comes to a stop during the 5.0 ms collision. Assume that during the impact the hand is independent of the arm and has a mass of 0.70 kg. What are the magnitudes of the (a) impulse and (b) average force on the hand from the target?

-29 Suppose a gangster sprays Superman’s chest with 3 g bullets at the rate of 100 bullets/min, and the speed of each bullet is 500 m/s. Suppose too that the bullets rebound straight back with no change in speed. What is the magnitude of the average force on Superman’s chest?

-30 Two average forces. A steady stream of 0.250 kg snowballs is shot perpendicularly into a wall at a speed of 4.00 m/s. Each ball sticks to the wall. Figure 9-49 gives the magnitude $F$ of the force on the wall as a function of time $t$ for two of the snowball impacts. Impacts occur with a repetition time interval $\Delta t = 50.0$ ms, last a duration time interval $\Delta t_d = 10$ ms, and produce isosceles triangles on the graph, with each impact reaching a force maximum $F_{max} = 200$ N. During each impact, what are the magnitudes of (a) the impulse and (b) the average force on the wall? (c) During a time in-
interval of many impacts, what is the magnitude of the average force on the wall?

>**31 Jumping up before the elevator hits.** After the cable snaps and the safety system fails, an elevator cab free-falls from a height of 36 m. During the collision at the bottom of the elevator shaft, a 90 kg passenger is stopped in 5.0 ms. (Assume that neither the passenger nor the cab rebounds.) What are the magnitudes of the (a) impulse and (b) average force on the passenger during the collision? If the passenger were to jump upward with a speed of 7.0 m/s relative to the cab floor just before the cab hits the bottom of the shaft, what are the magnitudes of the (c) impulse and (d) average force (assuming the same stopping time)?

>**32** A 5.0 kg toy car can move along an x axis; Fig. 9-50 gives $F_x$ of the force acting on the car, which begins at rest at time $t = 0$. The scale on the $F_x$ axis is set by $F_{p_x} = 5.0$ N. In unit-vector notation, what is $\vec{p}$ at (a) $t = 4.0$ s and (b) $t = 7.0$ s, and (c) what is $\vec{v}$ at $t = 9.0$ s?

>**33** Figure 9-51 shows a 0.300 kg baseball just before and just after it collides with a bat. Just before, the ball has velocity $\vec{v}_1$ of magnitude 12.0 m/s and angle $\theta_1 = 35.0^\circ$. Just after, it is traveling directly upward with velocity $\vec{v}_2$ of magnitude 10.0 m/s. The duration of the collision is 2.00 ms. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the impulse on the ball from the bat? What are the (c) magnitude and (d) direction of the average force on the ball from the bat?

>**34 Basilisk lizards can run across the top of a water surface** (Fig. 9-52). With each step, a lizard first slaps its foot against the water and then pushes it down into the water rapidly enough to form an air cavity around the top of the foot. To avoid having to pull the foot back up against water drag in order to complete the step, the lizard withdraws the foot before water can flow into the air cavity. If the lizard is not to sink, the average upward impulse on the lizard during this full action of slap, downward push, and withdrawal must match the downward impulse due to the gravitational force. Suppose the mass of a basilisk lizard is 90.0 g, the mass of each foot is 3.00 g, the speed of a foot as it slaps the water is 1.50 m/s, and the time for a single step is 0.600 s. (a) What is the magnitude of the impulse on the lizard during the slap? (Assume this impulse is directly upward.) (b) During the 0.600 s duration of a step, what is the downward impulse on the lizard due to the gravitational force? (c) Which action, the slap or the push, provides the primary support for the lizard, or are they approximately equal in their support?

>**35** Figure 9-53 shows an approximate plot of force magnitude $F$ versus time $t$ during the collision of a 58 g Superball with a wall. The initial velocity of the ball is 34 m/s perpendicular to the wall; the ball rebounds directly back with approximately the same speed, also perpendicular to the wall. What is $F_{\text{max}}$, the maximum magnitude of the force on the ball from the wall during the collision?

>**36** A 0.25 kg puck is initially stationary on an ice surface with negligible friction. At time $t = 0$, a horizontal force begins to move the puck. The force is given by $\vec{F} = (12.0 - 3.000t)i \text{ N}$, with $\vec{F}$ in newtons and $t$ in seconds, and it acts until its magnitude is zero. (a) What is the magnitude of the impulse on the puck from the force between $t = 0.500$ s and $t = 1.25$ s? (b) What is the change in momentum of the puck between $t = 0$ and the instant at which $\vec{F} = 0$?

>**37 SSM** A soccer player kicks a soccer ball of mass 0.45 kg that is initially at rest. The foot of the player is in contact with the ball for 3.0 $\times$ 10$^{-3}$ s, and the force of the kick is given by

$$F(t) = \left[6.0 \times 10^9 t - 2.0 \times 10^9 t^2\right] \text{N}$$

for 0 $\leq t \leq$ 3.0 $\times$ 10$^{-3}$ s, where $t$ is in seconds. Find the magnitudes of (a) the impulse on the ball due to the kick, (b) the average force on the ball from the player’s foot during the period of contact, (c) the maximum force on the ball from the player’s foot during the period of contact, and (d) the ball’s velocity immediately after it loses contact with the player’s foot.

>**38** In the overhead view of Fig. 9-54, a 300 g ball with a speed $\nu$ of 6.0 m/s strikes a wall at an angle $\theta$ of 30° and then rebounds with the...
same speed and angle. It is in contact with the wall for 10 ms. In unit-vector notation, what are (a) the impulse on the ball from the wall and (b) the average force on the wall from the ball?

sec. 9-7 Conservation of Linear Momentum

**42** An object, with mass \( m \) and speed \( v \) relative to an observer, explodes into two pieces, one three times as massive as the other; the explosion takes place in deep space. The less massive piece stops relative to the observer. How much kinetic energy is added to the system during the explosion, as measured in the observer’s reference frame?

**43** In the Olympics of 708 B.C., athletes competing in the standing long jump used handheld weights called halteres to lengthen their jumps (Fig. 9-56). The weights were swung up in front just before liftoff and then swung down and thrown backward during the flight. Suppose a modern 78 kg long jumper similarly uses two 5.50 kg halteres, throwing them horizontally to the rear at his maximum height such that their horizontal velocity is zero relative to the ground. Let his liftoff velocity be \( \vec{v} = (9.5 \hat{i} + 4.0 \hat{j}) \) m/s with or without the halteres, and assume that he lands at the liftoff level. What distance would the use of the halteres add to his range?

**44** In Fig. 9-57, a stationary block explodes into two pieces \( L \) and \( R \) that slide across a frictionless floor and then into regions with friction, where they stop. Piece \( L \), with a mass of 2.0 kg, encounters a coefficient of kinetic friction \( \mu_L = 0.40 \) and slides to a stop in distance \( d_L = 0.15 \) m. Piece \( R \) encounters a coefficient of kinetic friction \( \mu_R = 0.50 \) and slides to a stop in distance \( d_R = 0.25 \) m. What was the mass of the block?

![Fig. 9-57 Problem 44.](image)

**45** A 20.0 kg body is moving through space in the positive direction of an \( x \) axis with a speed of 200 m/s when, due to an internal explosion, it breaks into three parts. One part, with a mass of 10.0 kg, moves away from the point of explosion with a speed of 100 m/s in the positive \( y \) direction. A second part, with a mass of 4.00 kg, moves in the negative \( x \) direction with a speed of 500 m/s. (a) In unit-vector notation, what is the velocity of the third part? (b) How much energy is released in the explosion? Ignore effects due to the gravitational force.

**46** A 4.0 kg mess kit sliding on a frictionless surface explodes into two 2.0 kg parts: 3.0 m/s, due north, and 5.0 m/s, 30° north of east. What is the original speed of the mess kit?

**47** A vessel at rest at the origin of an \( xy \) coordinate system explodes into three pieces. Just after the explosion, one piece, of mass \( m \), moves with velocity \((-30 \hat{j})\) and a second piece, also of mass \( m \), moves with velocity \((-30 \hat{m}/s)\). The third piece has mass \( 3m \). Just after the explosion, what are the (a) magnitude and (b) direction of the velocity of the third piece?

**48** Particle \( A \) and particle \( B \) are held together with a compressed spring between them. When they are released, the spring pushes them apart, and they then fly off in opposite directions, free of the spring. The mass of \( A \) is 2.00 times the mass of \( B \), and the energy stored in the spring was 60 J. Assume that the spring has negligible mass and that all its stored energy is transferred to the particles. Once that transfer is complete, what are the kinetic energies of (a) particle \( A \) and (b) particle \( B \)?

sec. 9-9 Inelastic Collisions in One Dimension

**49** A bullet of mass 10 g strikes a ballistic pendulum of mass 2.0 kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet’s initial speed.

**50** A 5.20 g bullet moving at 672 m/s strikes a 700 g wooden block at rest on a frictionless surface. The bullet emerges, traveling in the same direction with its speed reduced to 428 m/s. (a) What is the resulting speed of the block? (b) What is the speed of the bullet–block center of mass?

**51** In Fig. 9-58a, a 3.50 g bullet is fired horizontally at two blocks at rest on a frictionless table. The bullet passes through block 1 (mass 1.20 kg) and embeds itself in block 2 (mass 1.80 kg). The blocks end up with speeds \( v_1 = 0.630 \) m/s and \( v_2 = 1.40 \) m/s (Fig. 9-58b). Neglecting the material removed from block 1 by the...
In Fig. 9-59, a 10 g bullet moving directly upward at 1000 m/s strikes and passes through the center of mass of a 5.0 kg block initially at rest. The bullet emerges from the block moving directly upward at 400 m/s. To what maximum height does the block then rise above its initial position?

In Fig. 9-61, a ball of mass \( m = 60 \) g is shot with speed \( v_i = 22 \) m/s into the barrel of a spring gun of mass \( M = 240 \) g initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase in thermal energy due to friction between the ball and the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b) What fraction of the initial kinetic energy of the ball is stored in the spring?

In Fig. 9-62, block 2 (mass 1.0 kg) is at rest on a frictionless surface and touching the end of an unstretched spring of spring constant 200 N/m. Block 1 (mass 2.0 kg), traveling at speed \( v_1 = 4.0 \) m/s, collides with block 2, and the two blocks stick together. When the blocks momentarily stop, by what distance is the spring compressed?

In Fig. 9-63, block 1 (mass 2.0 kg) is moving rightward at 10 m/s and block 2 (mass 5.0 kg) is moving rightward at 3.0 m/s. The surface is frictionless, and a spring with a spring constant of 1120 N/m is fixed to block 2. When the blocks collide, the compression of the spring is maximum at the instant the blocks have the same velocity. Find the maximum compression.

In Fig. 9-64, block A (mass 1.6 kg) slides into block B (mass 2.4 kg), along a frictionless surface. The directions of three velocities before (i) and after (f) the collision are indicated; the corresponding
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speeds are \( v_{Ai} = 5.5 \text{ m/s} \), \( v_{Bi} = 2.5 \) \text{ m/s} \), and \( v_{Bf} = 4.9 \text{ m/s} \). What are the (a) speed and (b) direction (left or right) of velocity \( \vec{v}_{Af} \)? (c) Is the collision elastic?

**61 SSM** A cart with mass 340 g moving on a frictionless linear air track at an initial speed of 1.2 m/s undergoes an elastic collision with an initially stationary cart of unknown mass. After the collision, the first cart continues in its original direction at 0.66 m/s. (a) What is the mass of the second cart? (b) What is its speed after impact? (c) What is the speed of the two-cart center of mass?

**62** Two titanium spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is 300 g, remains at rest. (a) What is the mass of the other sphere? (b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2.00 m/s?

**63** Block 1 of mass \( m_1 \) slides along a frictionless floor and into a one-dimensional elastic collision with stationary block 2 of mass \( m_2 = 3m_1 \). Prior to the collision, the center of mass of the two-block system had a speed of 3.00 m/s. Afterward, what are the speeds of (a) the center of mass and (b) block 2?

**64** A steel ball of mass 0.500 kg is fastened to a cord that is 70.0 cm long and fixed at the far end. The ball is then released when the cord is horizontal (Fig. 9-65). At the bottom of its path, the ball strikes a 2.50 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.

**65 SSM** A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the original direction but with one-fourth of its original speed. (a) What is the mass of the other body? (b) What is the speed of the two-body center of mass if the initial speed of the 2.0 kg body was 4.0 m/s?

**66** Block 1, with mass \( m_1 \) and speed 4.0 m/s, slides along an \( x \) axis on a frictionless floor and then undergoes a one-dimensional elastic collision with stationary block 2, with mass \( m_2 = 0.4m_1 \). The two blocks then slide into a region where the coefficient of kinetic friction is 0.50; there they stop. How far into that region do (a) block 1 and (b) block 2 slide?

**67** In Fig. 9-66, particle 1 of mass \( m_1 = 0.30 \) kg slides rightward along an \( x \) axis on a frictionless floor with a speed of 2.0 m/s. When it reaches \( x = 0 \), it undergoes a one-dimensional elastic collision with stationary particle 2 of mass \( m_2 = 0.40 \) kg. When particle 2 then reaches a wall at \( x = 70 \) cm, it bounces from the wall with no loss of speed. At what position on the \( x \) axis does particle 2 then collide with particle 1?

**68** In Fig. 9-67, block 1 of mass \( m_1 \) slides from rest along a frictionless ramp from height \( h = 2.50 \text{ m} \) and then collides with stationary block 2, which has mass \( m_2 = 2.00m_1 \). After the collision, block 2 slides into a region where the coefficient of kinetic friction \( \mu_k \) is 0.500 and comes to a stop in distance \( d \) within that region. What is the value of distance \( d \) if the collision is (a) elastic and (b) completely inelastic?

**69** A small ball of mass \( m \) is aligned above a larger ball of mass \( M = 0.63 \text{ kg} \) (with a slight separation, as with the baseball and basketball of Fig. 9-68a), and the two are dropped simultaneously from a height of \( h = 1.8 \text{ m} \). (Assume the radius of each ball is negligible relative to \( h \).) (a) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of \( m \) results in the larger ball stopping when it collides with the small ball? (b) What height does the small ball then reach (Fig. 9-68b)?

**70** In Fig. 9-69, puck 1 of mass \( m_1 = 0.20 \text{ kg} \) is sent sliding across a frictionless lab bench, to undergo a one-dimensional elastic collision with stationary puck 2. Puck 2 then slides off the bench and lands a distance \( d \) from the base of the bench. Puck 1 rebounds from the collision and slides off the opposite edge of the bench, landing a distance \( 2d \) from the base of the bench. What is the mass of puck 2? (Hint: Be careful with signs.)

**sec. 9.11 Collisions in Two Dimensions

**71 ILW** In Fig. 9-21, projectile particle 1 is an alpha particle and target particle 2 is an oxygen nucleus. The alpha particle is scattered at angle \( \theta_1 = 64.0^\circ \) and the oxygen nucleus recoils with speed \( 1.20 \times 10^6 \text{ m/s} \) and at angle \( \theta_2 = 51.0^\circ \). In atomic mass units, the mass of the alpha particle is 4.00 \( u \) and the mass of the oxygen nucleus is 16.0 \( u \). What are the (a) final and (b) initial speeds of the alpha particle?

**72** Ball \( B \), moving in the positive direction of an \( x \) axis at speed \( v \), collides with stationary ball \( A \) at the origin. \( A \) and \( B \) have different masses. After the collision, \( B \) moves in the negative direction of the \( y \) axis at speed \( v/2 \). (a) In what direction does \( A \) move? (b)
Show that the speed of A cannot be determined from the given information.

**Problem 73** After a completely inelastic collision, two objects of the same mass and same initial speed move away together at half their initial speed. Find the angle between the initial velocities of the objects.

**Problem 74** Two 2.0 kg bodies, A and B, collide. The velocities before the collision are \( v_A = (15\hat{i} + 30\hat{j}) \) m/s and \( v_B = (-10\hat{i} + 5.0\hat{j}) \) m/s. After the collision, \( v'_A = (-5.0\hat{i} + 20\hat{j}) \) m/s. What are (a) the final velocity of B and (b) the change in the total kinetic energy (including sign)?

**Problem 75** A projectile proton with a speed of 500 m/s collides elastically with a target proton initially at rest. The two protons then move along perpendicular paths, with the projectile path at 60° from the original direction. After the collision, what are the speeds of (a) the target proton and (b) the projectile proton?

**Section 9-12 Systems with Varying Mass: A Rocket**

**Problem 76** A 6090 kg space probe moving nose-first toward Jupiter at 105 m/s relative to the Sun fires its rocket engine, ejecting 80.0 kg of exhaust at a speed of 253 m/s relative to the space probe. What is the final velocity of the probe?

**Problem 77 SSM** In Fig. 9-70, two long barges are moving in the same direction in still water, one with a speed of 10 km/h and the other with a speed of 20 km/h. While they are passing each other, coal is shoveled from the slower to the faster one at a rate of 1000 kg/min. While they are passing each other, coal is shoveled from the slower to the faster one at a rate of 1000 kg/min.

**Problem 78** Consider a rocket that is in deep space and at rest relative to an inertial reference frame. The rocket’s engine is to be fired for a certain interval. What must be the rocket’s mass ratio (ratio of initial to final mass) over that interval if the rocket’s original speed relative to the inertial frame is to be equal to (a) the exhaust speed (speed of the exhaust products relative to the rocket) and (b) 2.0 times the exhaust speed?

**Problem 79 SSM ILW** A rocket that is in deep space and initially at rest relative to an inertial reference frame has a mass of \( 2.55 \times 10^5 \) kg, of which \( 1.81 \times 10^5 \) kg is fuel. The rocket engine is then fired for 250 s while fuel is consumed at the rate of 480 kg/s. The speed of the exhaust products relative to the rocket is 3.27 km/s. (a) What is the rocket’s thrust? After the 250 s firing, what are (b) the mass and (c) the speed of the rocket?

**Additional Problems**

**Problem 80** An object is tracked by a radar station and determined to have a position vector given by \( r = (3500 - 160t)i + 2700j + 300k \), with \( r \) in meters and \( t \) in seconds. The radar station’s x-axis points east, its y-axis north, and its z-axis vertically up. If the object is a 250 kg meteorological missile, what are (a) its linear momentum, (b) its direction of motion, and (c) the net force on it?

**Problem 81** The last stage of a rocket, which is traveling at a speed of 7600 m/s, consists of two parts that are clamped together: a rocket case with a mass of 290.0 kg and a payload capsule with a mass of 150.0 kg. When the clamp is released, a compressed spring causes the two parts to separate with a relative speed of 910.0 m/s. What are the speeds of (a) the rocket case and (b) the payload after they have separated? Assume that all velocities are along the same line. Find the total kinetic energy of the two parts (c) before and (d) after they separate. (e) Account for the difference.

**Problem 82 Pancake collapse of a tall building**. In the section of a tall building shown in Fig. 9-71, the infrastructure of any given floor \( K \) must support the weight \( W \) of all higher floors. Normally the infrastructure is constructed with a safety factor \( s \) so that it can withstand an even greater downward force of \( sW \). If, however, the support columns between \( K \) and \( L \) suddenly collapse and allow the higher floors to free-fall together onto floor \( K \) (Fig. 9-71b), the force in the collision can exceed \( sW \) and, after a brief pause, cause \( K \) to collapse onto floor \( J \), which collapses on floor \( I \), and so on until the ground is reached. Assume that the floors are separated by \( d = 4.0 \text{ m} \) and have the same mass. Also assume that when the floors above \( K \) free-fall onto \( K \), the collision lasts 1.5 ms. Under these simplified conditions, what value must the safety factor \( s \) exceed to prevent pancake collapse of the building?

**Problem 83** “Relative” is an important word. In Fig. 9-72, block \( L \) of mass \( m_L = 1.00 \text{ kg} \) and block \( R \) of mass \( m_R = 0.500 \text{ kg} \) are held in place with a compressed spring between them. When the blocks are released, the spring sends them sliding across a frictionless floor. (The spring has negligible mass and falls to the floor after the
blocks leave it.) (a) If the spring gives block \( L \) a release speed of 1.20 m/s relative to the floor, how far does block \( R \) travel in the next 0.800 s? (b) If, instead, the spring gives block \( L \) a release speed of 1.20 m/s relative to the velocity that the spring gives block \( R \), how far does block \( R \) travel in the next 0.800 s?

84 Figure 9-73 shows an overhead view of two particles sliding at constant velocity over a frictionless surface. The particles have the same mass and the same initial speed \( v = 4.00 \text{ m/s} \), and they collide where their paths intersect. An \( x \) axis is arranged to bisect the angle between their incoming paths, such that \( \theta = 40.0^\circ \). The region to the right of the collision is divided into four lettered sections by the \( x \) axis and four numbered dashed lines. In what region or along what line do the particles travel if the collision is (a) completely inelastic, (b) elastic, and (c) inelastic? What are their final speeds if the collision is (d) completely inelastic and (e) elastic?

85 Speed deamplifier. In Fig. 9-74, block 1 of mass \( m_1 \) slides along an \( x \) axis on a frictionless floor at speed \( 4.00 \text{ m/s} \). Then it undergoes a one-dimensional elastic collision with stationary block 2 of mass \( m_2 = 2.00m_1 \). Next, block 2 undergoes a one-dimensional elastic collision with stationary block 3 of mass \( m_3 = 2.00m_2 \). (a) What then is the speed of block 3? (b) Are (b) the speed, (c) the kinetic energy, and (d) the momentum of block 3 greater than, less than, or the same as the initial values for block 1?

Fig. 9-74  Problem 85.

86 Speed amplifier. In Fig. 9-75, block 1 of mass \( m_1 \) slides along an \( x \) axis on a frictionless floor with a speed of \( v_{i1} = 4.00 \text{ m/s} \). Then it undergoes a one-dimensional elastic collision with stationary block 2 of mass \( m_2 = 0.500m_1 \). Next, block 2 undergoes a one-dimensional elastic collision with stationary block 3 of mass \( m_3 = 0.500m_2 \). (a) What then is the speed of block 3? (b) Are (b) the speed, (c) the kinetic energy, and (d) the momentum of block 3 greater than, less than, or the same as the initial values for block 1?

Fig. 9-75  Problem 86.

87 A ball having a mass of 150 g strikes a wall with a speed of 5.2 m/s and rebounds with only 50% of its initial kinetic energy. (a) What is the speed of the ball immediately after rebounding? (b) What is the magnitude of the impulse on the wall for the ball? (c) If the ball is in contact with the wall for 7.6 ms, what is the magnitude of the average force on the ball from the wall during this time interval?

88 A spacecraft is separated into two parts by detonating the explosive bolts that hold them together. The masses of the parts are 1200 kg and 1800 kg; the magnitude of the impulse on each part from the bolts is 300 N·s. With what relative speed do the two parts separate because of the detonation?

89 SSM A 1400 kg car moving at 5.3 m/s is initially traveling north along the positive direction of a \( y \) axis. After completing a 90° right-hand turn in 4.6 s, the inattentive operator drives into a tree, which stops the car in 350 ms. In unit-vector notation, what is the impulse on the car (a) due to the turn and (b) due to the collision? What is the magnitude of the average force that acts on the car during the turn and (d) during the collision? (c) What is the direction of the average force during the turn?

90 ILW A certain radioactive (parent) nucleus transforms to a different (daughter) nucleus by emitting an electron and a neutrino. The parent nucleus was at rest at the origin of an \( xy \) coordinate system. The electron moves away from the origin with linear momentum \((-1.2 \times 10^{-25} \text{ kg} \cdot \text{m/s})\); the neutrino moves away from the origin with linear momentum \((-6.4 \times 10^{-23} \text{ kg} \cdot \text{m/s})\). What are the (a) magnitude and (b) direction of the linear momentum of the daughter nucleus? (c) If the daughter nucleus has a mass of 5.8 × 10^{-26} \text{ kg}, what is its kinetic energy?

91 A 75 kg man rides on a 39 kg cart moving at a velocity of 2.3 m/s. He jumps off with zero horizontal velocity relative to the ground. What is the resulting change in the cart’s velocity, including sign?

92 Two blocks of masses 1.0 kg and 3.0 kg are connected by a spring and rest on a frictionless surface. They are given velocities toward each other such that the 1.0 kg block travels initially at 1.7 m/s toward the center of mass, which remains at rest. What is the initial speed of the other block?

93 SSM A railroad freight car of mass 3.18 × 10^4 \text{ kg} collides with a stationary caboose car. They couple together, and 27.0% of the initial kinetic energy is transferred to thermal energy, sound, vibrations, and so on. Find the mass of the caboose.

94 An old Chrysler with mass 2400 kg is moving along a straight stretch of road at 80 km/h. It is followed by a Ford with mass 1600 kg moving at 60 km/h. How fast is the center of mass of the two cars moving?

95 SSM In the arrangement of Fig. 9-21, billiard ball 1 moving at a speed of 2.2 m/s undergoes a glancing collision with identical billiard ball 2 that is at rest. After the collision, ball 2 moves at speed 1.1 m/s, at an angle of \( \theta_2 = 60^\circ \). What are (a) the magnitude and (b) the direction of the velocity of ball 1 after the collision? (c) Do the given data suggest the collision is elastic or inelastic?

96 A rocket is moving away from the solar system at a speed of \( 6.0 \times 10^3 \text{ m/s} \). It fires its engine, which ejects exhaust with a speed of \( 3.0 \times 10^3 \text{ m/s} \) relative to the rocket. The mass of the rocket at this time is \( 4.0 \times 10^4 \text{ kg} \), and its acceleration is \( 2.0 \text{ m/s}^2 \). (a) What is the thrust of the engine? (b) At what rate, in kilograms per second, is exhaust ejected during the firing?

97 The three balls in the overhead view of Fig. 9-76 are identical. Balls 2 and 3 touch each other and are aligned perpendicular to the path of ball 1. The velocity of ball 1 has magnitude \( v_0 = 10 \text{ m/s} \) and is directed at the contact point of balls 1 and 2. After the collision, what are the (a) speed and (b) direction of the velocity of ball 2, the (c) speed and (d) direction of the velocity of ball 3, and the (e) speed and (f) direction of the velocity of ball 1? (Hint: With friction absent, each impulse is directed along the line connecting the centers of the colliding balls, normal to the colliding surfaces.)
98 A 0.15 kg ball hits a wall with a velocity of \( (5.00 \, \text{m/s})\hat{i} + (6.50 \, \text{m/s})\hat{j} \). It rebounds from the wall with a velocity of \( (2.00 \, \text{m/s})\hat{i} + (3.50 \, \text{m/s})\hat{j} + (-3.20 \, \text{m/s})\hat{k} \). What are (a) the change in the ball’s momentum, (b) the impulse on the ball, and (c) the impulse on the wall?

99 In Fig. 9-77, two identical containers of sugar are connected by a cord that passes over a frictionless pulley. The cord and pulley have negligible mass, each container and its sugar together have a mass of 500 g, the centers of the containers are separated by 50 mm, and the containers are held fixed at the same height. What is the horizontal distance between the center of container 1 and the center of mass of the two-container system (a) initially and (b) after 20 g of sugar is transferred from container 1 to container 2? After the transfer and after the containers are released, (c) in what direction and (d) at what acceleration magnitude does the center of mass move?

100 In a game of pool, the cue ball strikes another ball of the same mass and initially at rest. After the collision, the cue ball moves at 3.50 m/s along a line making an angle of 22.0° with the cue ball’s original direction of motion, and the second ball has a speed of 2.00 m/s. Find (a) the angle between the direction of motion of the second ball and the original direction of motion of the cue ball and (b) the original speed of the cue ball. (c) Is kinetic energy (of the centers of mass, don’t consider the rotation) conserved?

101 In Fig. 9-78, a 3.2 kg box of running shoes slides on a horizontal frictionless table and collides with a 2.0 kg box of ballet slippers initially at rest on the edge of the table, at height \( h = 0.40 \, \text{m} \). The speed of the 3.2 kg box is 3.0 m/s just before the collision. If the two boxes stick together because of packing tape on their sides, what is their kinetic energy just before they strike the floor?

102 In Fig. 9-79, an 80 kg man is on a ladder hanging from a balloon that has a total mass of 320 kg (including the basket passenger). The balloon is initially stationary relative to the ground. If the man on the ladder begins to climb at 2.5 m/s relative to the ladder, (a) in what direction and (b) at what speed does the balloon move? (c) If the man then stops climbing, what is the speed of the balloon?

103 In Fig. 9-80, block 1 of mass \( m_1 = 6.6 \, \text{kg} \) is at rest on a long frictionless table that is up against a wall. Block 2 of mass \( m_2 \) is placed between block 1 and the wall and sent sliding to the left, toward block 1, with constant speed \( v_{2i} \). Find the value of \( m_2 \) for which both blocks move with the same velocity after block 2 has collided once with block 1 and once with the wall. Assume all collisions are elastic (the collision with the wall does not change the speed of block 2).

104 The script for an action movie calls for a small race car (of mass 1500 kg and length 3.0 m) to accelerate along a flattop boat (of mass 4000 kg and length 14 m), from one end of the boat to the other, where the car will then jump the gap between the boat and a somewhat lower dock. You are the technical advisor for the movie. The boat will initially touch the dock, as in Fig. 9-81; the boat can slide through the water without significant resistance; both the car and the boat can be approximated as uniform in their mass distribution. Determine what the width of the gap will be just as the car is about to make the jump.

105 SSM A 3.0 kg object moving at 8.0 m/s in the positive direction of an \( x \) axis has a one-dimensional elastic collision with an object of mass \( M \), initially at rest. After the collision the object of mass \( M \) has a velocity of 6.0 m/s in the positive direction of the axis. What is mass \( M \)?

106 A 2140 kg railroad flatcar, which can move with negligible friction, is motionless next to a platform. A 242 kg sumo wrestler runs at 5.3 m/s along the platform (parallel to the track) and then jumps onto the flatcar. What is the speed of the flatcar if he then (a) stands on it, (b) runs at 5.3 m/s relative to it in his original direction, and (c) turns and runs at 5.3 m/s relative to the flatcar opposite his original direction?

107 SSM A 6100 kg rocket is set for vertical firing from the ground. If the exhaust speed is 1200 m/s, how much mass must be ejected each second if the thrust (a) is to equal the magnitude of the gravitational force on the rocket and (b) is to give the rocket an initial upward acceleration of 21 m/s²?

108 A 500.0 kg module is attached to a 400.0 kg shuttle craft, which moves at 1000 m/s relative to the stationary main spaceship. Then a small explosion sends the module backward with speed 100.0 m/s relative to the new speed of the shuttle craft. As measured by someone on the main spaceship, by what fraction did the kinetic energy of the module and shuttle craft increase because of the explosion?

109 SSM (a) How far is the center of mass of the Earth–Moon system from the center of Earth? (Appendix C gives the masses of Earth and the Moon and the distance between the two.) (b) What percentage of Earth’s radius is that distance?

110 A 140 g ball with speed 7.8 m/s strikes a wall perpendicularly and rebounds in the opposite direction with the same speed. The
collision lasts 3.80 ms. What are the magnitudes of the (a) impulse and (b) average force on the wall from the ball?

111 SSM A rocket sled with a mass of 2900 kg moves at 250 m/s on a set of rails. At a certain point, a scoop on the sled dips into a trough of water located between the tracks and scoops water into an empty tank on the sled. By applying the principle of conservation of linear momentum, determine the speed of the sled after 920 kg of water has been scooped up. Ignore any retarding force on the scoop.

112 SSM A pellet gun fires ten 2.0 g pellets per second with a speed of 500 m/s. The pellets are stopped by a rigid wall. What are (a) the magnitude of the momentum of each pellet, (b) the kinetic energy of each pellet, and (c) the magnitude of the average force on the wall from the stream of pellets? (d) If each pellet is in contact with the wall for 0.60 ms, what is the magnitude of the average force on each pellet during contact? (e) Why is this average force so different from the average force calculated in (c)?

113 A railroad car moves under a grain elevator at a constant speed of 3.20 m/s. Grain drops into the car at the rate of 540 kg/min. What is the magnitude of the force needed to keep the car moving at constant speed if friction is negligible?

114 Figure 9-82 shows a uniform square plate of edge length $6d = 6.0$ m from which a square piece of edge length $2d$ has been removed. What are (a) the $x$ coordinate and (b) the $y$ coordinate of the center of mass of the remaining piece?

![Fig. 9-82](image)

115 SSM At time $t = 0$, force $\vec{F}_1 = (-4.0\, \text{i} \, \text{N}) + (5.0\, \text{j} \, \text{N})$ acts on an initially stationary particle of mass $2.00 \times 10^{-3}$ kg and force $\vec{F}_2 = (2.0\, \text{i} \, \text{N}) - (4.0\, \text{j} \, \text{N})$ acts on an initially stationary particle of mass $4.00 \times 10^{-3}$ kg. From time $t = 0$ to $t = 2.00$ ms, what are the (a) magnitude and (b) angle (relative to the positive direction of the $x$ axis) of the displacement of the center of mass of the two-particle system? (c) What is the kinetic energy of the center of mass at $t = 2.00$ ms?

116 Two particles $P$ and $Q$ are released from rest 1.0 m apart. $P$ has a mass of 0.10 kg, and $Q$ a mass of 0.30 kg. $P$ and $Q$ attract each other with a constant force of $1.0 \times 10^{-2}$ N. No external forces act on the system. (a) What is the speed of the center of mass of $P$ and $Q$ when the separation is 0.50 m? (b) At what distance from $P$’s original position do the particles collide?

117 A collision occurs between a 2.00 kg particle traveling with velocity $\vec{v}'_1 = (-4.00 \, \text{m/s})$ + $(5.00 \, \text{m/s})$ and a 4.00 kg particle traveling with velocity $\vec{v}'_2 = (6.00 \, \text{m/s})$ + $(-2.00 \, \text{m/s})$. The collision connects the two particles. What then is their velocity in (a) unit-vector notation and as a (b) magnitude and (c) angle?

118 In the two-sphere arrangement of Fig. 9-20, assume that sphere 1 has a mass of 50 g and an initial height of $h_1 = 9.0$ cm, and that sphere 2 has a mass of 85 g. After sphere 1 is released and collides elastically with sphere 2, what height is reached by (a) sphere 1 and (b) sphere 2? After the next (elastic) collision, what height is reached by (c) sphere 1 and (d) sphere 2? (Hint: Do not use rounded-off values.)

119 In Fig. 9-83, block 1 slides along an $x$ axis on a frictionless floor with a speed of 0.75 m/s. When it reaches stationary block 2, the two blocks undergo an elastic collision. The following table gives the mass and length of the (uniform) blocks and also the locations of their centers at time $t = 0$. Where is the center of mass of the two-block system located (a) at $t = 0$, (b) when the two blocks first touch, and (c) at $t = 4.00$ s?

<table>
<thead>
<tr>
<th>Block</th>
<th>Mass (kg)</th>
<th>Length (cm)</th>
<th>Center at $t = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>5.0</td>
<td>$x = -1.50$ m</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>6.0</td>
<td>$x = 0$</td>
</tr>
</tbody>
</table>

![Fig. 9-83](image)

120 A body is traveling at 2.0 m/s along the positive direction of an $x$ axis; no net force acts on the body. An internal explosion separates the body into two parts, each of 4.0 kg, and increases the total kinetic energy by 16 J. The forward part continues to move in the original direction of motion. What are the speeds of (a) the rear part and (b) the forward part?

121 An electron undergoes a one-dimensional elastic collision with an initially stationary hydrogen atom. What percentage of the electron’s initial kinetic energy is transferred to kinetic energy of the hydrogen atom? (The mass of the hydrogen atom is 1840 times the mass of the electron.)

122 A man (weighing 915 N) stands on a long railroad flatcar (weighing 2415 N) as it rolls at 18.2 m/s in the positive direction of an $x$ axis, with negligible friction. Then the man runs along the flatcar in the negative $x$ direction at 4.00 m/s relative to the flatcar. What is the resulting increase in the speed of the flatcar?