**Physics 4A**

**Error Analysis or Experimental Uncertainty**

- **Error**
  - In Physics language, error does not mean a mistake.
  - **Error** refers to the difference between an observed or measured result and the true value.
  - No measurement, no matter how carefully made, can be completely free of errors.
  - There are two types of experimental error: **systematic errors** and **random errors**.

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**Systematic Error**

- **Systematic errors** refer to errors that result from mistakes inherent in a particular apparatus (i.e., bad equipment).
  - **A slow clock**
  - **A short meter stick**

- When systematic errors are present, the measured results are always shifted away from the true results in a given direction (i.e., too high or too low).

- **Systematic Error**
  - There is some control over systematic errors (i.e., using better equipment).
  - Systematic errors are usually very hard to detect and extremely hard to evaluate.
  - In lab, we will assume that all forms of systematic error have been identified and minimized.
**Random Error**

random (statistical) errors $\Rightarrow$ experimental uncertainties associated with random fluctuations of any measurement apparatus

$\Rightarrow$ random errors usually result from instrumental uncertainties and/or statistical fluctuations

$\Rightarrow$ random errors can be reduced by repeating the experiment many times and averaging the results

$\Rightarrow$ random errors can be treated mathematically using statistics

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**Mean, $\sigma$, and SE**

$\Rightarrow$ in lab, we will be using three quantities to express the experimental result of a set of measurements of some quantity $x$:

Mean ($\bar{x}$) $\Rightarrow$ the average of all measurements (gives your best estimate of the value of the result)

Standard deviation ($\sigma_x$) $\Rightarrow$ indicates how spread out your different measurements were

Standard Error (SE) $\Rightarrow$ gives the best measure of the uncertainty in the mean

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**Mean (Average)**

$\Rightarrow$ Suppose that I gave the same test to two different classes. If I wanted to know how well each class did, what number would I use to compare the two classes?

$\Rightarrow$ I would probably use the average or mean from each class.

$\Rightarrow$ However, there is a major problem with only looking at the average. What is it???
Deviation from the Mean

⇒ What we would like is some indication of how spread out the test scores are from each class.

⇒ One possibility would be to calculate the average deviation, where the deviation \( d \) for each value is defined as:

\[
\text{deviation} = \text{value} - \text{average}
\]

⇒ However, there is a major problem with calculating the average deviation. What is it???

⇒ The average deviation is always zero!!!

<table>
<thead>
<tr>
<th>Test Scores</th>
<th>Deviation</th>
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<tbody>
<tr>
<td>79</td>
<td>4</td>
</tr>
<tr>
<td>78</td>
<td>3</td>
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<tr>
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<td>-1</td>
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<tr>
<td>70</td>
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<table>
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<th>Test Scores</th>
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<tr>
<td>57</td>
<td>-18</td>
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</table>

Standard Deviation

⇒ The standard deviation \( \sigma \) is the quantity used to describe how spread out the values are in a given set of data.

⇒ The standard deviation \( \sigma \) is defined as follows:

\[
\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (d_i)^2} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}
\]

1) First, find the mean (average) of all values: \( \bar{x} \)

2) Then, for each value, find the deviation of that value from the mean and square it:

\( (x_i - \bar{x})^2 \)
Standard Deviation

\[ \sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (d_i)^2} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{x})^2} \]

3) Then, find the average of all of the deviations squared (dividing by N-1 instead of N):

\[ \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{x})^2 \]

4) Finally, take the square root of the average of the deviations squared:

\[ \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{x})^2} \]

The sum of all deviations squared is 58

\[ 58 / (10-1) = \frac{58}{9} = 6.44 \]

\[ \sqrt{6.4} = 2.5 \]

⇒ The standard deviation for class 1 is \( \sigma = 2.5 \)

Standard Deviation

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<th>Test Scores</th>
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<th>Deviation^2</th>
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</tr>
<tr>
<td>70</td>
<td>-5</td>
<td>25</td>
</tr>
</tbody>
</table>

⇒ The standard deviation for class 2 is \( \sigma = 17 \)

Standard Deviation

The sum of all deviations squared is 2560

\[ 2560 / (10-1) = 284 \]

\[ \sqrt{284} = 17 \]

⇒ The standard deviation \( \sigma \) gives us an idea of how spread out the values in a given data set are.

small \( \sigma \) ⇒ values are close together
large \( \sigma \) ⇒ values are spread out

⇒ Statistically, ~68% (95%) of the values in a data set should fall within 1\( \sigma \) (2\( \sigma \)) of the mean. This means that ~68% of the values should fall within the range: mean – \( \sigma \) and mean + \( \sigma \) and ~95% of the values should fall within mean ± 2\( \sigma \).
Standard Deviation

(Class 1)

average = 75 \hspace{0.5cm} \sigma = 2.5

\Rightarrow \text{~68% of the value should fall within the range: } 75 - 2.5 \text{ and } 75 + 2.5

\Rightarrow \text{~68% of the values should fall between 72.5 and 77.5}

(Class 2)

average = 75 \hspace{0.5cm} \sigma = 17

\Rightarrow \text{~68% of the value should fall within the range: } 75 - 17 \text{ and } 75 + 17

\Rightarrow \text{~68% of the values should fall between 58 and 92}

\Rightarrow \text{Statistically, \sim 68% of the values in a data set should fall within } 1\sigma \text{ of the average, \sim 95% of the values in a data set should fall within } 2\sigma \text{ of the average, and \sim 99% of the values in a data set should fall within } 3\sigma \text{ of the average.}

Standard Error

\Rightarrow \text{The standard deviation of a set a means (i.e. from all lab groups) is called the standard error.}

\Rightarrow \text{The standard error gives the best estimate of the uncertainty of the mean from any individual lab group.}

\Rightarrow \text{Statistical theory tells us that even when we have only a single set of measurements, we can still estimate the uncertainty in the mean.}
**Standard Error**
⇒ For a single set of measurements, the standard error is defined as:

\[ SE = \frac{\sigma}{\sqrt{N}} \]

\( \sigma = \text{standard deviation} \)
\( N = \text{number of measurements} \)

⇒ In lab, we will report the result of a set of measurements of a quantity \( x \) as

\[ x = \bar{x} \pm 2SE \]

**Are Results Consistent with Theory?**
⇒ In 4A, we will use the following criteria to determine whether an experimental measurement of \( x \) is consistent with the theoretical prediction \( x_{\text{thy}} \):

\[ \bar{x} - 2SE \leq x_{\text{thy}} \leq \bar{x} + 2SE \]

then the experiment result and the theoretical prediction are consistent.

⇒ That is, if the theoretical result falls within two standard errors of the mean, the experimental result is consistent with the model.