Math 152  
Final Exam Review

Below are problems that will help you prepare for the Final Exam. You may bring a one-sided 8 ½ x 11 sheet with any formulas, procedures, or helpful hints you may need for the exam. Also, be sure to bring a scientific calculator (non-graphing), straightedge, pencil, eraser, and a rested, well-prepared mind.

1. Graph the functions \( f(x) = 2^x \) and \( g(x) = 2^{x-1} \) using integer values for \( x \) ranging from -2 to 2.

2. a) Use the formula \( A = P \left(1 + \frac{r}{n}\right)^{nt} \) to find the amount of money accumulated if $5000 is invested at a rate of 5.5% for 5 years and compounded semiannually.
   b) Use the formula \( A = Pe^{rt} \) to find the amount of money accumulated if $14,000 is invested at a rate of 6.85% for 10 years and compounded continuously.

3. Given \( f(x) = x^2 + 3 \) and \( g(x) = 4x - 1 \) find
   a) \( (f \circ g)(x) \)
   b) \( (g \circ f)(x) \)
   c) \( (f \circ g)(3) \)

4. Given \( f(x) = 2 - 5x \) and \( g(x) = \frac{2 - x}{5} \), determine if the functions are inverses of each other.

5. Assume that \( f(x) = \sqrt{x+2} \) is one-to-one over its domain, \( \{x | x \geq -2\} \) (Therefore, it has an inverse).
   a) Find \( f^{-1}(x) \).
   b) Confirm that \( f \) and \( f^{-1} \) are inverses by showing that \( f(f^{-1}(x)) = f^{-1}(f(x)) = x \).

6. Write the equation in equivalent exponential form. \( 3 = \log_4 x \)

7. Write the equation in equivalent logarithmic form. \( b^4 = 625 \)

8. Evaluate without using a calculator.
   a) \( \log_3 \sqrt{3} \)
   b) \( \ln \frac{1}{e} \)
   c) \( \log 1000 \)

9. Use inverse logarithmic properties to simplify the expressions.
10. Expand the logarithmic expressions.
   a) $\log_6 (36x^3)$
   b) $\log_4 \left( \frac{\sqrt{x}}{64} \right)$

11. Condense the logarithmic expressions.
   a) $\log 3 - 4 \log x$
   b) $\frac{1}{2} \ln x - \ln y$

12. Use the common or natural logarithm with the change of base property and a calculator to find $\log_6 72,348$.

13. Solve by expressing each side as a power of the same base and equating the exponents.
   a) $2^{4x-2} = 64$
   b) $9^x = \frac{1}{27}$

14. Solve by taking the natural logarithm ($\ln$) of both sides.
   a) $8^x = 12,143$
   b) $30e^{0.045x} = 90$

15. Solve the equations by using the definition of the logarithm.
   a) $\log x = 2$
   b) $\log_4 (3x - 5) = 3$

16. The function $W(x) = 0.37 \ln x + 0.05$ models the average walking speed, $W(x)$, in feet per second, of residents in a city whose population is $x$ thousand. Visitors to New York City frequently feel they are moving too slowly to keep pace with New Yorkers’ average walking speed of 3.38 feet per second. What is the population of New York City? Round to the nearest thousand.

   Make sure review OTHER applications from with similar properties!

17. Find the distance between the points $(-2, -3)$ and $(3, 9)$.

18. Find the midpoint between the points $(2, 6)$ and $(-12, 4)$.

19. Write an equation in standard form for a circle with a center $(-2, 4)$ and a radius 6.

20. Find the center and radius for a circle whose equation is $(x + 5)^2 + (y - 2)^2 = 9$

21. Graph the ellipse described by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$.
22. Use vertices and asymptotes to graph the hyperbola described by the equation
\[
\frac{x^2}{16} - \frac{y^2}{49} = 1
\]

23. Solve the system.
\[
\begin{align*}
5y &= x^2 - 1 \\
x - y &= 1
\end{align*}
\]

24. Solve the following.
a) \(3x + 7 > 4\) or \(6 - x < 1\)
b) \(\frac{5}{x - 3} = 1 + \frac{30}{x^2 - 9}\)

25. Graph the function.
\(f(x) = -\frac{2}{3}x + 4\)

\((3x^3 - 19x^2 + 17x + 4) ÷ (3x - 4)\)

27. Simplify.
\[\sqrt[3]{4x^2y^5} \cdot \sqrt[4]{4xy^2}\]

28. Factor.
a) \(12x^3 - 36x^2 + 27x\)  
b) \(x^3 - 2x^2 - 9x + 18\)

29. Rationalize the denominator.
\[\frac{11}{5 + \sqrt{3}}\]

30. Two cars leave the same place at the same time, traveling in opposite directions. The rate of the faster car exceeds that of the slower car by 10 miles per hour. After 2 hours, the cars are 180 miles apart. Find the rate of each car.

31. You have 72 feet of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area.

Make sure review OTHER applications from with similar properties!

32. Solve.
a) \(5(x + 2) = 3x - 7\)  
b) \(x^2 + 5x + 6 = 0\)

33. Find the domain of the function.
\(f(x) = \frac{2x - 3}{x^2 - 5x + 6}\)