Chapter 1 — Real Numbers and Expressions

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Answers
Section 1.1  Real Numbers

1.1 — Real Numbers Worksheet

Examples:

Given the set of numbers: \( \{1, -713, -\sqrt{2}, 0, -2, -\frac{1}{3}, 17, 0.15, -3\frac{1}{2} \} \) list those that belong to the set of

a) Natural numbers

Natural numbers are the “counting numbers.” \( \{1, 17\} \)

b) Whole numbers

Whole numbers are the natural numbers as well as 0. \( \{0, 1, 17\} \)

c) Integers

The set of integers includes 0, the counting numbers, and their opposites. \( \{-713, -2, 0, 1, 17\} \)

d) Rational numbers

Rational numbers are numbers that can be written as an integer over an integer. Rational numbers in decimal form either end or have a pattern. Integers and fractions are rational. \( \{1, -713, 0, -2, -\frac{1}{3}, 17, 0.15, -3\frac{1}{2} \} \)

e) Irrational numbers

Irrational numbers in decimal form do not end and/or do not have a pattern. \( \{-\sqrt{2}\} \)

f) Real numbers

Real numbers include all of the rational and irrational numbers. \( \{1, -713, -\sqrt{2}, 0, -2, -\frac{1}{3}, 17, 0.15, -3\frac{1}{2}\} \)

Homework

1. Define each set listed below in your own words. You may use sentences or a list of numbers.
   a) Natural numbers
   b) Whole numbers
   c) Integers
   d) Rational numbers
   e) Irrational numbers
   f) Real numbers

2. State whether each number is a natural number, whole number, integer, rational number, irrational number, and/or real number. Each number may have more than one answer.
   a) \(-3\)
   b) \(\sqrt{6}\)
   c) 0
   d) \(\frac{2}{3}\)
   e) \(\pi\)
   f) 7.45
   g) 13
   h) \(5\frac{1}{2}\)
   i) \(\sqrt{5}\)
   j) 8.984344679312098765...
   k) \(-2.1\overline{8}\)
3. Given the set of numbers: \( \{-0.3, 17, -56, 4\frac{1}{2}, \sqrt{7}, 0, 53, 9\sqrt{3} \} \) list those that belong to the set of
   a) Natural numbers
   b) Whole numbers
   c) Integers
   d) Rational numbers
   e) Irrational numbers
   f) Real numbers

4. Given the set of numbers: \( \{\sqrt{4}, 5\frac{1}{2}, -0.25, 11, \frac{1}{6}, \sqrt{10}, \pi \} \) list those that belong to the set of
   a) Natural numbers
   b) Whole numbers
   c) Integers
   d) Rational numbers
   e) Irrational numbers
   f) Real numbers

Determine whether each statement below is true or false.
5. Every rational number is a real number.
6. Every real number is irrational.
7. Every integer is a rational number.
8. All whole numbers are integers.
9. All irrational numbers are real.
10. All integers are natural numbers.

11. Define each word below in your own words and then give a few examples.
   a) Variable
   b) Constant
   c) Expression
   d) Equation
Section 1.2  Integers

Examples:

Perform each operation.

a) \(-8 + (-9)\)

To add two numbers with the same signs add their absolute values and attach the common sign to the sum.
A negative plus a negative equals a negative.

\(-17\)

b) \(-8 + (9)\)

To add two numbers with the opposite signs, subtract their absolute values and attach the sign of the larger absolute value to the difference.

*Note that this problem is the same as \(9 + (-8)\). For this example, because \(|9| > |-8|\), the answer is positive.*

\(1\)

c) \(8 + (-9)\)

*Note that this problem is the same as \(-9 + 8\). For this example, because \(|-9| > |8|\), the answer is negative.*

\(-1\)

d) \(-8 - (-9)\)

To subtract two numbers when at least one of them is negative, you may change the problem to “adding the opposite.”

Change the subtraction to an addition and the number after the subtraction to its opposite. Leave the first number alone.

*This problem becomes: \(-8 + (9)\) or \(-8 + 9\).*

\(1\)

e) \(-8 - (9)\)

*This problem becomes: \(-8 + (-9)\).*

\(-17\)

f) \(8 - (-9)\)

*This problem becomes: \(8 + (+9)\) or \(8 + 9\).*

\(17\)

g) \(-9 - (-8)\)

*This problem becomes: \(-9 + (+8)\) or \(-9 + 8\).*

\(1\)

h) \(-9 - (8)\)

*This problem becomes: \(-9 + (-8)\).*

\(-17\)

i) \(9 - (-8)\)

*This problem becomes: \(9 + (+8)\) or \(9 + 8\).*

\(17\)

j) \(-8(-9)\)

To multiply two numbers with the same sign, multiply their absolute values. The answer will ALWAYS be positive.
A negative times a negative equals a positive.

\(72\)
To multiply, two numbers with opposite signs, multiply their absolute values. The answer will ALWAYS be negative. A negative times a positive equals a negative.

Note that this problem is the same as \( 9 (\text{-}8) \).

\[ -72 \]

A positive times a negative equals a negative.

Note that this problem is the same as \( -9 (8) \).

\[ -72 \]

**Homework**

Perform each operation without using a calculator. If you have trouble with these problems, ask for a handout on integers for more practice.

1. \( -8 + (-13) \)
2. \( 4 + (-21) \)
3. \( -6 + 10 \)
4. \( -8 - (-13) \)
5. \( 4 - (-21) \)
6. \( -6 - 10 \)
7. \( -(2) \)
8. \( 3(-5) \)
9. \( -1(-9) \)
10. \( 16 - (-14) \)
11. \( 0(-15) \)
12. \( -80 + 50 \)
13. \( -4(60) \)
14. \( -75 + (-25) \)
15. \( -18 - 7 \)
16. \( -90(4) \)
17. \( 42 + (-22) \)
18. \( -56(-1) \)
19. \( -63 - (-23) \)
20. \( -5 - (-5) \)
21. \( -8 - 8 \)
Section 1.3  

Fractions

Examples:
Perform each operation.

a) \( \frac{14}{21} \cdot \frac{15}{30} \)

To multiply two fractions, it’s usually easiest to cancel factors common to a numerator and a denominator before multiplying numerator times numerator and denominator times denominator.

\[
\frac{2 \cdot 7}{3 \cdot 7} \cdot \frac{5 \cdot 3}{2 \cdot 3 \cdot 5} = \frac{2 \cdot 7 \cdot 5 \cdot 3}{3 \cdot 7 \cdot 2 \cdot 3 \cdot 5} = \frac{\cancel{2} \cdot \cancel{7} \cdot \cancel{5} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{7} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{5}}
\]

Sometimes it is easiest to “see” the factors by writing the prime factorization of each numerator and each denominator.

Cancel directly up and down or diagonally across the multiplication sign.

\[
\frac{1}{3}
\]

If “everything” goes away when canceling during a multiplication problem, the factor that remains is 1.

b) \( \frac{7}{21} \div \frac{10}{42} \)

To divide two fractions, you may change the problem to multiplying the dividend (the first fraction) by the reciprocal of the divisor (the second fraction), and then complete the multiplication problem.

\[
\frac{7}{21} \cdot \frac{42}{10} = \frac{7 \cdot 42}{21 \cdot 10} = \frac{1 \cdot \cancel{7} \cdot \cancel{42} \cdot \cancel{21} \cdot \cancel{10}}{\cancel{21} \cdot \cancel{10}} = \frac{\cancel{1} \cdot \cancel{2} \cdot 3 \cdot 7}{\cancel{2} \cdot 5}
\]

Write the prime factorization for each numerator and each denominator.

\[
\frac{7}{3 \cdot 7} \cdot \frac{2 \cdot 3 \cdot 7}{2 \cdot 5}
\]

Cancel directly up and down or diagonally across the multiplication sign. Notice that in this problem, the factor of 7 in the left denominator only cancels with one of the 7s in the numerator.

\[
\frac{7}{5}
\]

You may write this answer as the improper fraction \( \frac{7}{5} \) or as a mixed number \( 1 \frac{2}{5} \).

c) \( \frac{7}{15} - \frac{5}{9} \)

To subtract (or add) two fractions, find a common denominator, rewrite each fraction in an equivalent form with that common denominator, subtract (or add) the new numerators appropriately, keep the denominator the same as the common denominator, and then reduce/simplify the answer if possible. Do NOT cancel unless the final answer needs to be reduced/simplified.

\[
\frac{7}{15} - \frac{5}{9} = \frac{7 \cdot 3}{3 \cdot 5} - \frac{5 \cdot 3}{3 \cdot 5} = \frac{7 \cdot 3 - 5 \cdot 3}{3 \cdot 5} = \frac{\cancel{7} \cdot \cancel{3} - \cancel{5} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{5}}
\]

The common denominator is \( 3 \cdot 5 = 45 \).

\[
\frac{\cancel{7} \cdot \cancel{3} - \cancel{5} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{5}} = \frac{21 - 25}{45} = \frac{\cancel{21} - \cancel{25}}{\cancel{45}}
\]

Multiply each fraction by the appropriate form of 1, so that its denominator is the common denominator (45 for this problem).

\[
\frac{\cancel{21} - \cancel{25}}{\cancel{45}} = \frac{-4}{45}
\]

Notice that if the denominator changed, the numerator of that fraction also changed.

\[
\frac{-4}{45}
\]

This fraction does not reduce.
Homework

Perform each operation without using a calculator. If you have trouble with these problems, ask for a handout on fractions for more practice. All answers must be simplified.

1. \( \frac{7}{10} + \frac{3}{10} \)
2. \( \frac{4}{5} \cdot \frac{1}{5} \)
3. \( \frac{9}{13} \div \frac{3}{13} \)
4. \( \frac{3}{4} - \frac{3}{4} \)
5. \( 0 \left( -\frac{5}{14} \right) \)
6. \( \frac{2}{3} + \frac{5}{4} \)
7. \( \frac{2}{3} \cdot \frac{5}{4} \)
8. \( \frac{5}{6} \div \left( -\frac{25}{4} \right) \)
9. \( \frac{1}{6} \cdot 18 \)
10. \( -4 \left( -\frac{7}{12} \right) \)
11. \( \frac{8}{5} - \frac{7}{4} \)
12. \( -\frac{7}{8} - \left( -\frac{9}{10} \right) \)
13. \( \frac{39}{24} \cdot \frac{8}{52} \)
14. \( -\frac{3}{14} + \frac{2}{21} \)
15. \( -\frac{1}{6} \div \left( -\frac{21}{5} \right) \)
16. \( 4\frac{1}{8} \cdot \frac{4}{5} \)
17. \( 20\frac{1}{2} + 5\frac{3}{4} \)
18. \( 2\frac{1}{3} \cdot 12 \)
19. \[-7 \left( -\frac{5}{12} \right) \]

20. \[2\frac{4}{5} - 1\frac{3}{4} \]
Section 1.4 Exponents and Order of Operations

1.4 — Order of Operation Worksheet

Example:

Perform the operations—in the correct order.

\[ 6 \div 3 + 4 \left[ 9 + 2 (3 - 1)^2 \right] \]

In order of operation, grouping symbols come “first”, start inside the parentheses by subtracting 1 from 3.

\[ 6 \div 3 + 4 \left[ 9 + 2 (2)^2 \right] \]

Now, working inside the brackets, since that is a grouping symbol, follow the order of operation for addition, multiplication, and exponents, by doing the exponent first.

\[ 6 \div 3 + 4 \left[ 9 + 2 (4) \right] \]

Next, do the multiplication inside the brackets.

\[ 6 \div 3 + 4 [9 + 8] \]

Add 9 and 8.

\[ 6 \div 3 + 4 [17] \]

The inside of the bracket is now a single number, so the bracket is just a multiplication symbol. Division, addition, and multiplication remain. Do the division before the multiplication, since it is first from left to right. Next, do the multiplication.

\[ 2 + 4 [17] \]

Finally, add.

\[ 2 + 68 \]

70

Homework

1. Define each word below in your own words and then give a few examples.
   a) Exponent
   b) Base
   c) Squared
   d) Cubed

2. Calculate each. Pay attention to signs, parentheses, and whether the exponent is odd or even. All answers must be simplified.
   a) \(3^2\)
   b) \((-3)^2\)
   c) \((-3)^3\)
   d) \(-3^2\)
   e) \(-3^3\)

3. Calculate each. Pay attention to signs, parentheses, and whether the exponent is odd or even. All answers must be simplified.
   a) \(-4^2\)
   b) \(-4^3\)
   c) \((-4)^3\)
   d) \(4^2\)
   e) \((-4)^2\)
Perform the operations—in the correct order—without using a calculator. All answers must be simplified. Since all problems contain only constants, it’s best to not use the distributive property; instead follow order of operations. The distributive property will be discussed in the following section.

4. \(5 \cdot 3^2\)
5. \(35 - 6^2\)
6. \(\frac{1}{2}(27 - 17)\)
7. \(18 - 3(5 + 2)\)
8. \(6(11 - 10)^2\)
9. \(\frac{1}{4}(12 + \frac{8}{7})\)
10. \(\frac{1}{2}(6 - 4)^2\)
11. \(-20(6 - 4) + 5(9 - 11)\)
12. \(\frac{-10(6 + 4) - 30}{5}\)
13. \(\frac{2 - 8(16 - 12)}{1\frac{3}{4}}\)
14. \((\frac{1}{2})^2 - \frac{1}{3}\)
15. \(\frac{4}{3} \div \left(-\frac{11}{2} - \frac{7}{6}\right)\)
Section 1.5  Simplifying Expressions

Example:
Simplify.

a) \( \frac{1}{7}(9 + 3x) - 2(x - 4) \)

\[
\frac{1}{7}(9 + 3x) - 2(x - 4) \\
3 + x - 2x + 8 \\
-x + 11
\]

Distribute. Take the sign in front of the number you are distributing, so distribute \(-2\).

Notice: \(-2(-4) = +8\).

Now add like terms. \(x\) is the same as \(1x\).

The answer may be written as \(-x + 11\) or \(11 - x\).

Homework

1. Define each word below in your own words and then give a few examples.
   a) Distributive Property
   b) Term
   c) Constant
   d) Expression

First, state the number of terms in each expression as it is written. Second, if the expression has a factor to distribute, distribute and then state how many terms are in the expression.

2. a) \(4m - 2 + 3m\)
   b) \(2(7p - 3) + 11\)
   c) \(5x + 13 - 4(x + 1)\)
   d) \(-3(w - y) + 10(w - 6)\)

Simplify.

3. \(5x + 2x\)
4. \(13y - 4y\)
5. \(16x - x\)
6. \(4m - 2 + 3m\)
7. \(2(7p - 3) + 11\)
8. \(\frac{1}{4}(6x - 2) - \frac{5}{6}\)
9. \(15 - (y + 3)\)
10. \(12 - 4(w - 8)\)
11. \(\frac{7}{2} + \frac{1}{4}(18y + 5)\)
12. \(5x + 13 - 4(x + 1)\)
13. \(\frac{1}{7}(4x - 10) - 8(x - 12)\)
14. \(-20(w - 4) + 5(w - 11)\)
15. \(x + 4 - x - 11\)
16. \(-\frac{1}{3}(16 - 5y) + \frac{1}{3}(y + 9)\)
17. \(11x + 5 - (2x - 7)\)
18. \(-2k(-3) - 4(k - 5)\)
19. \(\frac{1}{2}(6y + 4) + 3 - 7(y + 5)\)
20. \(x + 0.24x\)
21. \(p - 0.15p\)
22. \(x + x\)
23. \(19 - 4(a + 10) - 2a\)
24. \(x - x\)
25. \(-3(w - y) + 10(w - 6)\)
26. \(4(xy - 5) + 14xy\)
27. \(-\frac{3}{2}(6n - 3) + n\)
28. \(12(4x + 2y - 1) + 15x - 3\)
29. \(24 - 20(a - 3)\)
30. \(\frac{3}{4}(6 - x) - \frac{5}{8}(3x + 1)\)
31. \(42 + 3(a - \frac{3}{4}c + 7) - (\frac{1}{2}a - 8)\)
Section 1.6 — Phrases into Algebraic Expressions

1.6 — Phrases into Algebraic Expressions Worksheet

Examples:
For each problem below, define a variable and then use that variable to write an algebraic expression for the requested quantity.

a) Santa Cruz County’s per capita taxable sales were 18% less than California in 2009.  
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   What were Santa Cruz County’s per capita taxable sales in 2009?

   California’s per capita taxable sales: \( x \)
   Define the quantity that the least is known about.

   Santa Cruz County’s per capita taxable sales: \( x - 0.18x \)
   Now write an algebraic expression for the requested quantity.

b) According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the weekly earnings (in US$) of a person with a Bachelor’s degree were $150 more than twice the weekly earnings of a person with less than a high school diploma. What were the weekly earnings of a person with a Bachelor’s degree? (Write an expression. Do not give a dollar amount.)

   Weekly earnings of a person with less than a high school diploma: \( x \)

   Weekly earnings of a person with a Bachelor’s degree: \( 2x + 150 \)

Homework

1. Write the mathematical operation (addition, subtraction, multiplication, or division) that each word below signifies.
   a) Quotient
   b) Sum
   c) Less
   d) More than
   e) Difference
   f) Product
   g) Decreased by

2. Write the mathematical expression for each English phrase.
   a) Twelve less three
   b) Sixteen less than two
   c) Eight more than eleven
   d) Twice a number
   e) Forty percent of a number
   f) Eleven subtracted from fifteen

For each problem below, define a variable and then use that variable to write an algebraic expression for the requested quantity.

3. Two numbers are consecutive integers. What are the numbers?

4. In five days in New York, each participant of the World Economic Forum spent on average…14 times what the average person in India makes in a year….  
   —Derrick Z. Jackson, Boston Globe, February 8, 2002
   What is the average amount spent by a participant at the World Economic Forum in 5 days?

5. [In 2011, the] median family income in Santa Cruz County was…$15,400 more than the California median.  
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   What was the median family income in Santa Cruz County in 2011?

6. Santa Cruz County saw a decrease of 13% in taxable sales from 2008 to 2009 in all jurisdictions.  
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   What were Santa Cruz County’s taxable sales in 2009?

7. Eduardo bought a pair of shoes that were 20% off of the regular price. If there is no tax, how much were his shoes?
8. One number is **three less than twice the other number**. What are the numbers?

9. Across the 34 countries that make up the [Organization for Economic Cooperation and Development’s] membership, the average income of the richest 10 percent of the population is **nine times that of the poorest 10 percent**.
   —Eric Pfanner, nytimes.com, January 24, 2012
   What is the average income of the richest 10 percent of the population?

10. In Santa Cruz County travel spending **decreased by $40,000,000** between 2008 and 2009.
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
    What was Santa Cruz County’s travel spending in 2009?

11. Dominique brought a new jacket. If sales tax was 8%, **what was the total cost of the jacket (including sales tax)**?

12. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the unemployment rate of a person with a high school diploma was **0.5 less than twice a person with a Bachelor’s degree**.
    What is the unemployment rate of a person a high school diploma?

13. [In Santa Cruz County, jobs] in…health service jobs **increased 26%** during the [past decade: 2000–2010].
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
    What is the number of jobs in Santa Cruz County in 2010?

14. More than 14% of [Santa Cruz County] CAP survey respondents had moved at least once in the past 12 months. The percentage was higher among Latino respondents compared to Caucasian respondents (33% and 9% respectively), a statistically significant difference.
    [The number of Caucasian respondents who reported moving at least once in the past year was less than the number of Latino respondents who reported moving at least once in the past year.]
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
    What was the number of Caucasian respondents who reported moving at least once in the last year?

15. Two numbers are **consecutive odd integers**. What are the numbers?

16. [The 2010 Santa Cruz County] median home sales price has **decreased by...$225,000** [since 2007.]
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
    What is the 2010 Santa Cruz County median home sales price?

17. The average rent [for a three-bedroom dwelling] in Santa Cruz County has been steadily increasing, showing a **16% increase** [between 2008 and 2011].
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
    What is the average rent for a three-bedroom dwelling in Santa Cruz County in 2011?

18. **$75 is to be split between Martita and Nicole.** They will not necessarily receive the same amount. How much money does each person receive?

19. While more than 57% of CAP survey respondents felt they had job opportunities in Santa Cruz County in 2011, this varied by income level. Of the respondents who indicated they earned less than $35,000 per year, 51% felt they had opportunities to work in the area compared to 69% of respondents who earned $65,500 per year or more, a statistically significant difference.
    [The number of respondents who earned less than $35,000 per year and felt they had job opportunities in Santa Cruz County was less than the number of respondents who earned $65,500 per year or more and felt they had job opportunities in Santa Cruz County.]
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
    What was the number of respondents who earned less than $35,000 per year and felt they had job opportunities in Santa Cruz County?

20. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the weekly earnings (in US$) of a person with a Master’s degree were **$20 more than twice the weekly earnings of a person with a high school diploma**.
    What were the weekly earnings of a person with a Master’s degree?

21. One number is **fourteen more than half the other number**. What are the numbers?
Chapter 2 – Linear Equations

Section 2.1 – Additive Property of Equality
Section 2.2 – Multiplicative Property of Equality
Section 2.3 – Linear Equations in One-Variable
Section 2.4 – Linear Equations in One-Variable with Fractions
Section 2.5 – Linear Inequalities in One-Variable
Answers
Section 2.1  Additive Property of Equality

Examples:
Solve each equation.
\[ a) \quad -18 + x = -57 \]
\[ \begin{align*}
-18 + x &= -57 \\
+18 &= +18 \\
x &= -39 
\end{align*} \]
\[ \text{Add 18 to both sides of the equation to “undo” subtracting 18 from } x. \]
\[ b) \quad 20 - 34 = x + 7 \]
\[ \begin{align*}
20 - 34 &= x + 7 \\
-14 &= x + 7 \\
-7 -7 &= \underbrace{-21}_{= x} 
\end{align*} \]
\[ \text{First combine like-terms on the left side, by subtracting 34 from } 20. \]
\[ \text{Subtract 7 from both sides of the equation to “undo” adding 7 to } x. \]

Homework
1. In your own words, describe a linear equation. How does one recognize that an equation is linear? Give two examples of linear equations.
2. What is the inverse operation of addition?
3. What is the inverse operation of subtraction?

Solve the equation or simplify the expression.
4. \[ a) \quad x + 3 = 8 \]
   \[ b) \quad y - 14 = 67 \]
   \[ c) \quad x + 5 = -8 \]
   \[ d) \quad 9 = k - 16 \]
   \[ e) \quad 16 - 7x + 5 + 2x \]

Solve each equation. Show a check for every third problem.
5. \[ y + 7 = 12 \]
6. \[ p - 35 = -6 \]
7. \[ p + 4 = -17 \]
8. \[ 8 = z + 14 \]
9. \[ 6 - 5 = m \]
10. \[ 3 = 48 + n \]
11. \[ -7 + x = -7 \]
12. \[ z - 36 = 100 \]
13. \[ -11 + w = 15 \]
14. \[ -10 = x + 18 \]
15. \[ 1.4 = m - 2.9 \]
16. \[ -27 + x = -9 \]
17. \( 56 = 12 + c \)
18. \( 32 + p = -32 \)
19. \( y + 62 = 19 \)
20. \( -81 = x + 2 \)
21. \( x - 13 = -21 \)
22. \( m + 34 = -10 \)
23. \( 9 - 15 = x + 2 \)
Section 2.2  Multiplicative Property of Equality

Examples:
Solve each equation.

a) $-20x = 5$

$-20x = 5 \Rightarrow \frac{-20x}{-20} = \frac{5}{-20}$

Divide both sides of the equation by $-20$ to “undo” multiplying $x$ by $-20$.

$x = \frac{1}{4}$

Simplify the fraction $\frac{5}{-20}$ to $\frac{1}{4}$.

b) $-2 = \frac{x}{8}$

$-2 = \frac{x}{8} \Rightarrow -2 \cdot 8 = \frac{x}{8} \cdot -8$

The negative sign in front of the fraction can be applied to either the numerator or the denominator. But not both at the same time. In this problem, it’s easiest to apply it to the denominator, since that is a constant factor.

$-8(-2) = \frac{x}{8} \cdot -8$

Multiply both sides of the equation by $-8$ to “undo” dividing $x$ by $-8$.

$-8(-2) = \frac{x}{8} \cdot -8$

$16 = x$

Homework

1. What is the inverse operation of multiplication?

2. What is the inverse operation of division?

Solve the equation or simplify the expression.

3. a) $3x = 18$
   b) $2x + 18 + x$
   c) $-2p = 10$
   d) $-4y - 5 + 3y$
   e) $3x - x = 14$

Solve each equation. Show a check for every third problem. Be sure to notice the operation in the equation before you solve.

4. $15m = 45$
5. $-20p = 40$

6. \[
\frac{r}{3} = -10
\]

7. $-12 = 2x$
8. $5y = 7$

9. $24 = -6w$
10. $-9 = \frac{p}{3}$

11. $8x = 2$
12. $-12a = -48$
13. $6 = -9z$
14. $1.3y = -26$
15. $-2 = \frac{x}{6}$
16. $w - 75 = -25$
17. $7 = -4m$
18. $-16c = 20$
19. $-\frac{x}{7} = 2$
20. $-20 = 6w$
21. $-14x = 7$
22. $14 = -\frac{a}{2}$
23. $-16 = -4p$
24. $8n = 3$
25. $22 = -10 + y$
26. $9 = 18z$
27. $-15 = -10x$
28. $\frac{k}{10} = -5$
Section 2.3  Linear Equations in One-Variable

2.3 — Linear Equations in One-Variable Worksheet

Examples:

Solve each equation.

a) \(7x - 15x = 2 - 3(x - 6)\)

\[\begin{align*}
7x - 15x &= 2 - 3(x - 6) \\
-8x &= 2 - 3x + 18 \\
-8x &= 20 - 3x
\end{align*}\]

\[\begin{align*}
\text{Combine like-terms.} \\
-8x &= 20 - 3x \\
+3x &= +3x \\
-5x &= 20
\end{align*}\]

\[\begin{align*}
\text{Add } 3x \text{ to both sides of the equation to get the variable terms on one side of the equation and the constant terms on the other side of the equation.} \\
-5x &= 20 \\
\frac{-5x}{-5} &= \frac{20}{-5} \\
x &= -4
\end{align*}\]

b) \(7x - x - 30 = -2(-3x + 15)\)

\[\begin{align*}
7x - x - 30 &= -2(-3x + 15) \\
6x - 30 &= 6x - 30
\end{align*}\]

\[\begin{align*}
\text{This equation is an identity, since both sides are exactly the same. To satisfy the equation, } x \text{ can be any real number. If you notice this at this point, you may stop solving and write the answer.} \\
6x - 30 &= 6x - 30 \\
-6x &= -6x \\
-30 &= -30
\end{align*}\]

Since there are no variables left and the statement is true, the answer is “all real numbers”.

Subtract 6x from both sides of the equation to get the variable terms on one side of the equation and the constant terms on the other side of the equation.

Since there are no variables left and the statement is false, the answer is “no solution.”

Caspers
Homework

1. In your own words, describe how to solve a linear equation.

2. In your own words, describe how to determine if a linear equation has no solution.

3. In your own words, describe how to determine if all real numbers are solutions to a linear equation.

Solve the equation or simplify the expression.

4. 
   a) \(3x + 2x = 40\)
   b) \(2x + 3x + 40\)
   c) \(-2y + 5y = 17\)
   d) \(24 = 5x - 1\)

Solve each equation. Show a check for every third problem. Give simplified answers.

5. \(2x - 5 = 11\)

6. \(6 + 8 = 3y - 1\)

7. \(-14p + 6 = 13\)

8. \(-19 = 4w - 6\)

9. \(12 = -20x + 2\)

10. \(4 - 23 + 5x = -16\)

11. \(5(x - 2) = 45\)

12. \(25 = 14(x - 1)\)

13. \(3 + 2(x + 1) = -17\)

14. \(-11 = 25 - 3(x + 2)\)

15. \(7a - 3(a + 4) = 13 + 3\)

16. \(5x - x + 2(x + 4) = 26\)

17. \(18 = 6 - 12(x + 4)\)

18. \(32 = 7x + 5 - 2(x - 1)\)

19. \(13x + 6 - (x + 8) = 54\)

20. \(5x + 3 = 2x\)

21. \(9x - x = -16\)

22. \(8y + 5 = -13y + 4\)

23. \(4 - x = x - 4\)

24. \(x - 10 = -x + 10\)

25. \(5x - x + 8 = 2(2x + 4)\)

26. \(-4x + 12 = -7x - 12\)

27. \(x - 13 = x + 13\)

28. \(7p = 4(p - 3)\)
29. \[ 14x - (x + 5) = 3x \]
30. \[ 18x - 5x = -2 - 4(x + 8) \]
31. \[ 5x - 9(x + 2) = 10 - 4(x + 7) \]
32. \[ 11 - 6(m - 5) = 13 + 2(m - 9) \]
33. \[ 4x - 10x = 8 - 3(2x + 1) \]
34. \[ 17 - 4(2x - 6) - 13 = 7x + x - 6 \]
35. \[ 11x - x + 8 = 12 - 9(x + 3) \]
36. \[ 7 + 2(5x - 3) = 4x - (6x - 1) \]
37. \[ 16 = 4(n - 7) \]
38. \[ 14 - 5(x + 1) = 23 \]
39. \[ 17x - 2x + 12 = 9x + 6(x + 2) \]
40. \[ 29 - 2(z - 6) - z = 17 + 3(z - 8) \]
41. \[ 15 - (x + 12) = x - 3 \]
42. \[ 8(x - 3) + 31 = x + 7(x + 1) \]
43. \[ 5x - x + 12 = 2x - 6(x - 2) \]
Section 2.4  Linear Equations in One-Variable with Fractions

2.4 — Equations and Expressions Worksheet

Example:
Solve each equation.

a) \[ \frac{4}{3} \left( \frac{1}{2} x - \frac{9}{10} \right) = x - 5 \]

\[ \frac{4}{3} \left( \frac{1}{2} x - \frac{9}{10} \right) = x - 5 \]

\[ \frac{4}{3} \cdot \frac{1}{2} x - \frac{4}{3} \cdot \frac{9}{10} = x - 5 \]

\[ \frac{2}{3} x - \frac{3}{2} \cdot \frac{9}{10} = x - 5 \]

\[ \frac{6}{3} x - \frac{6}{3} \cdot \frac{9}{10} = \frac{6}{1} x - \frac{6}{1} \cdot \frac{5}{1} \]

\[ \frac{2}{1} x - \frac{3}{1} \cdot \frac{3}{1} \cdot \frac{2}{1} = \frac{6}{1} x - \frac{6}{1} \cdot \frac{5}{1} \]

\[ 4x - 9 = 6x - 30 \]

\[ 4x - 9 = 6x - 30 \]

\[ -4x \]

\[ -9 = 2x - 30 \]

\[ -9 = 2x - 30 \]

\[ +30 \]

\[ 21 = 2x \]

\[ 21 \]

\[ \frac{21}{2} = x \]

\[ \frac{21}{2} = x \]

Distribute \( \frac{4}{3} \).

To multiply both terms on the left side of the equation by \( \frac{4}{3} \), cancel factors common to numerator and denominator in each multiplication.

The LCD of simplified equation is 6. Since this is an equation, you may multiply each side of the equation by the LCD to “get rid” of the fractions.

Multiply all terms on both sides of the equation by 6 or \( \frac{6}{1} \).

Again, cancel factors common to numerator and denominator in each multiplication. It may be helpful to write integers with a denominator of 1 (as with \( x \) and \( 5 \) in this problem).

Subtract \( 4x \) from both sides of the equation to get the variable terms on one side of the equation.

Add 30 to both sides of the equation to get the constant terms on one side of the equation, as well as to “undo” subtracting 30.

Divide both sides of the equation by 2, to “undo” multiplying \( x \) by 2.

Homework

1. In your own words, describe how to solve a linear equation that contains fractions.

First, state the number of terms in each equation as it is written. Second, if the equation has a factor to distribute, distribute and then state how many terms are in the equation.

2. a) \[ -\frac{2}{3} w = -2 \]
Solve each equation. Show a check for every third problem. Give simplified answers.

3. \[ \frac{1}{2}x - \frac{5}{6} = \frac{1}{4} \]

4. \[ \frac{3}{5} = \frac{7}{10}y - \frac{2}{5} \]

5. \[ 3p + \frac{1}{7} = \frac{3}{14} \]

6. \[ -\frac{3}{4} = \frac{6}{5}w - 2 \]

7. \[ \frac{1}{2}(8x - 2) = 7 \]

8. \[ \frac{8}{15} = \frac{3}{5}(x + 5) \]

9. \[ 4 + \frac{2}{9}(x + 1) = \frac{2}{3} \]

10. \[ \frac{3}{4}\left(\frac{6}{5}x - \frac{2}{5}\right) = \frac{1}{2} \]

11. \[ \frac{8}{3}x - \frac{1}{4}(x + 5) = 2 \]

12. \[ 5 + \frac{6}{7}\left(x - \frac{1}{2}\right) = -\frac{5}{14} \]

13. \[ \frac{1}{8}m - 3 = 5m + \frac{5}{6} \]

14. \[ \frac{1}{13} - \frac{5}{26}x = \frac{1}{2}x - \frac{1}{13} \]

15. \[ 11x + \frac{2}{3}x = \frac{1}{4} - 2x \]

16. \[ \frac{3}{4} + \frac{1}{2}x = \frac{1}{2}x - \frac{3}{4} \]

17. \[ 2\left(\frac{4}{5}z + 3\right) = \frac{1}{10}z - 4 \]

18. \[ \frac{1}{2}(20x - 5) = \frac{5}{6}(12x - 3) \]

19. \[ \frac{12}{25} - \frac{4}{15}(n + 3) = 4n + 7 \]

20. \[ 4 - \frac{1}{3}(6x + 4) = -\frac{2}{9}(9x - 12) \]
Section 2.5 Linear Inequalities in One-Variable

Example:
Solve each equation.

\[ a) \quad 22 - \frac{1}{2}(10x - 8) \leq -3x - 10 \]

\[
22 - 5x + 4 \leq -3x - 10 \\
26 - 5x \leq -3x - 10 \\
26 - 5x \leq -3x - 10 \\
+3x \quad +3x \\
26 - 2x \leq -10 \\
26 - 2x \leq -10 \\
-26 \quad -26 \\
-2x \leq -36 \\
\frac{-2x}{-2} \quad \frac{-36}{-2} \\
x \geq 18
\]

Distribute \( \frac{1}{2} \).

Combine like terms.

Add \( 3x \) to both sides of the equation to get the variable terms on one side of the equation.

Subtract 26 from both sides of the equation to get the constant terms on one side of the equation, as well as to “undo” adding 26.

Divide both sides of the equation by \(-2\) to “undo” multiplying \( x \) by \(-2\) AND switch the direction of the inequality sign. You must switch the inequality sign whenever you divide (or multiply) by a negative value in order to solve an inequality.

The solution is “all real numbers that are greater or equal to 18.”

Homework

Define each word below in your own words and then give a few examples.

1. a) Expression
   b) Equation
   c) Inequality

2. Describe in your own words how to solve an inequality.

3. Describe in your own words how to determine when you switch the direction of an inequality sign when solving an inequality.

4. Describe in your own words when you would use a closed (solid) circle/dot when graphing an inequality.

5. Describe in your own words when you would use an open (hollow) circle/dot when graphing an inequality.

6. Describe in your own words how to determine which interval an inequality represents, in other words, “which way the line goes,” when graphing an inequality.

Graph each inequality on a number line.

7. \( x > -4 \)

8. \( x < -\frac{1}{2} \)

9. \( x \geq 10 \)

10. \( x \leq -8 \)

11. \( 5 \geq x \)

12. \(-2 < x \)

13. \(-3 > x \)

14. \( x > 0 \)

15. \( x \leq 0 \)
Solve each inequality. Graph the solution on a number line for every fifth problem. Check every fifth problem.

16. \(3x - 4 > 8\)
17. \(-2x + 5 \geq 11\)
18. \(14 - x < 6\)
19. \(3p + 5 \geq -19\)
20. \(12 \geq 6x + 7\)
21. \(\frac{1}{4}x + 5 \leq \frac{1}{2}\)
22. \(7 - a \geq \frac{3}{5}\)
23. \(\frac{2}{7} < \frac{1}{4}x - 3\)
24. \(2(y - 6) \leq -14\)
25. \(5 - 3(x - 7) > 19\)
26. \(22 \geq 6 - 4(x + 1)\)
27. \(\frac{3}{5}(6w + 4) \leq -8\)
28. \(\frac{4}{5} > \frac{1}{4}(5 - \frac{2}{3}x)\)
29. \(\frac{3}{5}(x + \frac{10}{3}) \leq -\frac{1}{10}\)
30. \(3(5z + 10) \leq 9z\)
31. \(5x - 7 < 3x + 12\)
32. \(4 - x < 5x + 2\)
33. \(\frac{1}{7}x - 3 \geq \frac{4}{3}x + 5\)
34. \(\frac{7}{8}x + \frac{1}{6} > \frac{3}{4}x - \frac{5}{12}\)
35. \(17 - 5(2q - 4) \leq q - 18\)
36. \(4 + 7(x - 2) \leq 5x - 12\)
37. \(2m - 8 \leq 2m - 8\)
38. \(4 - 3x < 4 - 3x\)
39. \(10 - 5x \leq 10 + 5x\)
40. \(3x - 10 < 3x - 8\)
41. \(n + 4 < n - 11\)
42. \(\frac{2}{3}x - \frac{1}{2} > \frac{2}{3}x - \frac{1}{3}\)
43. \(5x - (3x + 4) \leq -11(5 - x) - 9x\)
44. \(\frac{1}{2}(y + \frac{4}{5}) \leq \frac{1}{10}y - \frac{1}{4}(3y + 2)\)
45. \(3(x - 10) - 7x \leq 9x - 13x + 24\)
46. \(\frac{1}{8}(\frac{2}{3}c + \frac{4}{3}) \geq \frac{1}{3}c - \frac{1}{3}(c - \frac{2}{3})\)
47. \( \frac{1}{2} \left( \frac{3}{4} x + \frac{3}{4} \right) < \frac{1}{2} x - \frac{1}{4} (x - \frac{3}{4}) \)
48. \( 14 + 7(3x - 1) > 20x \)
49. \( \frac{3}{4} \left( \frac{1}{2} x + 8 \right) \leq -\frac{7}{11} x \)
50. \( 44 - 2(w - 6) \geq -w + 5 \)
51. \( \frac{4}{3} \left( \frac{5}{3} x + 3 \right) \leq \frac{1}{13} x - \frac{1}{3} (x + 2) \)
Chapter 3 — Application Problems of Linear Equations in One-Variable

Section 3.1 — Ratios and Proportions
Section 3.2 — Formulas
Section 3.3 — Literal Equations
Section 3.4 — Geometry Application Problems
Section 3.5 — General Application Problems of Linear Equations in One Variable
Section 3.6 — Distance, Rate, Time Application Problems

Answers
Section 3.1  Ratios and Proportions

3.1 — Ratios and Proportions Worksheet

Example:
Write a ratio, solve that ratio, and then give an answer with the appropriate units.

a) In 2010, at Cabrillo College, for every 100 faculty members, approximately 12 were Latino. If there are 20 full-time mathematics faculty members at Cabrillo College, how many would you expect to be Latino? Round your answer to the nearest person.

Given ratio: \( \frac{100 \text{ total faculty members}}{12 \text{ Latino faculty members}} \)

Find ratio: \( \frac{20 \text{ total math faculty members}}{x \text{ Latino math faculty members}} \)

Set up the “find” ratio with the same corresponding units as the “given” ratio.

\[
\frac{100}{12} = \frac{20}{x}
\]

Set the ratios equal to each other.

This problem may also be written as \( \frac{20}{x} = \frac{100}{12} \) if you reduce the ratio on the left side of the proportion.

Solve by cross-multiplying.

\[
100 \cdot x = 20 \cdot 12
\]

\[
100x = 240
\]

Divide each side by 100.

\[
x = \frac{12}{5} \text{ or } 2.4
\]

About 2 math faculty would be Latino.

Homework

1. In your own words, describe a ratio. Give an example of a ratio.

2. In your own words, describe a rate. Give an example of a rate.

3. What is the difference between a ratio and a rate?

4. In your own words, describe a proportion. Give an example of a proportion.

5. When can you use cross multiplication to solve a linear equation?

6. In your own words, describe how to solve a proportion.

Express each as a simplified ratio or rate.

7. a) In 2010, the federal government spent 27.4¢ on the military for every 3.3¢ on education.
   Source: nationalpriorities.org

   b) In 1995, it cost $10 per unit to attend Cabrillo College.

   c) In 2012, it cost $36 per unit to attend Cabrillo College.

For each problem, write a ratio, solve that ratio, and then give an answer with the appropriate units.

8. In 2010, Cabrillo College’s student body was comprised of 1 Native American student for every 57 Caucasian students. If there are 2 Native American students in a class, how many Caucasian students would you expect there to be?
   Source: Cabrillo College Factbook 2011
9. “About four of ten [California community college] students who focused on transfer courses in their first year stayed on track and also took a majority of transfer courses in their second year.”
   If there are 39 students in a transfer course, how many would you expect to “stay on track and take a majority of transfer courses in their second year”? Round your answer to the nearest person.
   —Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

10. In 2010, Cabrillo College’s student body was comprised of 29 Latino students for every 57 Caucasian students. If there are 23 Caucasian students in a class, how many Latino students would you expect there to be? Round your answer to the nearest student.
   —Source: Cabrillo College Factbook 2011

11. In 2003, for every 100 California residents, approximately 6 are African-American. If there are approximately 2.5 million California community college students, how many would you expect to be African-American? Round your answer to the nearest person.
   The actual 2003 ratio of African-American community college students to the total number of community college students was close to 2:25 (or 8:100).
   —Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

12. In 2003, for every 100 California residents, approximately 6 are African-American. In a group of 50 UC students, how many would you expect to be African-American?
   The actual 2003 ratio of African-American UC students to the total number of UC students was 3:100.
   —Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

13. In 2003, for every 100 California residents, approximately 9 are Asian/Pacific Islander. In a group of 40 community college students, how many would you expect to be Asian/Pacific Islander?
   The actual 2003 ratio of Asian/Pacific Islander California community college students to the total number of California community college students was 13:100.
   —Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

14. A recipe for 6 servings of French onion soup requires ½ cup of thinly sliced onions. If Alexander is making 15 servings, how many cups of onions will he need? Give your answer as a mixed number.

15. A recipe for cookies calls for 2 tablespoons of sugar for every 3/4 cup of flour. If you use 3/4 cup of flour, how much sugar would you need to follow the recipe? Give your answer as a fraction.

16. Garrett is 6 feet tall and his current shadow is 2.5 feet long. If the shadow of the building his is standing next to is 8 feet long, how tall is the building? Give your answer accurate to 1 decimal place.

17. On a California map, the distance between Watsonville and Davenport is 0.75 inch. If the scale on the map is 1.5 inches to 56 miles, how many miles is it between Watsonville and Davenport?

18. Lynn is asked to give a patient 0.7 grams of meprobamate per square meter of body surface. The patient’s body surface is 0.6 square meter. How many grams of meprobamate should be given to the patient? Give your answer accurate to 1 decimal place.
Section 3.2  Formulas

3.2 — Formulas Worksheet

Examples:
Use the formula to find the value of the variable indicated. When necessary, round your answer to the nearest hundredth.

a) \[ y = mx + b, \text{ find } x \text{ when } y = 11, \ m = 2 \text{ and } b = 4 \]

\[
\begin{align*}
y &= mx + b \\
11 &= 2x + 4 \\
11 - 4 &= 2x - 4 \\
7 &= 2x \\
\frac{7}{2} &= x
\end{align*}
\]

Plug in the given values for the corresponding variables. Solve for \( x \).

The answer may be written as \( \frac{7}{2} \) or \( 3 \frac{1}{2} \) or \( 3.5 \).

b) Bonnie lent her brother \$4000 \text{ for a period of 2 years.} \text{ At the end of the 2 years, her brother repaid the } \$4000 \text{ plus } \$640 \text{ interest.} \text{ What simple interest rate did her brother pay?}

\[
\begin{align*}
i &= prt \\
640 &= 4000 \cdot r \cdot 2 \\
640 &= 8000r \\
\frac{640}{8000} &= \frac{8000r}{8000} \\
0.08 &= r
\end{align*}
\]

For this problem: principle \( p = 4000 \), time \( t = 2 \), and interest \( i = 640 \).

Plug in the given values for the corresponding variables. The simple interest rate was 8%.

Homework
Use the given values and the formula to find the missing value. Give simplified answers.

1. \( P = 4s \); if \( s = 3 \), find \( P \).
2. \( P = a + b + c \); if \( a = 4 \), \( b = 3 \), and \( c = 5 \), find \( P \).
3. \( P = a + b + c \); if \( a = 8 \), \( b = 6 \), and \( P = 24 \), find \( c \).
4. \( I = prt \); if \( p = 100 \), \( t = 2 \), and \( I = 6 \), find \( r \).
5. \( A = \frac{1}{2}bh \); if \( b = 12 \) and \( h = 7 \), find \( A \).
6. \( P = 2w + 2l \); if \( w = 5 \) and \( P = 26 \), find \( l \).
7. \( D = \frac{m}{v} \); if \( v = 2 \) and \( D = 12 \), find \( m \).
8. \( V = lwh \); if \( l = \frac{1}{3} \), \( w = 6 \), and \( h = 1 \), find \( V \).
9. \( d = rt \); if \( d = 200 \) and \( r = 40 \), find \( t \).
10. \( F = ma \); if \( m = 30 \) and \( a = 4 \), find \( F \).
11. \( P = \frac{f}{a} \); if \( f = 12 \) and \( a = 60 \), find \( P \).

12. \( I = \frac{V}{R} \); if \( I = 30 \) and \( R = 2 \), find \( V \).

13. \( R = \frac{VC}{T} \); if \( R = \frac{1}{10} \), \( T = 5 \), and \( C = 4 \), find \( V \).

14. \( H = U + pV \); if \( H = 45 \), \( U = 15 \), and \( V = 6 \), find \( p \).

15. \( G = H - TS \); if \( G = 80 \), \( T = 12 \), and \( S = 3 \), find \( H \).

16. \( V = V_0 + at \); if \( V = 75 \), \( V_0 = 0 \), and \( a = 20 \), find \( t \).

17. \( P = Hpg \); if \( P = 20 \), \( H = 3 \), and \( p = 2 \), find \( g \).

18. \( V = V_0 + 0.6tc \); if \( V = 42 \) and \( t_c = 18 \), find \( V_0 \).

19. \( N = n_2 - n_1 \); if \( N = 8 \) and \( n_2 = 13 \), find \( n_1 \).

20. \( A = \pi r^2 \); if \( r = 3 \), find \( A \). (Remember: \( \pi \approx 3.14 \))

Use the given value for \( x \) to find each value of \( y \).

21. \( x + y = 15 \); if \( x = 4 \), find \( y \).

22. \( y = \frac{1}{3}x + 6 \); if \( x = 0 \), find \( y \).

23. \( y = \frac{3}{2}x - 4 \); if \( x = -2 \), find \( y \).

24. \( y = 4x \); if \( x = -3 \), find \( y \).

25. \( x = 5y - 2 \); if \( x = 8 \), find \( y \).

Use the given value for \( y \) to find each value of \( x \).

26. \( x + y = 2 \); if \( y = -3 \), find \( x \).

27. \( y = \frac{1}{4}x - 1 \); if \( y = 0 \), find \( x \).

28. \( y = 3x \); if \( y = 5 \), find \( x \).

29. \( y = x \); if \( y = -6 \), find \( x \).

30. \( x = 3 - y \); if \( y = 4 \), find \( x \).

Determine which formula to use to solve each problem. Give simplified answer with the appropriate units.

31. Eduardo invested $800 in an account earning 5% simple interest. How much interest will he earn after 4 years?

32. Xavier lent his brother $1000 for a period of 2 years. At the end of the 2 years, his brother repaid the $1000 plus $75 interest. What simple interest rate did his brother pay?

33. The top of a rectangular dining room table measures 8 feet by 3 feet. Find the area of the tabletop.

34. Susan plans to fence in a triangular garden whose sides measure 10 feet, 12 feet, and 8 feet. How much fencing will she need to go around her garden?

35. The iPhone 4S rectangular display screen is approximately 2.25 inches by 2.75 inches. Find the area of the window.

36. The iPhone 4S is 4.5 inches in length, 2.31 inches in width, and 0.37 inches in height. Find the volume of this rectangular solid. Round your answer to 2 decimal places.
37. Maria invested $500 in an account earning 2% simple interest. How many years will it take for her to earn $40 interest?

38. The circular lid of a canning jar has a diameter of 3 inches. Find the area of the top of the lid. Round your answer to 3 decimal places.

39. Desmond’s rectangular sandbox is 4 feet by 6 feet. Find the area of the sandbox. How much sand (volume) is needed to fill the sandbox with sand that is 1.5 feet deep?

40. What volume of water is needed to fill up a spherical balloon that has a radius of 2 inches? Give both an exact answer and an approximate answer rounded to 2 decimal places.
Section 3.3  Literal Equations

Examples:
Solve each equation for the variable indicated.

a) \( P = 2w + 2l \); solve for \( l \)

\[
\begin{align*}
P &= 2w + 2l \\
\frac{P - 2w}{2} &= \frac{2l}{2} \\
\frac{P - 2w}{2} &= l
\end{align*}
\]

For this problem, \( l \) is the variable, so \( P \) and \( w \) are considered constants.

Subtract \( 2w \) from both sides of the equation to get the “constants” on one side of the equation.

Divide both sides of the equation by 2, to “undo” multiplying \( l \) by 2.

The answer may be written as \( l = \frac{P - 2w}{2} \) or \( l = \frac{P}{2} - w \).

Solve each equation for \( y \).

b) \( 2x + 4y = 7 \)

\[
\begin{align*}
2x + 4y &= 7 \\
-2x &\quad \quad -2x \\
4y &= 7 - 2x \\
\frac{4y}{4} &= \frac{7 - 2x}{4} \\
y &= \frac{7}{4} - \frac{1}{2}x
\end{align*}
\]

For this problem, \( y \) is the variable, so \( x \) is considered a “constant”.

Subtract \( 2x \) from both sides of the equation to get the “constants” on one side of the equation.

Divide each term on both sides of the equation by 4, to “undo” multiplying \( y \) by 4.

This answer is fine for this chapter, but later it will be better to write the \( x \)-term before the true constant.

Homework

1. In your own words, describe how solve a linear literal equation (formula) for a specific variable.

Solve each equation for the variable indicated.

2. \( A = lw \); solve for \( w \).
3. \( P = 4s \); solve for \( s \).
4. \( P = a + b + c \); solve for \( a \).
5. \( I = prt \); solve for \( r \).
6. \( A = \frac{1}{2}bh \); solve for \( h \).
7. \( P = 2w + 2l \); solve for \( l \).
8. \( D = \frac{m}{v} \); solve for \( m \).
9. \( V = lwh \); solve for \( h \).
10. \( d = rt \); solve for \( r \).
11. \( F = ma \); solve for \( a \).
12. \( P = \frac{f}{a} \); solve for \( a \).
13. \( I = \frac{V}{R} \); solve for \( V \).
14. \( R = \frac{VC}{T} \); solve for \( C \).
15. \( H = U + pV \); solve for \( U \).
16. \( G = H - TS \); solve for \( S \).
17. \( V = V_0 + at \); solve for \( a \).
18. \( P = Hpg \); solve for \( g \).
19. \( V = V_0 + 0.6t_c \); solve for \( V_0 \).
20. \( N = n_2 - n_1 \); solve for \( n_2 \).

Solve each equation for \( y \).
21. \( 4 = 2x + y \)
22. \( 6x + 2y = 30 \)
23. \( x - y = 7 \)
24. \( 4 + 8x = 2y \)
25. \( 3x + 4y = 5 \)
26. \( \frac{x}{3} y = 4 \)
27. \( y + 3 = 2x \)
28. \( 5y - 2 = 15x \)
29. \( x - 2y = 0 \)
30. \( 10x - 2y = 8 \)
Section 3.4  Geometry Application Problems

3.4 — Geometry Application Problems Worksheet

Example:
Solve by defining a variable, writing an equation, solving that equation, and stating the answer. Give simplified answers with units when appropriate.

a) One angle of a triangle is 40° larger than the smallest angle, and the third angle is 6 times as large as the smallest angle. Find the measure of the three angles.

The measure of the smallest angle: \( x \)

The measure of “one” angle: \( x + 40 \)

The measure of the third angle: \( 6x \)

The sum of the measures of the three angles in a triangle is 180°.

Define what the problem asks to find.

Use the appropriate geometric fact to write an equation.

\[
\begin{align*}
x + (x + 40) + 6x &= 180 \\
8x + 40 &= 180 \\
8x &= 140 \\
x &= 17.5
\end{align*}
\]

The smallest angle is 17.5°, the one angle is 17.5 + 20 = 37.5°, and the largest angle is \( 6(17.5) = 105° \).

Homework

1. How can you find the perimeter of any polygon?
2. What is the formula for the area of a rectangle?
3. What is the formula for the area of a triangle?
4. What is the formula for the area of a circle?
5. What is the sum of the measurements of two complementary angles?
6. What is the sum of the measurements of two supplementary angles?
7. What is the sum of the measurements of all three angles in a triangle?
8. What is an isosceles triangle?
9. What is the sum of the measurements of all four angles in a quadrilateral (square, diamond, rectangle, trapezoid, etc.)?
Solve by defining a variable, writing an equation, solving that equation, and stating the answer. Give simplified answers with units when appropriate.

10. Martita is planning to build a desk for her daughter. The top of the desk will be a rectangle, whose **width is three inches less than twice the length**. If the perimeter of the desktop is **156 inches**, what are the dimensions (width and length) of the desktop?

11. One angle of a triangle is **15°** larger than the smallest angle, and the third angle is **4 times** as large as the smallest angle. **Find the measure of the three angles. Give your answers accurate to 1 decimal place.**

12. One angle of a triangle is **14°** less than twice the smallest angle, and the third angle is **20°** larger than the smallest angle. **Find the measure of the three angles. Give your answers accurate to 1 decimal place.**

13. Victor is building a sand box for his niece. He plans to make it in the shape of an isosceles triangle. Two of the sides measure the same and the **third is three feet less than the measure of one of the others.** The perimeter of the sand box is **30 feet.** **Find the measure of the three sides.**

14. Oliver has a candy box that is an isosceles triangle. **Two of the angles measure the same** and the **third is 39° more than the measure of one of the others.** **Find the measure of the three angles.**

15. A “to-go” container from a restaurant has ends that are trapezoids. The measures of the two bottom angles are the same. **Each top angle measures 8° less than the measure of one of the bottom angles.** **Find the measure of each angle.**

16. Emily plans to run a drill for her soccer team that requires four equal areas, shown below (where the overall width is divided into four). She plans to separate the areas by fencing. **The length of the each fenced-in area is to be 10 feet greater than the width of each fenced-in area.** The **total amount of fencing available is 310 feet.** Find the length and width of each fenced-in area.

17. Angles A and B are **supplementary** angles, and the **measure of angle B is 132° less than the measure angle A.** **Find the measures of angle A and B.**

18. Angles A and B are **complementary** angles, and the **measure of angle B is 20° more than the measure of angle A.** **Find the measures of angle A and B.**

19. A bookcase is to have three shelves as shown. **The height of the bookcase is to be two feet less than the width,** and only **26 feet of lumber is available.** **What should be the width and height of the bookcase?**
Section 3.5  General Application Problems of Linear Equations in One Variable

3.5 — General Application Problems of Linear Equations in One-Variable Worksheet

Example:
Define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units when appropriate.

a) The hourly self-sufficiency wage for a single adult in Santa Cruz County increased 33% [between 2003 and 2011].
—Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
If the hourly self-sufficiency wage for a single adult in Santa Cruz County in 2011 was $15.28, what was the hourly self-sufficiency wage for a single adult in Santa Cruz County in 2003? Round your answer to the nearest cent.

Hourly self-sufficiency wage for a single adult in SC County in 2003: x

The hourly self-sufficiency wage increased 33% from 2003 to 2011.

Define what the problem asks to find.

Use the appropriate statement from the problem to write an equation.

the hourly self-sufficiency wage in 2003 + 33% of the hourly self-sufficiency wage in 2003 = the hourly self-sufficiency wage in 2011

\[ x + 0.33x = 15.28 \]

Solve the equation.

\[ 1.33x = 15.28 \]

\[ x \approx 11.488721 \ldots \]

The hourly self-sufficiency wage for a single adult in Santa Cruz County in 2003 was approximately $11.49.

Homework

For each problem below, define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units when appropriate.

1. More than half of Caucasian [Santa Cruz County] CAP respondents reported saving money for the future through…retirement compared to…21% [of Latinos] who saved through retirement, a statistically significant difference between Latinos and Caucasians. [The number of Caucasian respondents who reported saving through retirement was 27 more than seven times the number of Latino respondents who reported saving through retirement. Thirty respondents who reported saving through retirement did not identify as Caucasian or Latino.]
—Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
If the total number of respondents who reported saving through retirement was 321, how many identified as Latino and how many identified as Caucasian?

2. Two numbers are even consecutive integers. If the sum of the numbers is 318, what are the numbers?

3. [In 2011, the] median family income in Santa Cruz County was…$15,400 more than the California median.
—Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
In 2011, if the sum of the median family income in Santa Cruz County and the median family income in the California is $156,200, what was the median family income in California and what was the median family income in Santa Cruz County?

4. Eduardo bought a pair of shoes that were 20% off of the regular price. If he paid a total of $40 and there is no tax, how much were his shoes?
5. Santa Cruz County saw a decrease of 13% in taxable sales from 2008 to 2009 in all jurisdictions.  
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

If Santa Cruz County’s taxable sales in 2009 were $2,638,469, what were Santa Cruz County’s taxable sales in 2008? Round your answer to the nearest dollar.

6. Across the 34 countries that make up the [Organization for Economic Cooperation and Development’s] membership, the average income of the richest 10 percent of the population is nine times that of the poorest 10 percent.
   —Eric Pfanner, nytimes.com, January 24, 2012

If this same pattern of disparity is applied to the entire world and the sum of the income of the richest 10 percent of the population and the income of the poorest 10 percent of the population is $46,400, what is the average income of the poorest 10 percent of the population?

7. The wealthiest 10 percent of Americans…earned 11.5 times…made by those living near or below the poverty line in 2008.
   —Associated Press, msnbc.msn.com, November 29, 2009

If the difference of the income of the wealthiest 10 percent of Americans and the income of Americans living near or below the poverty line is $126,000, what is the average income of the poorest 10 percent of Americans and the richest 10 percent of Americans?

8. Dominique brought a new jacket. If sales tax was 8% and the total amount she paid (including the sales tax) was $35.90, what was the cost of the jacket before tax? Round your answer to the nearest cent.

9. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the unemployment rate (%) of a person with a high school diploma was 0.5 less than twice a person with a Bachelor’s degree.
   If the difference of the unemployment rate of a person with a high school diploma and the unemployment rate of a person with a Bachelor’s degree was 4.9, what is the unemployment rate of a person with a Bachelor’s degree and what is the unemployment rate of a person with a high school diploma?

10. More than 14% of [Santa Cruz County] CAP survey respondents had moved at least once in the past 12 months. The percentage was higher among Latino respondents compared to Caucasian respondents (33% and 9% respectively), a statistically significant difference.  
    The number of Caucasian respondents who reported moving at least once in the past year was ten less than the number of Latino respondents who reported moving at least once in the past year. Five respondents who reported moving at least once in the past year did not identify as Caucasian or Latino.
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

In 2011, if the total number of respondents who reported moving at least once in the last year was 101, how many identified as Latino and how many identified as Caucasian?

11. Two numbers are consecutive odd integers. If the sum of the numbers is 380, what are the numbers?

12. The 2010 Santa Cruz County] median home sales price has decreased by…$225,000 [since 2007.]
    —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

If the Santa Cruz County median home sale price in 2010 was $347,000, what is the Santa Cruz County median home sales price in 2007?

13. $75 is to be split between Martita and Nicole. If Martita gets $12 more than twice as much as Nicole, how much money does each person receive?

14. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the weekly earnings (in U$) of a person with a Master’s degree were $20 more than twice the weekly earnings of a person with a high school diploma.
   If the difference of the weekly earnings of a person with a Master’s degree and the weekly earnings of a person with less than a high school diploma was $646, what were the weekly earnings of a person with a high school diploma and what were the earnings of a person with a Master’s degree?
15. While more than 57% of CAP survey respondents felt they had job opportunities in Santa Cruz County in 2011, this varied by income level. Of the respondents who indicated they earned less than $35,000 per year, 51% felt they had opportunities to work in the area compared to 69% of respondents who earned $65,500 per year or more, a statistically significant difference.

[The number of respondents who earned less than $35,000 per year and felt they had job opportunities in Santa Cruz County was 23 less than the number of respondents who earned $65,500 per year or more and felt they had job opportunities in Santa Cruz County. Ninety-two respondents earned between $35,000 and $65,500 per year.]

—Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

In 2011, if the total number of respondents who felt they had job opportunities in Santa Cruz County was 379, how many earned $65,500 per year or more and how many earned less than $35,000 per year?

16. A larger number is fourteen more than half the smaller number. If the difference between the numbers is 10, what are the numbers?

17. In California, the average price for a gallon of gas in 2007 was $3.29. If the average price for a gallon of gas has increased about $0.20 per year, what was the price for a gallon in 2012?

18. A sales associate at Walmart makes an average of $12.74 per hour (walmartstores.com). How many hours would a Walmart employee have to work in order to make $672.25, the 2011 Santa Cruz County self-sufficiency weekly wage for a single adult (appliedsurveyresearch.org)? Round your answer to the nearest hour.

19. While Santa Cruz County’s population has grown consistently over the past decade, it is growing less than half as fast as the state of California and was at 272,201 individuals as of January 2010.

—city-data.com

If Santa Cruz County’s population is growing by about 7,000 per year, when will the population reach 300,000?

20. Verizon Wireless offers a single-line plan that costs $39.99 plus $0.45 per minute for overage minutes. Zack has this plan and also pays a “Messaging” fee of $10. If his bill this month was $85.99 (not including taxes), how many overage minutes did he use?

21. The average monthly rent [for a three-bedroom dwelling] in Santa Cruz County has been steadily increasing, showing a 16% increase [between 2008 and 2011].

—Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

If the difference in the average monthly rent for a three-bedroom dwelling in Santa Cruz County between 2011 and 2008 was approximately $375, what is the average monthly rent for a three-bedroom dwelling in Santa Cruz County in 2008 and what is the average monthly rent for a three-bedroom dwelling in Santa Cruz County in 2011? Round your answers to the nearest dollar.
Section 3.6  Distance, Rate, Time Application Problems

3.6 — Distance, Rate, Time Application Problems Worksheet

Example:
Define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units when appropriate. You may want to use a chart to organize the information in the problem.

a) Two stunt planes plan to fly towards each other, with one passing just below the other. The orange plane’s rate is 15 mph less than the rate of the purple plane. If they start out 2.5 miles apart, and it takes 2 minutes for them to meet (pass), what is the purple plane’s rate and what is the orange plane’s rate?

the rate of purple plane: \( x \)
the rate of orange plane: \( x - 15 \)

Define what the problem asks to find.

Fill in the distance/rate/time chart with the givens and variables.

Note that 2 minutes is \( \frac{2}{60} = \frac{1}{30} \) hour.

<table>
<thead>
<tr>
<th>Planes</th>
<th>Rate</th>
<th>Time</th>
<th>Distance = Rate \cdot Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purple</td>
<td>( x )</td>
<td>( \frac{1}{30} )</td>
<td>( \frac{1}{30} x )</td>
</tr>
<tr>
<td>Orange</td>
<td>( x - 15 )</td>
<td>( \frac{1}{30} )</td>
<td>( \frac{1}{30} (x - 5) )</td>
</tr>
</tbody>
</table>

Unused fact/picture:

They start out 2.5 miles apart.

“the purple plane’s distance” + “the orange plane’s distance” = 2.5 miles

\[
\frac{1}{30} x + \frac{1}{30} (x - 5) = 2.5
\]

Solve the equation:

\[
\frac{1}{30} x + \frac{1}{30} (x - 5) = 2.5
\]

\[
\frac{1}{30} x + \frac{1}{30} x - \frac{1}{6} = 2.5
\]

\[
\frac{2}{30} x - \frac{1}{6} = 2.5
\]

\[
\frac{1}{15} x - \frac{1}{6} = 2.5
\]

\[
30 \cdot \frac{1}{15} x - 30 \cdot \frac{1}{6} = 30 (2.5)
\]

\[
2x - 5 = 75
\]

\[
2x = 80
\]

\[
x = 40
\]

The purple plane’s rate is 40 mph and the orange plane’s rate is \( 40 - 15 = 25 \) mph.

Homework

1. Define “distance” in your own words and give some examples of units for distance.

2. Define “time” in your own words and give some examples of units for time.

3. Define “rate” in your own words and give some examples of units for rate.

4. How can you find a distance given a rate and a time?

5. Describe how to use a “chart” to organize information in a distance/rate/time application problem.
Use the distance/rate/time formula to answer each question.

6. It took Ramon 30 minutes to drive 20 miles. What was his average speed?

7. If you travel in a bike at 8 mph for half an hour, how far have you traveled?

8. Stuart plans to drive from Sacramento to Los Angeles (approximately 385 miles). If he drives an average of 70 mph, how long will it take?

For each problem below, define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units when appropriate. You may want to use a chart to organize the information in the problem.

9. A rescue boat with paramedics is traveling north at 50 mph to meet a sailboat with an injured passenger that is traveling south at 30 mph. If the distance between the boats when they started traveling towards each other is 10 miles, how long will it take for them to meet? After you figure out your answer in hours, give your answer in minutes accurate to 1 decimal place.

10. Elyzabeth is pulling her son in a wagon at 2 mph. Shannon begins riding her bike alongside the wagon and is traveling at 5 mph. How long will it take for Elyzabeth and Shannon to be 1.5 miles apart? Give your answer accurate to 1 decimal place.

11. Two cars leave a jazz festival, one heading north and the other heading south. The northbound car is traveling 19 mph faster than the southbound car. If the cars are 260 miles apart after 2 hours, how fast is each car traveling? Give your answer accurate to 1 decimal place.

12. California’s planned 220 mph high-speed train system will cost less than half as much as building more freeway lanes and airport runways and will increase mobility while cutting air pollution and reducing the greenhouse gas emissions that cause global warming. —cahighspeedrail.ca.gov

If a high-speed train traveling at an average of 220 mph leaves Sacramento and arrives in Los Angeles, and at the same time an Amtrak train traveling at an average of 100 mph left Los Angeles and arrived in Sacramento, the Amtrak’s trip would take 2.1 hours longer than the high-speed train’s trip. (Trips do not include stops and distance in either direction is the same.) How long does it take the high-speed train to make its trip? Give your answer accurate to 2 decimal places.

13. One afternoon, it took John 15 minutes to mow his family’s lawn with a riding-lawnmower that he borrowed from his neighbor. Two weeks later, it took his sister Greta 2.5 hours to use a gas-powered push lawnmower to mow the same lawn. If John’s rate was 22.5 mph faster than Greta’s rate, what were each of their mowing rates? Give your answer accurate to 1 decimal place.

14. Two friends, Derrick and Sirrom, are 2.2 miles away from each other on opposite ends of a park. They start riding their bikes towards each other at the same time. If Derrick is riding at 12 mph and Sirrom is riding at 10 mph, how long will it take for them to meet? Give your answer accurate to 1 decimal place.

15. Sisters JoJo and Jasmine decide to have a bike race. They both start at the same time from the same point and bike in the same direction down the same street. JoJo bikes 70 meters per minute. After 4 minutes, they are 6 meters apart. Determine the speed at which Jasmine is biking. Give your answer accurate to 1 decimal place.

Extra: What is JoJo’s winning time for a 100 meter bike race? Give your answer accurate to 2 decimal places.
Chapter 4 – Graphing Linear Equations in Two-Variables

Section 4.1 – The Rectangular (Cartesian) Coordinate System
Section 4.2 – Graphing Linear Equations in Two-Variables
Section 4.3 – Intercepts and Slope
Section 4.4 – Equations of Lines
Section 4.5 – Application Problems of Linear Equations in Two-Variables

Answers
Section 4.1  The Rectangular (Cartesian) Coordinate System

4.1 — The Rectangular (Cartesian) Coordinate System Worksheet

Examples:
Draw a rectangular (Cartesian) coordinate system (graph). Label the axes with the appropriate variable and units, and then graph each point below on that same graph, using both the ordered pair and letter as a label.

Points are called ordered pairs, as the numbers are in order so that the first number indicates the independent variable (often $x$) and the second number indicates the dependent variable (often $y$).

On the grid, the first number (independent variable) indicates a horizontal (right/left) movement. “$x$ movements” are right if the number is positive and left if the number is negative.

On the grid, the second number (dependent variable) indicates a vertical (up/down) movement. “$y$ movements” are up if the number is positive and down if the number is negative.

a) $A (4, 6)$
To graph this point, from the origin move right (positive) 4 and up (positive) 6.

b) $B (0, -8)$
To graph this point, from the origin do not move right or left as $x$ is 0 and down (negative) 8.

c) $C (-3, 5)$
To graph this point, from the origin move left (negative) 3 and up (positive) 5.

---

Homework
1. Which variable ($x$ or $y$) usually represents the independent variable? What axis is used to represent the independent variable (horizontal or vertical)? What kind of movement does the value of this variable represent (left/right or up/down)?

2. Which variable ($x$ or $y$) usually represents the dependent variable? What axis is used to represent the dependent variable (horizontal or vertical)? What kind of movement does the value of this variable represent (left/right or up/down)?

3. Why are the points on a rectangular (Cartesian) coordinate system called “ordered pairs”?

4. Draw a rectangular (Cartesian) coordinate system (graph). Label the axes with the appropriate variable and quadrants.
5. **Determine the correct ordered pair for each point graphed below.**

   ![Graph](image)

6. **Determine the correct ordered pair for each point graphed below. Pay attention to the units!**

   ![Graph](image)

For each problem, draw a rectangular (Cartesian) coordinate system (graph). Label the axes with the appropriate variable and units, and then graph each point below on that same graph, using both the ordered pair and letter as a label. You should have one graph for each separate problem, so three graphs for problems 7–9.

7. a) \( (0, 2) \)
   b) \( \left( \frac{1}{2}, -3 \right) \)
   c) \( (-4, 1) \)
   d) \( (-3, -5) \)
   e) \( (-4, 0) \)

8. a) \( (40, 10) \)
   b) \( (-10, 30) \)
   c) \( (50, 0) \)
   d) \( (20, -\frac{70}{2}) \)
   e) \( (0, -20) \)

9. a) \( \left(-\frac{1}{4}, 1 \right) \)
   b) \( (0, \frac{3}{4}) \)
   c) \( (-\frac{2}{3}, 0) \)
Section 4.2  Graphing Linear Equations in Two-Variables

4.2 — Graphing Linear Equations in Two-Variables Worksheet

Example:

Graph each equation by plotting at least three points

a) \( \frac{1}{3}x + y = -2 \)

A graph of an equation is an illustration of all of the solutions (in this case, ordered pairs) that satisfy that equation.

To find points, choose either an \( x \)-value (or a \( y \)-value), plug it into the equation, and then find its “partner”.

Make the choices easy to graph, so choose numbers that are close to the origin and think ahead about the computation needed to obtain the “partner”.

For this problem, it’s easiest to choose \( x \)-values that are divisible by 3, since that is the denominator of the coefficient of \( x \).

Choosing \( x = 0 \), will yield:

\[
\frac{1}{3}(0) + y = -2
\]

\[
0 + y = -2
\]

\[
y = -2
\]

This corresponds to the point \((0, -2)\).

Choosing \( x = 3 \), will yield:

\[
\frac{1}{3}(3) + y = -2
\]

\[
1 + y = -2
\]

\[
y = -3
\]

This corresponds to the point \((3, -3)\).

Choosing \( x = -3 \), will yield:

\[
\frac{1}{3}(-3) + y = -2
\]

\[
-1 + y = -2
\]

\[
y = -1
\]

This corresponds to the point \((-3, -1)\).

Now graph each of the points and draw a straight line through them.
Homework

1. In your own words, describe how to recognize that an equation’s graph will be a straight line.

2. In your own words, describe how to check if a point is a solution to a linear equation in two variables.

3. In your own words, describe how to determine if a linear equation’s graph is a “slanted”, horizontal, or vertical line, just by looking at the equation.

4. In your own words, describe how to graph a linear equation in two variables by plotting points.

Determine which of the points below are solutions to the given equation. Graph each of the points that are solutions and then draw a line through them.

5. \(2x - 3y = 10\)
   a) \((5, 0)\)
   b) \((-\frac{5}{3}, -5)\)
   c) \((0, 5)\)
   d) \((-1, -4)\)
   e) \((1, 3)\)

Determine which of the points below are solutions to the given equation. Graph each of the points that are solutions and then draw a line through them.

6. \(y = \frac{1}{2}x + 1\)
   a) \((-2, 1)\)
   b) \((-2, 0)\)
   c) \((0, 1)\)
   d) \((-1, 3)\)
   e) \((-4, -1)\)

Determine which of the points below are solutions to the given equation. Graph each of the points that are solutions and then draw a line through them.

7. \(y = -2\)
   a) \((-2, 0)\)
   b) \((0, -2)\)
   c) \((-3, 1)\)
   d) \(\left(\frac{1}{2}, -2\right)\)
   e) \((-4, -2)\)

Graph each equation by plotting at least three points.

8. \(2x - y = 4\)
9. \(2y - x = 6\)
10. \(y = 4\)
11. \(y = x + 3\)
12. \(4x - 2y = 12\)
13. \(x = \frac{1}{3}y + 5\)
14. \(y = 2x\)
15. \( x = 4y \)
16. \( x = -1 \)
17. \( y = \frac{2}{3}x - 1 \)
18. \( 2x = y + 3 \)
19. \( 2y = x + 1 \)
20. \( 5x - 2y = 20 \)
21. \( 2y = -6 \)
22. \( 2 = 4x \)
23. \( 3x + 2y = 6 \)
24. \( x + y = 0 \)
25. \( 3x - 3y = 3 \)
26. \( 2y - 4x = 8 \)
27. \( \frac{1}{2}x + y = 5 \)
28. \( \frac{1}{4}x + \frac{1}{3}y = 1 \)
29. \( \frac{2}{3}x = \frac{1}{2}y - 4 \)
30. \( x = \frac{5}{3}y + 1 \)
Section 4.3  Intercepts and Slope

4.3 — Intercept and Slope Worksheet

Examples:

Find the \(x\)-intercept, the \(y\)-intercept, and a different third “check” point, then graph the line.

a) \(2x - 3y = 12\)

Finding the \(x\)-intercept and the \(y\)-intercept is the same as finding other points, except you must use specific values for \(x\) and \(y\). Remember, the \(x\)-intercept is a point different than the \(y\)-intercept, unless the line passes through the origin.

To find the \(x\)-intercept, \(y = 0\) and solve for \(x\):

\[
\begin{align*}
2x - 3(0) &= 12 \\
2x &= 12 \\
x &= 6
\end{align*}
\]

This means the \(x\)-intercept is the point \((6, 0)\). Note: this point is always on the \(x\)-axis.

To find the \(y\)-intercept, \(x = 0\) and solve for \(y\):

\[
\begin{align*}
2(0) - 3y &= 12 \\
-3y &= 12 \\
y &= -4
\end{align*}
\]

This means the \(y\)-intercept is the point \((0, -4)\). Note: this point is always on the \(y\)-axis.

To find a “check” point, choose any value for \(x\) or \(y\) (other than the values already found), and find its “partner”. This point checks the work used to find the intercepts. If all three points line up, then most likely, the work was correct.

Choosing \(x = 3\), will yield:

\[
\begin{align*}
2(3) - 3y &= 12 \\
6 - 3y &= 12 \\
-3y &= 6 \\
y &= -2
\end{align*}
\]

This corresponds to the point \((3, -2)\).

Now graph each of the points and draw a straight line through them.
Determine the slope between the pair of points.

b) \( \left( \frac{2}{3}, -2 \right) \) and \( \left( -\frac{4}{9}, \frac{1}{6} \right) \)

Slope is the change in \( y \) over the change in \( x \). The formula for slope is \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

Let \( \left( \frac{2}{3}, -2 \right) = (x_1, y_1) \) and \( \left( -\frac{4}{9}, \frac{1}{6} \right) = (x_2, y_2) \). Using the formula:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1/6 - (-2)}{-4/9 - 2/3}
\]

Now, simplify the expression. There are different ways to simplify. One way is below.

\[
= \frac{\frac{1}{6} + 2}{-\frac{4}{9} - \frac{2}{3}}
\]

The LCD of the top is 6. The LCD of the bottom is 9.

\[
= \frac{\frac{1}{6} + \frac{4}{6}}{-\frac{4}{9} - \frac{6}{9}} \rightarrow \frac{\frac{1}{6} + \frac{12}{9}}{-\frac{4}{9} - \frac{6}{9}} \rightarrow \frac{\frac{13}{6}}{-\frac{10}{9}}
\]

This fraction can be written as a division problem.

\[
= \frac{13}{6} \cdot \left( -\frac{10}{9} \right) \rightarrow \frac{13}{6} \cdot \left( -\frac{9}{10} \right) \rightarrow \frac{13}{6} \cdot \left( -\frac{3}{10} \right) \rightarrow -\frac{39}{20}
\]

The slope is \(-\frac{39}{20}\).

**Homework**

1. In your own words, describe how to find the **y-intercept** of a linear equation in two-variables.

2. In your own words, describe how to find the **x-intercept** of a linear equation in two-variables.

3. In your own words, describe how to find the **slope** of a line between two points.

4. What is the slope of any **horizontal** line?

5. What is the slope of any **vertical** line?

6. In your own words, describe how to use slope to determine if two lines are **parallel**.

7. In your own words, describe how to use slope to determine if two lines are **perpendicular**.

**Find** **x-intercept**, **y-intercept** **and** **slope** **of** **each** **line** **graphed**.

8. \( y = -2x + 4 \)
9. \( y = 4 \)

10. \( x - 2y = 6 \)

11. \( 2x = -4 \)
12. $-2x + 3y = 120$

13. $y = -\frac{1}{3}x - 2$

14. $10x - y = 20$
Find the x-intercept and the y-intercept. If either does not exist, state so.

15. \( x - y = -3 \)
16. \( 11x + 4y = 88 \)
17. \( 3y = 60 \)
18. \( y = -\frac{5}{9}x - 4 \)
19. \( 7x + 14 = 0 \)

Find the x-intercept, the y-intercept, and a different third “check” point, then graph the line.

20. \( 5x - 3y = 15 \)
21. \( x + 2y = 10 \)
22. \( y = -2x - 2 \)
23. \( x = -4 \)
24. \( 2x + 3y = 60 \)
25. \( 2y = 20 \)
26. \( x = 4y - 5 \)

Determine the slope between each pair of points.

27. \((0, -2) \text{ and } (4, 10)\)
28. \((11, -3) \text{ and } (7, 1)\)
29. \((13, 5) \text{ and } (-4, 5)\)
30. \((-9, 0) \text{ and } (6, 5)\)
31. \((-8, 4) \text{ and } (-2, 3)\)
32. \((7, -2) \text{ and } (8, 4)\)
33. \((6, -8) \text{ and } (6, 9)\)
34. \((-1, -5) \text{ and } (-2, -3)\)
35. \((1, 0) \text{ and } (4, 0)\)
36. \((-2, 4) \text{ and } (4, 14)\)
37. \((7, 10) \text{ and } (11, 8)\)
38. \((0, -1) \text{ and } (0, 6)\)
39. \((\frac{7}{2}, \frac{1}{2}) \text{ and } (\frac{1}{2}, \frac{3}{2})\)
40. \((-\frac{1}{2}, -\frac{1}{2}) \text{ and } (1, -1)\)
41. \((\frac{5}{7}, 4) \text{ and } (\frac{1}{6}, 4)\)
42. \((5, -\frac{3}{2}) \text{ and } (3, \frac{1}{2})\)
For each problem below, the slopes of two distinct lines are listed. Use the slopes to determine whether each pair of lines is parallel, perpendicular, or neither parallel or perpendicular.

43. Line 1: \( m_1 = 2 \)  \hspace{1cm} Line 2: \( m_2 = -2 \)

44. Line 1: \( m_1 = \frac{1}{3} \)  \hspace{1cm} Line 2: \( m_2 = -3 \)

45. Line 1: \( m_1 = \frac{5}{7} \)  \hspace{1cm} Line 2: \( m_2 = \frac{7}{5} \)

46. Line 1: \( m_1 = \frac{1}{2} \)  \hspace{1cm} Line 2: \( m_2 = \frac{1}{2} \)

47. Line 1: \( m_1 = 4 \)  \hspace{1cm} Line 2: \( m_2 = \frac{1}{4} \)

Use the pair of points given for each line to find the slope of that line and then determine if the pair of lines is parallel, perpendicular, or neither parallel or perpendicular.

48. Line 1: \( (6, 4) \) and \( (2, 3) \)  \hspace{1cm} Line 2: \( (2, 11) \) and \( (1, 3) \)

49. Line 1: \( (2, 12) \) and \( (8, 9) \)  \hspace{1cm} Line 2: \( (-3, 6) \) and \( (1, 4) \)

50. Line 1: \( (7, 5) \) and \( (4, 3) \)  \hspace{1cm} Line 2: \( (-5, -10) \) and \( (-3, -7) \)

51. Line 1: \( (8, 11) \) and \( (14, 11) \)  \hspace{1cm} Line 2: \( (5, -3) \) and \( (5, 8) \)
Section 4.4  Equations of Lines

4.4 — Equations of Lines Worksheet
4.4 — Equations of Lines Practice Worksheet

Examples:

Graph the equation.

a)  \(2y = -3x + 2\)

Use the \(y\)-intercept and the slope to graph the line. Solve the equation for \(y\) to get it into slope-intercept form \(y = mx + b\), so that the \(y\)-intercept and slope can be easily read from the equation.

\[
2y = -3x + 2
\]

\[
y = -\frac{3}{2}x + 1
\]

Divide all terms on both sides by 2, to isolate \(y\) on one side of the equation.

Notice that if \(x = 0\), then \(y = 1\). This gives the \(y\)-intercept: \((0, 1)\).

The slope is \(m = -\frac{3}{2}\). This means that a second point on a line may be found by starting at any point on the line and moving:

- \(\frac{3}{2}\) units down
- \(\frac{3}{2}\) units right

Graph the point found: \((0, 1)\) and then use the slope to find at least two other points.

Find an equation of the line described by the set of characteristics. Although you do not necessarily need to start with slope-intercept form, your final answer should be written in slope-intercept form.

b) The line goes through the points \((-3, 1)\) and \((2, -9)\).

To write an equation of the line, first find the slope.

Using the slope formula and letting \((-3, 1) = (x_1, y_1)\) and \((2, -9) = (x_2, y_2)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9 - 1}{2 - (-3)} = \frac{-10}{5} = -2
\]

Now, since the \(y\)-intercept is not given, one way to find an equation is to use point-slope form: \(y - y_1 = m(x - x_1)\).

Either point may now be considered \((x_1, y_1)\).

Choosing the first point \((-3, 1) = (x_1, y_1)\) yields:

\[
y - 1 = -2(x - (-3))
\]

The above is an equation of the line, but it is not in slope-intercept form. To get the equation into slope intercept from, solve for \(y\).

\[
y - 1 = -2(x - 3)
\]

Change subtracting \(-3\) to adding 3.

\[
y - 1 = -2(x + 3)
\]

Distribute \(-2\).

\[
y - 1 = -2x - 6
\]

Add 1 to both sides of the equation.

\[
y = -2x - 5
\]

The slope-intercept form of the equation of the line is \(y = -2x - 5\).
Homework

1. What is **slope-intercept form** of a linear equation?

2. What is **point-slope form**?

3. In your own words, describe the steps you need to perform on an equation in order to “read” the slope and the **y-intercept** off of a given linear equation in two variables.

4. In your own words, describe how to use the slope and the **y-intercept** of a linear equation in two variables to graph the equation of that line.

5. What is the **slope** of any **horizontal** line? What is the general form of the **equation** of any **horizontal** line?

6. What is the **slope** of any **vertical** line? What is the general form of the **equation** of any **vertical** line?

7. If you plan to use slope-intercept form of a linear equation to write the equation of a line, what information about that line must you find before you write the equation of that line?

8. If you plan to use point-slope form of a linear equation to write the equation of a line, what information about that line must you find before you write the equation of that line?

Without graphing, find the **slope** and **y-intercept** of each line whose equation is below.

9. \( y = -2x - 7 \)

10. \( y = x - 3 \)

11. \( y = 5x + \frac{3}{4} \)

12. \( 3x + y = 11 \)

13. \( 10x + 2y = 30 \)

14. \( x = -2y + 3 \)

15. \( y = -3 \)

16. \( y - 6 = 7x \)

17. \( y - 1 = -3(x + 5) \)

Find **x-intercept**, **y-intercept**, **slope** and **equation** of each line graphed.

18. [Diagram of a graphed line with labeled intercepts and slope]
19.

20.

21.

22.
23. Graph each equation.

24. \( y = 2x - 4 \)

25. \( y = \frac{1}{2} x + 1 \)

26. \( y = -x + 3 \)

27. \( y = -\frac{3}{4} x \)

28. \( y = 3 \)

29. \( 8 = -2x \)

30. \( 3x + y = 2 \)

31. \( y = x \)

32. \( 2x - 3y = 2 \)

33. \( y = \frac{5}{2} \)

34. \( 40x + 30y = 120 \)

35. \( x - 5y = 2 \)

36. \( 2x + 2y = -1 \)

37. \( x = -4y + 2 \)

38. \( 8 - 3x = y \)

39. \( 9 - 3y = 5x \)

40. \( \frac{1}{2} x - y = \frac{2}{3} \)

41. \( 1 + \frac{3}{2} y = \frac{6}{3} x \)

Find an equation of the line described by each set of characteristics. Although you do not necessarily need to start with slope-intercept form, your final answer should be written in slope-intercept form when possible.

42. The line has slope \( m = \frac{3}{7} \) and goes through the point \((0, 6)\).

43. The line has slope \( m = -3 \) and goes through the point \((0, \frac{1}{2})\).

44. The line has slope \( m = 1 \) and goes through the point \((0, -3)\).
45. The line has slope \( m = 0 \) and goes through the point \((5, 2)\).

46. The line has slope \( m = \text{undefined} \) and goes through the point \((-4, 1)\).

47. The line has slope \( m = 4 \) and goes through the point \((2, -1)\).

48. The line has slope \( m = -\frac{4}{2} \) and goes through the point \((-6, 5)\).

49. The line has slope \( m = \frac{3}{4} \) and goes through the point \((0, 10)\).

50. The line goes through the points \((0, -2)\) and \((4, 10)\).

51. The line goes through the points \((3, 0)\) and \((0, -4)\).

52. The line goes through the points \((0, 5)\) and \((2, 7)\).

53. The line goes through the points \((7, 2)\) and \((15, 2)\).

54. The line goes through the points \((-3, 9)\) and \((-5, 10)\).

55. The line goes through the points \((\frac{1}{4}, 2)\) and \((\frac{5}{4}, 6)\).

56. The line goes through the points \((-13, 2)\) and \((-13, 8)\).

57. The line is parallel to a line with slope \( m = 5 \) and goes through the point \((0, -4)\).

58. The line is parallel to a line with slope \( m = -3 \) and goes through the point \((1, -2)\).

59. The line is parallel to the line \(y = 3x - 2\) and goes through the point \((0, 1)\).

60. The line is parallel to the line \(y = x\) and goes through the point \((-6, 3)\).

61. The line is parallel to the line \(2y + x = 5\) and goes through the point \((0, -10)\).

62. The line is parallel to the line \(y - 2x = 8\) and goes through the point \((5, -7)\).

63. The line is perpendicular to a line with slope \( m = 7 \) and goes through the point \((0, 12)\).

64. The line is perpendicular to a line with slope \( m = -\frac{2}{3} \) and goes through the point \((6, -2)\).

65. The line is perpendicular to the line \(y = 2x - 4\) and goes through the point \((0, -13)\).

66. The line is perpendicular to the line \(y = 4x\) and goes through the point \((2, 0)\).

67. The line is perpendicular to the line \(3y + 4x = 12\) and goes through the point \((0, 11)\).

68. The line is perpendicular to the line \(y + x = 4\) and goes through the point \((-3, -4)\).
Section 4.5  Application Problems of Linear Equations in Two-Variables

4.5 — Application Problems of Linear Equations in Two-Variables Worksheet

Examples:
The graph below shows the relationship between Fahrenheit temperature and Celsius temperatures

a)  Determine the slope of the line.

Pick any two points on the line and use the slope formula.

Choosing \((0, 32) = (x_1, y_1)\) and \((-20, -4) = (x_2, y_2)\)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 32}{-20 - (0)} = \frac{-36}{-20} = \frac{9}{5}
\]

With units: \(\frac{9 \text{ degrees Fahrenheit}}{5 \text{ degrees Celsius}}\)

or for every increase of 9°F, there is a 5°C increase.

b)  Determine the equation of the line.

The \(y\)-intercept is \((0, 32)\), and the slope is \(\frac{9}{5}\), so the equation of the line is:

\[y = \frac{9}{5}x + 32\]  
or (to keep track of which degree is \(x\) or \(y\)): \[F = \frac{9}{5}C + 32\]

c)  Use the equation you obtained in part b) to find the Fahrenheit temperature when the Celsius temperature is 20°C.

Plugging \(C = 20\) into the equation \(F = \frac{9}{5}C + 32\) yields:

\[F = \frac{9}{5}(20) + 32\]

20°C is the same as 68°F.

d)  Use the equation your equation to determine the Celsius temperature when the Fahrenheit temperature is 14°F.

Plugging \(F = 14\) into the equation \(F = \frac{9}{5}C + 32\) yields:

\[14 = \frac{9}{5}C + 32\]

\[-18 = \frac{9}{5}C\]

\[\frac{5}{9}(-18) = \frac{9}{5}C\]

\[-10 = C\]

14°F is the same as -10°C.

Homework

For each line, state the slope with correct units. Give simplified answers.

1.  feet

seconds
2. dollars

For each problem below, find a linear equation that models the relationship between the two quantities, and then use that equation to answer the question(s).

5. George is selling raffle tickets. He begins with no money, since he hasn't sold any tickets. After he has sold 5 tickets, he has $2.50. The graph that illustrates this example is in problem #2.
   a) Write a linear equation for the amount of money he has in terms of the number of tickets sold.
   b) How many tickets must he sell in order to have $45.00?
6. Quinn plans to rent a moving van. The rental company has told her that there is an initial flat fee of $20, and that if she drives 15 miles, the cost is $25.
   a) Write a linear equation for the cost in terms of the number of miles driven.
   b) What is the cost for driving 21 miles?
   c) How many miles did she drive the van, if the cost was $32?

7. For many years, people have recognized a relationship between the temperature (Fahrenheit) and the rate at which crickets are chirping (number of chirps in a minute). If the temperature is 50°, there are about 40 chirps in a minute. If the temperature is 75°, there are about 140 chirps in a minute. Note: this is a linear model only for temperatures between 40° and 90°.
   a) Write a linear equation for the number of chirps in a minute in terms of the temperature (Fahrenheit).
   b) If the temperature is 85°, approximately how many chirps in a minute will there be?
   c) If there are 80 chirps in a minute, approximately what is the temperature?

8. Jordan sells handmade necklaces. Jordan has found that more necklaces can be sold by lowering the price. If the price for each necklace is $15, Jordan can sell 50 necklaces. If the price for each necklace is $12, Jordan can sell 75 necklaces.
   a) Write a linear equation for number of necklaces sold in terms of price of each necklace.
   b) If each necklace costs $9, how many necklaces would sell?
   c) How much money would Jordan have in total by selling the number of necklaces you found in part (b) for $9 each?
   d) If Jordan wants to sell 125 necklaces, how much should each one cost?
   e) How much money would Jordan have in total by selling 125 necklaces for the cost you found in part (d)?
   f) Does Jordan make more money selling 50 necklaces for $15 each or 75 necklaces for $12 each?

9. During a summer backpacking trip in the Sierras, Celeste noticed that as she climbed higher (her altitude increased), it got colder (the temperature decreased). At an altitude of 4,000 feet, the temperature was 85°F. At an altitude of 6,000 feet, the temperature was 80°F.
   a) Write a linear equation for the temperature (°F) in terms of the altitude (feet).
   b) At 8,000 feet, what can Celeste expect the temperature to be?
   c) If the temperature was 90°, what would be Celeste’s expected altitude?
Chapter 5 — Systems of Linear Equations in Two-Variables

Section 5.1 — Solving Systems of Linear Equations by Graphing
Section 5.2 — The Substitution Method
Section 5.3 — The Addition/Elimination Method
Section 5.4 — Application Problems of Systems of Linear Equations
Answers
Section 5.1 Solving Systems of Linear Equations by Graphing

5.1 – Solving Systems of Linear Equations by Graphing Worksheet

Example:
Solve the system by **graphing**. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check. You must show a graph and state an answer for the problem.

a) \[ 4x + 2y = 8 \]
\[ 3x - y = 1 \]

To solve a system of linear equations by graphing, first graph each equation on the same set of axes.

First, graph the top equation. This will be the “**blue**” equation/line.

\[ 4x + 2y = 8 \]

There are different ways to graph this equation. One may graph by solving for \(y\) and using the slope and \(y\)-intercept, or graph by plotting points. It is written in standard form, so finding the intercepts is the easiest.

To find the \(x\)-intercept, plug in \(y = 0\) and solve for \(x\): \[ 4x = 8 \]
\[ x = 2 \]

To find the \(y\)-intercept, plug in \(x = 0\) and solve for \(y\): \[ 4(0) + 2y = 8 \]
\[ 2y = 8 \]
\[ y = 4 \]

Next, graph the bottom equation. This will be the “**red**” equation/line.

\[ 3x - y = 1 \]

This equation is written in standard form, but the intercepts contain fraction-values. To graph, choose to solve for \(y\), in order to write in slope-intercept form.

\[ -y = -3x + 1 \]
\[ y = 3x - 1 \]

Subtract \(3x\) from both sides of the equation.

Divide all terms on both sides by \(-1\). Now, the equation is in slope-intercept form.

For the “**red**” line, the \(y\)-intercept is \((0, -1)\). The slope is \(m = -3\).

Now, use the graph to find the solution to the system. Finding a solution to a system means finding an ordered pair (point) that satisfies BOTH equations.

In this case, the two lines intersect and that intersection point (which is the only point on BOTH lines, is the only solution to the system.

Description of the graph: **Two lines intersect.**

Characteristics of lines: The lines have different slopes—hence, they “**cross**”.

Type of system: **Consistent**

Answer: \((1, 2)\) The point of intersection is the solution to the system.

“**Blue**” check: “**Red**” check:

\[ 4(1) + 2(2) \neq 8 \]
\[ 4 + 4 \neq 8 \]
\[ 8 = 8 \]

\[ 3(1) - 2 \neq 1 \]
\[ 3 - 2 \neq 1 \]
\[ 1 = 1 \]
Homework

1. In your own words, describe how to check if a given point is a solution to a system of linear equations in two variables.

2. In your own words, describe how to use graphing to solve a system of linear equations in two variables.

3. What are the three different types of systems of linear equations in two variables?

4. When asked to solve a system of linear equations in two variables, what are the three appropriate types of answers?

5. How many solutions does a consistent system of linear equations in two variables have? Draw a sketch of an example of a graph of this type of system.

6. How many solutions does an inconsistent system of linear equations in two variables have? Draw a sketch of an example of a graph of this type of system.

7. How many solutions does a dependent system of linear equations in two variables have? Draw a sketch of an example of a graph of this type of system.

8. In your own words, describe how to determine without graphing if a system of linear equations in two variables is consistent.

9. In your own words, describe how to determine without graphing if a system of linear equations in two variables is inconsistent.

10. In your own words, describe how to determine without graphing if a system of linear equations in two variables is dependent.

Determine whether the given point is a solution to the system of linear equations.

11. \[\begin{align*}
y &= 5x - 4 \\
y &= -2x - 2
\end{align*}\] \((2, 6)\)

12. \[\begin{align*}
x - \frac{1}{2}y &= -5 \\
5x + 3y &= -3
\end{align*}\] \((-3, 4)\)

13. \[\begin{align*}
4x + 3 &= y \\
\frac{1}{2}y - 2 &= x
\end{align*}\] \((\frac{1}{2}, 5)\)

14. \[\begin{align*}
2x - 6y &= 13 \\
x - 3y &= 11
\end{align*}\] \((0, 8)\)

Determine the number of solutions each system graphed below has. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

15.
Solve each system by graphing. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check every other even problem. You must show a graph and state an answer for each problem.

20. \(y = -2x + 3\)
   \(y = 2x - 1\)

21. \(y = 3x - 2\)
   \(y = -\frac{1}{2}x + 5\)

22. \(y = -\frac{2}{3}x\)
   \(y - 2x = 8\)

23. \(-2x + 3y = 12\)
   \(y = \frac{2}{3}x - 1\)

24. \(y = 3\)
   \(x - 2y = -2\)

25. \(y = \frac{1}{2}x - 1\)
   \(y - 3x = 9\)

26. \(3x + y = 5\)
   \(y = -\frac{1}{2}x - 3\)

27. \(y = -2x + 4\)
   \(6x + 3y = 12\)

28. \(2x = -4\)
   \(y = x - 3\)

29. \(x + y = 4\)
   \(y = -x\)

30. \(y = 6x + 3\)
   \(6x + 3y = -15\)

Without graphing, determine the number of solutions each system has. If there is one solution, state so—you do not need to find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

31. \(y - 8x = 3\)
   \(2y = 16x - 2\)

32. \(5y = 3x\)
   \(y = 2x - 3\)

33. \(2x = 6\)
   \(11y + 22 = 0\)

34. \(-4x + 3y = 3\)
   \(6y = 8x + 6\)

35. \(y = -5\)
   \(2y = 14\)
Section 5.2  The Substitution Method

Example:
Solve each system by the Substitution Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

a) \[4x + 2y = 8\]
\[3x - y = 1\]

First, isolate a variable (either \(x\) or \(y\)) in one of the equations. If one of the variables has a coefficient of 1 or \(-1\), choose that variable to isolate.

For this system, the easiest variable to isolate is the \(y\) in the bottom equation.

\[-y = -3x + 1\] Subtract 3\(x\) from both sides of the equation.
\[y = 3x - 1\] Divide all terms on both sides by \(-1\). Now, \(y\) is isolated.

Next, plug in the expression you obtained by isolating into the OTHER equation for the corresponding variable.

For this system, plug in \(3x - 1\) for the \(y\) in the top (other) equation.

\[4x + 2(3x - 1) = 8\] Parentheses are used around the expression since it contains more than one term. Notice that after this substitution, the remaining equation only has one type of variable, in this case: \(y\). Solve for it.

\[4x + 6x - 2 = 8\]
\[10x - 2 = 8\]
\[10x = 10\]
\[x = 1\]

Now, plug the value found in step 2, to ANY equation with both variables to find value of the other variable in the system.

For this system, it’s easiest to plug 1 into the bottom equation for \(x\).

\[3(1) - y = 1\]
\[3 - y = 1\]
\[-y = -2\]
\[y = 2\]

The solution to this system is \((1, 2)\).

If the system were graphed, the two lines would intersect at the point \((1, 2)\). This is a consistent system with one solution.

Homework

1. What are the three different types of systems of linear equations in two variables?

2. When asked to solve a system of linear equations in two variables, what are the three appropriate types of answers?

3. When asked to solve a system of linear equations in two variables by the Substitution Method, how do you choose which variable in which equation to isolate?

4. When there is no solution to a system of linear equations in two variables, what does the last step of the Substitution Method look like?

5. When there are an infinite number of solutions to a system of linear equations in two variables, what does the last step of the Substitution Method look like?
Solve each system by the **Substitution Method**. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check every other odd problem. You must show substitution and state an answer for each problem.

6. \[ y = x + 8 \]
   \[ 2x + 3y = 9 \]

7. \[ x = 11 - 3y \]
   \[ 3x - 5y = -23 \]

8. \[ 5x - 2y = 14 \]
   \[ y = -3x - 7 \]

9. \[ 2y + x = -3 \]
   \[ x = 13 - 2y \]

10. \[ x - 3y = -1 \]
    \[ -2x + 5y = -1 \]

11. \[ 5x - 2y = 39 \]
    \[ 3x + y = 19 \]

12. \[ 2x - 4y = 6 \]
    \[ -x + 2y = -3 \]

13. \[ y = 3x - 13 \]
    \[ y = \frac{1}{2} x + 2 \]

14. \[ 3x = 6y + 9 \]
    \[ -4x + 8y = -12 \]

15. \[ 3x - 5y = 45 \]
    \[ 6x - 2y = 66 \]

16. \[ \frac{3}{2} x + \frac{1}{2} y = \frac{5}{2} \]
    \[ \frac{1}{4} x + \frac{3}{4} y = -\frac{1}{4} \]

17. \[ \frac{1}{2} x - \frac{1}{4} y = 1 \]
    \[ \frac{10}{3} x - \frac{2}{3} y = 5 \]

18. \[ \frac{1}{2} x + y = 4 \]
    \[ 3x - 2y = 12 \]

19. \[ 8x - 3y = 220 \]
    \[ 4x + 12y = 920 \]

20. \[ 0.15x - 0.10y = 6 \]
    \[ 0.3x + 0.1y = 3 \]

21. \[ 11x + 22y = -121 \]
    \[ 36x - 40y = -60 \]
Section 5.3 The Addition/Elimination Method

Example:
Solve the system by the Addition/Elimination Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

a) \[ \frac{2}{3}x + \frac{1}{3}y = \frac{4}{3} \]
\[ 3x - 2y = 6 \]

Since the top equation in this system contains fractions, multiply by it by the LCD = 3; yielding:
\[ 2x + y = 4 \]
\[ 3x - 2y = 6 \]

To solve by Addition/Elimination, first, choose which variable to eliminate. If one of the variables has a coefficient of 1 or –1, choose that variable to eliminate.

For this system, the easiest variable to eliminate is y.

Next, multiply one (or both) of the equations by the appropriate quantity, so that the coefficients of the variable to be eliminated are opposites.
For this system, to make the coefficients of y opposites, multiply the top equation by \( 2 \).
\[ 2 \left[ 2x + y = 4 \right] \quad \rightarrow \quad 4x + 2y = 8 \]
\[ 3x - 2y = 6 \]

Now, add the two equations together.
\[ 4x + 2y = 8 \]
\[ + 3x - 2y = 6 \]
\[ 7x = 14 \]

Solve for the remaining (non-eliminated) variable.
In this system, \( 7x = 14 \), so \( x = 2 \).

Finally, plug the value found into ANY equation with both variables to find value of the other variable in the system.
For this system, it’s easiest to plug 2 into the top equation (without fractions) for x.
\[ 2(2) + y = 4 \]
\[ 4 + y = 4 \]
\[ y = 0 \]

The solution to this system is \( (2, 0) \).

If the system were graphed, the two lines would intersect at the point \( (2, 0) \). This is a consistent system with one solution.

Homework

1. What are the three different types of systems of linear equations in two variables?
2. When asked to solve a system of linear equations in two variables, what are the three appropriate types of answers?
3. When asked to solve a system of linear equations in two variables by the Addition/Elimination Method, what is the first thing to check before you begin?
4. When asked to solve a system of linear equations in two variables by the Addition/Elimination Method, how do you choose which variable in which equation to eliminate?
5. When there is no solution to a system of linear equations in two variables, what does the last step of the Addition/Elimination Method look like?
6. When there are an infinite number of solutions to a system of linear equations in two variables, what does the last step of the Addition/Elimination Method look like?
Solve each system by the Addition/Elimination Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check every other odd problem. You must show addition/elimination and state an answer for each problem.

7. \[3x - y = 11\]
   \[2x + y = -1\]

8. \[x + 2y = 17\]
   \[-x + 5y = 39\]

9. \[3x + 2y = 12\]
   \[4x - y = 5\]

10. \[5x - 3y = -12\]
    \[8 - 2y = 5x\]

11. \[11x = 3y + 29\]
    \[2x + 6y = 38\]

12. \[2x - 4y = 10\]
    \[2y = x + 3\]

13. \[10y = 52 - 8x\]
    \[4x = 26 - 5y\]

14. \[3x - 8y = -11\]
    \[2x + 6y = 38\]

15. \[22 + 10y = 6x\]
    \[8x + 15y = 1\]

16. \[2x - 8y = 6\]
    \[3x = 12y + 9\]

17. \[10x = 15y + 8\]
    \[8x - \frac{18}{5} = 6y\]

18. \[7x = 9 - 2y\]
    \[4y = 11 - 14x\]

19. \[\frac{1}{2}x - \frac{5}{6}y = 24\]
    \[\frac{3}{2}x + \frac{1}{8}y = 3\]

20. \[\frac{1}{2}x + \frac{2}{3}y = 8\]
    \[\frac{3}{8}x + \frac{1}{6}y = \frac{10}{3}\]

21. \[1.25x + 2.35y = 47.75\]
    \[2.55x + 4.70y = 96\]

22. \[2x = 6\]
    \[11x - 17 = 8y\]
Section 5.4  Application Problems of Systems of Linear Equations

5.4 — Application Problems of Systems of Linear Equations Worksheet

Example:
For the problem below, define variables, write two equations, solve the system, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate. You may use a chart to organize your information, but it isn’t necessary.

a) Natalie owns a coffee shop. In her shop there are many varieties of coffee. One, an Ethiopian coffee sells for $7 per pound, and a second, a Colombian coffee, sells for $4 per pound. She’s found that some of her customers like a blend of the Ethiopian coffee and Colombian coffee. How much of each type of coffee should she mix to get 12 pounds of a mixture that sells for $6 per pound? Note: the price per pound of the mix is given, NOT the total value.

First, figure out what you are asked to find and what you are given.

Find:
Number of pounds of Ethiopian coffee:  \( x \)
Number of pounds of Colombian coffee:  \( y \)

Given:
Price per pound of Ethiopian coffee: $7 per pound
Price per pound of Colombian coffee: $4 per pound
Price per pound of mixture of both coffees: $6 per pound
Number of pounds in the mix of both coffees: 12 pounds

Sometimes it is helpful to organize this information in a chart. Notice that all quantities in an individual column have the same units. Price per pound is a type of rate.

<table>
<thead>
<tr>
<th>Title</th>
<th>Price per pound</th>
<th>Individual Amount</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethiopian</td>
<td>$7 per pound</td>
<td>( x )</td>
<td>7x</td>
</tr>
<tr>
<td>Colombian</td>
<td>$4 per pound</td>
<td>( y )</td>
<td>4y</td>
</tr>
<tr>
<td>Mix</td>
<td>$6 per pound</td>
<td>12 pounds</td>
<td>6(12)</td>
</tr>
</tbody>
</table>

Next, write two equations based on the non-rate columns.

Equation about the individual amounts:
number of pounds of Ethiopian coffee + number of pounds of Colombian coffee = number of pounds in the mix of both coffees
\[ x + y = 12 \]

Equation about total value:
total value of Ethiopian coffee + total value of Colombian coffee = total value in the mix of both coffees
\[ 7x + 4y = 72 \]

Now, solve the system. You may solve it by graphing, The Substitution Method, or The Addition/Elimination Method. The Addition/Elimination Method is shown below.

\[
\begin{align*}
7x + 4y &= 72 \\
-4x - 4y &= -48
\end{align*}
\]

\[
\begin{align*}
7x + 4y &= 72 \\
-4x - 4y &= -48 \\
3x &= 24
\end{align*}
\]

\[
\begin{align*}
3x &= 24 \\
x &= 8
\end{align*}
\]

Plugging in 8 for \( x \) into the top equation yields:
\[
\begin{align*}
8 + y &= 12 \\
y &= 4
\end{align*}
\]

Natalie should mix 8 pounds of French Roast and 4 pounds of Colombian.

Homework
1. In your own words, describe how you can recognize that an application problem may be modeled with a system of equations.
2. If you plan to use a chart to organize the information in this mixture-type of “system of linear equations application problem”, how many rows and columns will you need, and what will each be labeled?
For each problem below, define variables, write two equations, solve the system, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate. You may use a chart to organize your information, but it isn’t necessary.

3. Doug made a watermelon and strawberry salad for a potluck. Strawberries cost $3.25 per pound and watermelon costs $0.50 per pound. If he paid $7.00 to make a 3-pound salad, how many pounds of strawberries, and how many pounds of watermelon did he buy?

4. Ana has a 2003 VW Golf Tdi that she usually runs on biodiesel. During a recent trip to her biofuel supplier, she purchased a mix of B-99 and B-80. B-99 costs $4.55 per gallon and B-80 costs $4.25 per gallon. If she bought a total of 12 gallons of fuel and paid $4.35 per gallon for the fuel, how many gallons of B-99 did she buy, and how many gallons of B-80 did she buy?

5. During a chemistry class, a student needs to mix hydrochloric acid solutions: one containing 15% of hydrogen chloride with one containing 25% hydrogen chloride. How much of each solution should she mix to get 1.5 liters of hydrochloric acid solution that has 18% hydrogen chloride? Give your answer accurate to 2 decimal places.

6. Alejandro invested $4,000, part at 4% simple interest and the rest at 3% simple interest for a period of 1 year. If he received a total annual interest of $145 from both investments, how much did he invest at each rate?

7. Sodium hypochlorite solution is commonly known as bleach. Sahil has household bleach that is 2% sodium hypochlorite and chlorination bleach that is 12% sodium hypochlorite. He wants 125 mL of bleach-mix that contains 6.25 mL of sodium hypochlorite. How much household bleach and how much chlorination bleach should he use? Give your answer accurate to 1 decimal place.

8. At a fundraiser, Patrick bought drinks and raffle tickets for his family. Each drink cost $0.75 and each raffle ticket cost $1.20. If he purchased a total of 10 items and spent a total of $10.20, how many drinks, and how many raffle tickets did he buy?

9. Gerry owns a candy and nut shop. In the shop, there are both bulk and packaged candy and nuts. A customer comes into the shop and explains that he needs a large amount of a mixture of chocolate covered cherries and amaretto cordials. The customer explains that he plans to make individual packages of the mixture and give each package to a guest at a dinner party. The chocolate covered cherries sell for $7.50 per pound and the amaretto cordials sell for $6.00 per pound. How many pounds of amaretto cordials must be mixed with 12 pounds of chocolate covered cherries to obtain a mixture that sells for $6.50 per pound? How many pounds of mixture will there be? Note: the price per pound of the mix is given, not the total value of the mix. Also, only the number of pounds of chocolate covered cherries is given, not the number of pounds in the mix.

10. The label on a 12-ounce can of frozen concentrate Hawaiian Punch indicates that when the can of concentrate is mixed with 3 cans (36-ounces) of cold water the resulting mixture is 10% fruit juice. Find the percent of pure juice in the can of concentrate.
Chapter 6 — Exponents and Polynomials

Section 6.1 — Exponents
Section 6.2 — Negative Exponents
Section 6.3 — Polynomials
Section 6.4 — Addition and Subtraction of Polynomials
Section 6.5 — Multiplication of Polynomials
Section 6.6 — Division of Polynomials

Answers
Section 6.1 Exponents

Examples:
Simplify each expression.

a) \((-5)^2\)

\((-5)^2\)

In this expression, \(-5\) is the base and 2 is the exponent. The parentheses around \(-5\) include the negative sign in the base.

To simplify:
\((-5)\cdot(-5) \rightarrow 25\)

b) \(-5^2\)

\(-5^2\)

In this expression, 5 is the base and 2 is the exponent. There is a multiplication by \(-1\) as well. There are no parentheses, so the negative sign is not part of the base.

The expression may be written as:
\(-1 \cdot 5^2\)

To simplify:
\(-1 \cdot 5 \cdot 5 \rightarrow -25\)

c) \(\left(\frac{-78x^{12}y^5}{52x^3y^{10}}\right)^2\)

\(\left(\frac{-78x^{12}y^5}{52x^3y^{10}}\right)^2\)

In this expression, it is easiest to simplify the inside of the parentheses before applying the “outside” exponent.

First, simplify the fraction \(\frac{-78}{52}\) by dividing 26 into both \(-78\) and 52.

\(\left(\frac{-3x^9y^{10}}{2x^3y^{10}}\right)^2\)

Next, cancel three factors of \(x\) from the numerator and denominator, and then cancel five factors of \(y\) from the numerator and denominator. It may help to visualize the cancellation like this:

\(\left(\frac{-3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}{2 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}\right)^2\)

\(\frac{(-3)^2 \cdot (x^9)^2}{2^2 \cdot (y^5)^2}\)

Now, apply the outside exponent to each factor in the numerator and denominator, by squaring \(-3\) and 2, and multiplying the exponents on \(x\) and \(y\).

\(\frac{9x^{18}}{4y^{10}}\)

Homework

1. When multiplying like-bases, what operation can you perform on the exponents to simplify the expression?

2. When dividing like-bases, what operation can you perform on the exponents to simplify the expression?

3. How do you determine if the factors in a simplification of a quotient of like-bases are in the numerator or the denominator?

4. When raising a power to a power, what operation can you perform on the exponents to simplify the expression?
5. When simplifying a quotient that is raised to a power, what is often the easiest step to take first?

6. If an expression is raised to the 0 power, what is its value?

7. Simplify each expression.
   a) $3^2$
   b) $(-3)^2$
   c) $-3^2$
   d) $(-3)^3$
   e) $-3^3$

8. Simplify each expression.
   a) $-4^2$
   b) $(-4)^3$
   c) $-4^3$
   d) $(-4)^2$
   e) $4^2$

Simplify.
9. $x^4x^5$
10. $y^6/y^4$
11. $z^8/z^{11}$
12. $(x^4)^5$
13. $18^0$
14. $7^9/7^3$
15. $d^{15}/d^{18}$
16. $x^8y^8$
17. $4^24^5$
18. $z/z^{12}$
19. $(-2^4)^{10}$
20. $y^6/y^4$
21. $x^{345}x^2$
22. $(y^{345})^2$
23. \( \frac{c^2}{c^{45}} \)
24. \( \frac{d^{45}}{d^2} \)
25. \( x^9 y^3 x \)
26. \( \frac{xy^6}{x^5 y^4} \)
27. \( (y^2 x^8)^3 \)
28. \( \frac{w^{14} y^5}{w^{15} y^5} \)
29. \( \frac{-6x^{10}}{4x^7} \)
30. \( x^2 y^5 x^{17} y^3 \)
31. \( (7x^{11})^0 \)
32. \( (2x^4)^3 \)
33. \( x^2 y^5 xy^3 \)
34. \( (3xy^5)^2 \)
35. \( \frac{12c^2 d^7}{4c^{13} d} \)
36. \( \frac{20a^{14}}{40a^7} \)
37. \( \frac{5x^3}{20x^7} \)
38. \( \frac{9y^{10}}{-3y} \)
39. \( \frac{4m^{20} n^4}{16m^{20} n^{16}} \)
40. \( \frac{20p^{12} q^8}{5p^5 q^8} \)
41. \( \left( \frac{2x^3}{y^2} \right)^4 \)
42. \( \left( \frac{3m}{6n^3} \right)^2 \)
43. \( \left( \frac{16x^8}{-8x^6} \right)^3 \)
44. \( \left( \frac{20y^3}{18y^2} \right)^2 \)
45. \( \left( \frac{9p^{15}q^3}{18p^{15}q^7} \right)^5 \)
46. \( \left( \frac{14x^3}{16x^2} \right)^0 \)
47. \( \left( \frac{-9m^{13}}{-45m^{21}} \right)^3 \)
48. \( \left( \frac{-121a^8c^5}{44a^9c^{12}} \right)^2 \)
49. \( (5x^3)(4x^2) \)
50. \( -(2y^4)(3y^2) \)
51. \( (10m^6)(5m^2)^3 \)
52. \( (3x^3y)^4 \left( 3x^7y^5 \right) \)
53. \( (-3xy^4)^2 \left( 14x^3y^{11} \right)^0 \)
54. \( (-3n^6)^2 \left( -2n^{10} \right)^3 \)
55. \( (4y^2z)^3 \left( 2yz^4 \right)^2 \)
Section 6.2  Negative Exponents

Examples:
Simplify each expression.

a) \((-5)^{-2}\)

In this expression, \(-5\) is the base and \(-2\) is the exponent. The parentheses around \(-5\) include the negative sign in the base. The negative exponent moves the factors to the denominator.

To simplify:
\[
\frac{1}{(-5)(-5)} \rightarrow \frac{1}{25}
\]

b) \(-5^{-2}\)

In this expression, \(5\) is the base and \(-2\) is the exponent. There is a multiplication by \(-1\) as well. There are no parentheses, so the negative sign is not part of the base. The negative exponent moves the factors to the denominator.

The expression may be written as:
\[-1 \cdot 5^{-2}\]

To simplify:
\[-1 \cdot \frac{1}{5 \cdot 5} \rightarrow -\frac{1}{25}\]

c) \(\frac{-12x^8 y^{-6}}{6x^{-4} y^{-3}}\)

In this expression, it is easiest to “move” the factors with negative exponents first. That means that \(x^{-4}\) and \(y^{-3}\) move to the numerator, and \(y^{-6}\) moves to the denominator. The exponents on these factors become their opposites during the “move”.

\[
\frac{-12x^8 y^{-6}}{6x^{-4} y^{-3}} \quad \text{Now, simplify the fraction } -\frac{12}{6} \text{ to } -2.
\]

\[
\frac{-2x^8 x^4 y^3}{6y^6} \quad \text{Next, combine the factors of } x \text{ in the numerator by adding the exponents and cancel three factors of } y \text{ from the numerator and denominator.}
\]

\[
\frac{-2x^{12}}{y^3}
\]

Homework

1. In your own words, describe what happens to the exponent of a factor when the factor is moved from the numerator to the denominator of a fraction?

2. In your own words, describe what happens to the exponent of a factor when the factor is moved from the numerator to the denominator of a fraction?
3. Simplify each expression.
   a) $5^{-2}$
   b) $(-5)^{-3}$
   c) $-5^{-3}$
   d) $(-5)^{-2}$
   e) $-5^{-2}$

4. Simplify each expression.
   a) $(-4)^{-3}$
   b) $-4^{-2}$
   c) $(-4)^{-2}$
   d) $4^{-2}$
   e) $-4^{-3}$

Simplify. Final answers should not contain negative exponents.

5. \(x^{-4}\)

6. \(2^{-5}\)

7. \(\frac{1}{x^{-3}}\)

8. \(-2^3\)

9. \(\frac{x}{y^{-3}}\)

10. \(\frac{m^{-2}}{n^3}\)

11. \(x^{-8}x^7\)

12. \(\frac{y^5}{y^{-3}}\)

13. \(\frac{z^{-8}}{z^{11}}\)

14. \((x^{-3})^4\)

15. \(\frac{6^{-3}}{6^{-8}}\)

16. \(\frac{d^{-12}}{d^{-12}}\)

17. \(xx^{-16}\)

18. \(4^{-1}4^7\)

19. \(\frac{z}{z^{-10}}\)

20. \((5^{-4})^{-10}\)
21. \( \frac{y^{-8}}{y} \)

22. \( x^{14} y^{-3} x \)

23. \( \frac{xy^{-3}}{x^{-7} y^9} \)

24. \( (y^x x^5)^{-3} \)

25. \( \frac{w^{14} y^5}{w^6 y^{-5}} \)

26. \( -\frac{14x^{10}}{4x^{-6}} \)

27. \( x^{-2} y^{-6} x^{18} y^3 \)

28. \( (17x^{-11})^0 \)

29. \( (-2x^4)^{-3} \)

30. \( -x^2 y^{-5} xy^{-3} \)

31. \( (-5xy^{-3})^2 \)

32. \( \frac{24c^3 d^{16}}{-6c^{-12} d} \)

33. \( \frac{60a^{-18}}{30a^{-9}} \)

34. \( \frac{4x^{-3}}{-20x^9} \)

35. \( \frac{-6y^{11}}{3y} \)

36. \( \frac{4m^{-22} n^{-4}}{12m^{-22} n^{-12}} \)

37. \( \frac{20p^{10} q^{-8}}{5p^{-3} q^8} \)

38. \( \left( \frac{2x^{-5}}{y^3} \right)^4 \)

39. \( \left( \frac{2m}{6n^7} \right)^{-2} \)

40. \( \left( \frac{8x^{12}}{4x^6} \right)^{-3} \)
41. \( \left( \frac{-25y^{-3}}{15y^{-2}} \right)^{-2} \)

42. \( \left( \frac{14p^{14}q^{-3}}{-28p^{14}q^6} \right)^{5} \)

43. \( \left( \frac{14x^{-3}}{6x^{-2}} \right)^{0} \)

44. \( \left( \frac{7m^{-11}}{35m^{-22}} \right)^{-3} \)

45. \( \left( \frac{-121a^{-6}c^{-5}}{66a^9c^{12}} \right)^{2} \)

46. \( (3x^{-3})(5x^{-2}) \)

47. \( (-2y^4)^{-3}(5y^2) \)

48. \( (-10m^6)(2m^{-5})^3 \)

49. \( (3x^{-4}y)^{-4}(-3x^6y^5) \)

50. \( (3xy^{-9})^{-2}(-24x^7y^{-14})^0 \)

51. \( (2n^3)^2(-3n^{-12})^{-3} \)
Section 6.3  Polynomials

6.3 — Polynomials Worksheet

Example:
For the polynomial given, find the degree of each term, the degree of the polynomial, the leading term, and the leading coefficient. If the polynomial has a specific name—monomial, binomial, or trinomial—give that name.

a) \(9x^6y - x^3y^2 + 12y^6\)

<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9x^6y)</td>
<td>7</td>
<td>9</td>
<td></td>
<td>9</td>
<td>trinomial</td>
</tr>
<tr>
<td>(-x^3y^2)</td>
<td>5</td>
<td>-1</td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>(12y^6)</td>
<td>6</td>
<td>12</td>
<td></td>
<td></td>
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</tbody>
</table>

Homework

1. In your own words, define a polynomial.

2. In your own words, define each word: monomial, binomial, trinomial.

3. In your own words, describe how you identify the degree of a polynomial.

4. In your own words, describe how you identify the leading term of a polynomial.

5. In your own words, define each word: constant, linear, quadratic, cubic.

For each polynomial given, find the degree of each term, the degree of the polynomial, the leading term, and the leading coefficient. If the polynomial has a specific name—monomial, binomial, or trinomial—give that name. You may use a chart like the one below for each polynomial, but it isn’t necessary, as long as you identify each answer.

<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
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<tr>
<td>6. (8x^4 + x^2)</td>
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<tr>
<td>7. (4x^3 - x^11 + 2x - 17)</td>
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<tr>
<td>8. (12x^4y + x^2y^2 - 7y^3)</td>
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<td></td>
</tr>
<tr>
<td>9. (y - 4y^2 + 15)</td>
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</tbody>
</table>

Arrange each polynomial in descending order. Give the degree of each polynomial and the leading coefficient.

10. \(y^3 - 14y^2 - 5y^6\)

11. \(x^{10} - 7x^2 + 12x + x^{12}\)
Section 6.4 Addition and Subtraction of Polynomials

Examples:
Perform the operation. Answers may be written in descending order of power, but it isn’t necessary.

a) \((7x^2 + 12x - 23) + (x^2 - 5x)\)

To add two polynomials, combine like-terms. Exponents on variables will NOT change. If terms are reordered, take the sign in front of the term with the term that’s moved.

\[
(7x^2 + 12x - 23) + (x^2 - 5x) \rightarrow 7x^2 + x^2 + 12x - 5x - 23 \rightarrow 8x^2 + 7x - 23
\]

b) Subtract \(-x^2 - 20x + 4\) from \(11x^2 + x - 5\)

When translating a statement that involves “subtract … from…”, polynomials are “switched” between the “math order” and the “English order”.

This problem becomes: \((11x^2 + x - 5) - (-x^2 - 20x + 4)\)

\[
11x^2 + x - 5 + x^2 + 20x - 4 \quad \text{Distribute the subtraction sign to all of the terms in the parentheses following it.}
\]

\[
12x^2 + 21x - 9 \quad \text{Combine like-terms.}
\]

Homework

1. In your own words, describe a “like-term”. What must be the same? What may be different?

2. In your own words, describe how to add two polynomials. What changes? What doesn’t change?

Perform the operation. Answers may be written in descending order of power, but it isn’t necessary.

3. \(6x + 3x\)

4. \(15y - 3y\)

5. \(4x + x - 7\)

6. \(8 + 11y - y\)

7. \(13x + 5 - 2x + 14\)

8. \(28x^2 - 4x^2 + 3x\)

9. \(7y - 45xy + 8x - 6y\)

10. \(13x^2y - 8xy + 4x^2y + 6x^2\)

11. \((3x - 8) + (11x + 2)\)

12. \((5m - 60) + (m^2 + 32m)\)

13. \((2m^2 + 8m - 9) + (m^2 + 7)\)

14. \((10a^2 - 6a - 14) + (a - 2)\)

15. \((y^2 - 15) + (3y^2 - y)\)

16. \((3x^2 + 27xy - 23) + (x^2 - 14xy)\)
17. \((-25x-13)+(x^2-1)\)
18. \((6x-12)-(8x+3)\)
19. \((24m-18)-(m^2+5m)\)
20. \((3n^2+10n-11)-(n^2+9)\)
21. \((13a^2-7a-8)-(a-16)\)
22. \((y^2-36)-(4y^2-y)\)
23. \((2x^2+20xy-28)-(x^2-16xy)\)
24. \((-29x-54)-(x^2-1)\)
25. \(\left(\frac{1}{2}m^2-\frac{1}{6}m-\frac{2}{3}\right)\left(\frac{1}{2}m^2+\frac{2}{3}\right)\)
26. \(\left(\frac{3}{4}x-18\right)-(\frac{3}{4}x^2+x)\)
27. \(\left(\frac{2}{3}a^2-\frac{4}{3}a-\frac{1}{7}\right)\left(\frac{3}{10}a-\frac{2}{3}\right)\)
28. Add \(x-4\) and \(5x^2+3x-7\).
29. Add \(y^2+13y\) and \(-4y-5\).
30. Subtract \(9x+4\) from \(3x+12\).
31. Subtract \(2y^2-8y\) from \(5y^2-16y\).
32. Subtract \(m^2+2m-18\) from \(4m^2+11m-9\).
33. Subtract \(-5x^2-27x+19\) from \(x^2+3x-4\).
Section 6.5  
Multiplication of Polynomials

Examples:
Perform each operation. Simplify answers (if not simplified after multiplying).

a)  \( \frac{1}{2} y^3 (4y^2 - y + 7) \)
To multiply a monomial by a polynomial, distribute the monomial to each term in the polynomial. Exponents on variables MAY change.

\( \frac{1}{2} y^3 (4y^2 - y + 7) \rightarrow \frac{1}{2} y^3 \cdot 4y^2 - \frac{1}{2} y^3 \cdot y + \frac{1}{2} y^3 \cdot 7 \rightarrow 2y^5 - \frac{1}{2} y^4 + \frac{7}{2} y^3 \)

b)  \((5x + 3y)(x - 2y)\)
To multiply two binomials, you may use the “FOIL method”. FOIL stands for the multiplications of the terms: First, Outer, Inner, and Last. “FOIL-ing” is the same thing as distributing each term in the first polynomial to each term in the second polynomial.

\((5x + 3y)(x - 2y) \rightarrow 5x \cdot x + 5x \cdot (-2y) + 3y \cdot x + 3y \cdot (-2y) \rightarrow 5x^2 - 10xy + 3xy - 6y^2 \rightarrow 5x^2 - 7xy - 6y^2\)

c)  \((x + 4)^2\)
To square a binomial, multiply it out using the “FOIL method”. There is a pattern. If you recognize it, you are welcome to use it.

\((x + 4)^2 \rightarrow (x + 4)(x + 4) \rightarrow x \cdot x + x \cdot 4 + 4 \cdot x + 4 \cdot 4 \rightarrow x^2 + 4x + 4x + 16 \rightarrow x^2 + 8x + 16\)

Homework

1. What property is most used when multiplying polynomials?

2. When computing the square of a binomial — for example, an expression of the form \((a + b)^2\) — what must you remember?

3. Compute each problem.
   a) \((x + 3)^2\)
   b) \((3x)^2\)
   c) \((x^2 - 4)^2\)
   d) \((4x^2)^2\)
   e) Using the above problems as examples, in your own words, describe how you can tell when you may use a “shortcut” exponent rule and when you must “FOIL”?

Perform each operation. Simplify answers (if not simplified after multiplying).

4. \(4(2x + 7)\)

5. \(-5(x + 3)\)

6. \(7(y^2 + 6y + 3)\)

7. \(10(x - 6)\)

8. \(-2(x^2 - 5x + 12)\)

9. \(4x(x + 8)\)

10. \(2y(y - 3)\)
11. $\frac{1}{2}x(18x+5)$
12. $3x^2(2x^2+x-6)$
13. $y^2(2y^3-y^2+5y-11)$
14. $xy(4x^2-7xy+y^2)$
15. $mn^2(5m^2+2mn)$
16. $(y+5)(y+9)$
17. $(d-3)(d-12)$
18. $(x-3)(x+7)$
19. $(k-\frac{1}{2})(k-\frac{3}{4})$
20. $(p+4)(p-5)$
21. $(x+1)(x+6)$
22. $(3y+1)(y+7)$
23. $(x+7)^2$
24. $(x-4)(2x+9)$
25. $(k^2-1)(4k^2-7)$
26. $(5y^2+4)(y^2-1)$
27. $(x-3)^2$
28. $(p^2+1)(p^2+8)$
29. $(x^2-12)(3x^2+2)$
30. $(2k-11)(3k-10)$
31. $(xy+3)(y-1)$
32. $(mp+14)(mp+10)$
33. $(x-y)(x+7y)$
34. $(2a-c)(5a-c)$
35. $(4x-y)(3x-y)$
36. $(2x+9)^2$
37. $(x^2+3y)(2x-y)$
38. $(2x-3y)^2$
39. \((x+5)(x^2+3x+10)\)

40. \((y-1)(y^2+4y+7)\)

41. \((x-6)(2x^2-8x+11)\)

42. \((p^2+2)(p^2+6p-20)\)

43. \((m^2-3)(2m^2+m-1)\)

44. \((x+\frac{1}{4})(8x^2+\frac{1}{2}x+24)\)

45. \((x+y)(x^2-xy+y^2)\)

46. \((a-b)(a^2+ab+b^2)\)

47. \((2x-y)(x^2+3xy-4y^2)\)

48. Multiply each pair of binomials, and then answer the last question.
   a) \((x+2)(x-2)\)
   b) \((y+5)(y-5)\)
   c) \((p-\frac{1}{3})(p+\frac{1}{3})\)
   d) \((2y+5)(2y-5)\)
   e) \((3a-1)(3a+1)\)
   f) In your own words, describe how the above problems similar before they are multiplied, how they similar after they are multiplied, and then describe the pattern.

49. Multiply each pair of binomials, and then answer the last question.
   a) \((x+1)^2\)
   b) \((y+3)^2\)
   c) \((k+\frac{1}{4})^2\)
   d) \((2a+7)^2\)
   e) \((4x+1)^2\)
   f) In your own words, describe how the above problems similar before they are multiplied, how they similar after they are multiplied, and then describe the pattern.

50. Multiply each pair of binomials, and then answer the last question.
   a) \((y-2)^2\)
   b) \((x-5)^2\)
   c) \((a-\frac{1}{3})^2\)
   d) \((2a-1)^2\)
   e) \((3x-2)^2\)
   f) In your own words, describe how the above problems similar before they are multiplied, how they similar after they are multiplied, and then describe the pattern.
Section 6.6  Division of Polynomials

Examples:

Perform each operation.

a) \( \frac{8x^6 - 2x^4 + 4x^3}{-4x^3} \)

To divide a polynomial by a monomial, divide the monomial into each term of the polynomial. Notice that after the division/simplification, there will be the same number of terms in the answer as there were in the polynomial.

\[
\frac{8x^6 - 2x^4 + 4x^3}{-4x^3} \rightarrow \frac{8x^6}{-4x^3} - \frac{2x^4}{-4x^3} + \frac{4x^3}{-4x^3} \rightarrow -2x^3 + \frac{1}{2}x - 1
\]

b) \( (x^2 + 5x + 8) ÷ (x - 2) \)

To divide a polynomial by a binomial, use polynomial long division. There is another way to divide polynomials by binomials of degree 1; this method will be covered in the next math course.

\[
\begin{align*}
(x^2 + 5x + 8) ÷ (x - 2) & \rightarrow \frac{x}{x - 2} \div \frac{x^2 + 5x + 8}{x - 2} \\
& \rightarrow x - \frac{(x^2 - 2x)}{7x} \\
& \rightarrow x - \frac{7x + 8}{22} \\
& \rightarrow x^2 + 5x + 8
\end{align*}
\]

Next, multiply that value by the binomial \( x - 2 \), and write it below the dividend inside the long division bar, so that like-terms are lined-up. Subtract that product from the polynomial \( x^2 - 5x + 8 \).

Now, figure out “what” times the first term, \( x \), of the divisor \( x - 2 \) will be \( 7x \), the first term of the dividend \( x^2 + 5x + 8 \). For this problem that value is 7, and it is the next term written on the top of the long division bar.

22 is the remainder. In the answer, it is written over the divisor \( x - 2 \).

The answer to \( (x^2 + 5x + 8) ÷ (x - 2) \) is \( x + 7 + \frac{22}{x - 2} \).

You may always check division problems by multiplying the divisor by the quotient and adding the remainder. Doing this should result in the dividend.

Check: \( (x - 2)(x + 7) + 22 \rightarrow x^2 + 5x - 14 + 22 \rightarrow x^2 + 5x + 8 \)

Homework

1. In your own words, describe the “easiest” way to divide a polynomial by a monomial.

2. When you divide a polynomial with \( n \) terms by a monomial, how many terms will you have in your quotient (answer)?

3. In your own words, describe how to divide a polynomial by a binomial.
Perform each operation.

4. \( \frac{4p + 12}{2} \)

5. \( \frac{21y + 12}{-7} \)

6. \( \frac{5n + 35}{5} \)

7. \( \frac{8x - 11}{4} \)

8. \( \frac{4k + 60}{-10} \)

9. \( \frac{10x - 12}{-6} \)

10. \( \frac{(12x^2 + 4x)}{(2x)} \)

11. \( \frac{(20y^3 + 40y^2 - 10y)}{(5y)} \)

12. \( \frac{(36p^3 + 18p^2 - 12p)}{(-6p)} \)

13. \( \frac{9x^6 - 45x^4 + 15x^2}{-3x^2} \)

14. \( \frac{(16k^3 + 20k^2 - 3k)}{(4k)} \)

15. \( \frac{6y^9 + 30y^7 - 18y^8 + 15y^3}{6y^5} \)

16. \( \frac{(4m^{10} - 14m^9 - 22m^5 + 2m^3)}{(-2m^3)} \)

17. \( \frac{(3x^{12} - 30x^{10} + 18x^6 + 12x^4)}{(-6x^2)} \)

18. \( \frac{14p^6 + 21p^5 - 7p^4 + 35p^3}{-7p^4} \)

19. \( \frac{15y^2 + 3y - 5}{-5y^2} \)

Perform each operation.

20. \( \frac{y^2 + 5y - 13}{y + 2} \)

21. \( \frac{k^2 + 8k + 14}{k + 3} \)

22. \( \frac{x^2 - 3x - 12}{x + 6} \)

23. \( \frac{m^2 + 10m + 7}{m - 3} \)

24. \( \frac{a^2 - 6a + 11}{a - 1} \)

25. \( \frac{p^2 + 4p - 21}{p + 7} \)
26. \( (3x^2 + 16x + 20) \div (x + 4) \)

27. \( \frac{4y^2 + 3y - 20}{y - 2} \)

28. \( (6k^2 + 23k + 21) \div (2k + 3) \)

29. \( (4x^2 - 8x + 10) \div (2x + 1) \)

30. \( (8m^2 - 26m + 15) \div (2m - 5) \)

31. \( \frac{5k^2 - 3k - 8}{5k + 2} \)

32. \( (p^3 + 6p^2 + 11p - 10) \div (p + 2) \)

33. \( (2y^3 + 10y^2 + 20y - 9) \div (y + 4) \)

34. \( (x^2 + 8) \div (x + 2) \)

35. \( (x^2 - 4) \div (x - 2) \)

36. \( (5x^2 + 7) \div (x + 3) \)

37. \( (x^3 - 8) \div (x - 2) \)
Chapter 7 – Factoring

Section 7.1 – Greatest Common Factor
Section 7.2 – Factoring by Grouping
Section 7.3 – Factoring Trinomials with a Leading Coefficient of 1
Section 7.4 – Factoring a Difference of Squares
Section 7.5 – Factoring Trinomials with a non-1 Leading Coefficient
Section 7.6 – General Factoring
Section 7.7 – Solving Quadratic Equations by Factoring
Section 7.8 – General Application Problems Involving Quadratic Equations

Answers
Section 7.1 Greatest Common Factor

Examples:
Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

a) \[40x^4y^2 - 8x^2y^3 + 16xy^4\]

To factor an expression means to write it as a product (multiplication). The first step is to determine if all terms have a Greatest Common Factor (GCF). If there is a GCF, factor it out by “un-distributing it”. When determining the variable part of a GCF, use the smallest exponent on that variable.

\[40x^4y^2 - 8x^2y^3 + 16xy^4 = 8xy^2(5x^2 - xy + 2y^2)\]

The GCF for this expression is \(8xy^2\) since all terms contain the factors of \(8\), \(x\), and \(y^2\).

b) \[8x(2y - 3) + (2y - 3)\]

Again, the first step when factoring is to determine if there is a Greatest Common Factor (GCF).

This expression has two terms: \(8x(2y - 3)\) and \((2y - 3)\). The GCF of these two terms is \((2y - 3)\). To factor this expression, it may be helpful to rewrite it:

\[8x(2y - 3) + 1(2y - 3)\]

In order to “see” what factor remains in the second term when factoring out the GCF.

\[8x(2y - 3) + 1(2y - 3)\]

\[(2y - 3)(8x + 1)\]

Homework
1. In general, what does it mean to factor an expression?
2. In your own words, describe how to determine the greatest common factor of the terms in an expression.
3. If an expression has no greatest common factor, is it factorable?
4. What is an expression that isn’t factorable called?
5. In your own words, describe how to determine how many factors (the exponent) of any variables are in a greatest common factor.

Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

6. \(4x + 12\)

7. \(10p + 15\)

8. \(6y + 3\)

9. \(18x - 6\)

10. \(21k - 14\)

11. \(36y - 60\)

12. \(88n + 44\)

13. \(36x + 49\)

14. \(27x + 54\)

15. \(2y^3 + 4y\)

16. \(16p^4 - 8p^3\)

17. \(25x^2 + 45x\)
18. $15a^3 - 117a^2$
19. $30y^6 - 9y^4$
20. $33x^5 + 11x^3 - 66x^2$
21. $35p^4 + 28p^3 - 7p^2$
22. $60x^3y^2 + 90xy$
23. $80y^3z^2 - 72yz^3$
24. $70a^6c^2 - 56a^4c^3 + 91a^7c^4$
25. $65p^5 + 13p^3q^2 + 39p^2q^4$
26. $48m^4n - 64m^3n^2 + 8mn$
27. $9x^4y^2 - 22x^3y^3 - 18x^2y^4$
28. $52w^4x^2 - 26w^2x^3 + 39w$
29. $45x^3y + 60x^2y^2 - 90xy^3$
30. $48a^4c + 24a^3c^3 - 12ac^4$
31. $3x + 6x^{3/2}$  
   *Hint: The GCF is $3x^{1/2}$.*
32. $14a^{3/2} + 2a^{5/2}$
33. $4x^{-2} + 8x^{-3}$  
   *Hint: The GCF is $4x^{-3}$.*
34. $25y^{-5} + 5y^{-4}$
35. $x(a + 5) + 4(a + 5)$
36. $y(z + 1) - 7(z + 1)$
37. $7p(q - 3) - 2(q - 3)$
38. $3d(c - 4) + (c - 4)$
39. $x(x + 9) + 2(x + 9)$
40. $m(m - 3) - (m - 3)$
Section 7.2  Factoring by Grouping

7.2 — GCF and Grouping Worksheet

Example:
Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

a) \(3x^2y^2 + 6y - 4x^2y - 8\)

To factor an expression, first check for a GCF of all of the terms.

For this expression, there is not a GCF for all terms.

Next, count the number of terms in the expression. If there are four terms, try factoring by grouping. There are other techniques for factoring expression with four terms, but this course does not cover those other methods.

\[
\begin{align*}
3x^2y^2 + 6y - 4x^2y - 8 & \quad \text{GCF is } 3y \quad \text{GCF is } 4 \\
3y(x^2y + 2) - 4(x^2y + 2) & \quad \text{Next, factor out each of these GCFs from the appropriate terms. Notice that for the second two terms: } -4x^2y - 8, \text{ since the third term was negative, } -4 \text{ was factored out. This leaves the expression with two terms that have a GCF of } (x^2y + 2). \\
(x^2y + 2)(3y - 4) & \quad \text{Now, factor out the GCF of } (x^2y + 2). \text{ This is the final answer. Notice that it is a product.}
\end{align*}
\]

Homework

1. In general, what does it mean to factor an expression?
2. What are the two types of factoring methods covered in Chapter 7 so far (from sections 7.1 & 7.2)?
3. When asked to factor an expression, do you determine when to try factoring by grouping?
4. In your own words, describe how to factor an expression by grouping.
5. What factoring method should you always look for first, even if you eventually use another factoring method?
6. In your own words, describe how to determine if an expression has been factored completely.
7. What do you call an expression that cannot be factored using any factoring method?

Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

8. \(6x^3 + 4x^2 + 15x + 10\)
9. \(12p^4 + 15p^3 + 8p + 10\)
10. \(5y^3 - 15y^2 + 7y - 21\)
11. \(6k^4 - k^2 + 24k^2 - 4\)
12. \(2n^3 + 6n^2 + 7n + 28\)
13. \(16x^3 + 6x^2 - 56x - 21\)
14. \(ac + c + 2a + 2\)
15. \(18w^3 - 72w^2 + 10w - 40\)  \text{Hint: GCF first! Watch for this in all problems.}
16. \(36y^3 + 63y^2 - 8y - 14\)
17. \(22x^3 + 6x^2 + 11x + 3\)

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18. $30p^2 - 80p - 9p + 24$
19. $9m^3 + 81m^2 + m + 9$
20. $x^4 - 7x^2 + 7x^2 - 7$
21. $3p^2q^2 + 6pq - 4pq - 8$
22. $26m^3 + 2m^2 - 13m - 1$
23. $6x^2y + 60xy - 3xy - 30y$
24. $3a^3 + 24a^2 + 21a^2 + 168a$
25. $x^2y^2 + x^2y + 2x^2 + 3x$
26. $8p^3 - 2p^2 - 12p - 3$
27. $8y^3 - 2y^2 - 12y + 3$
28. $3x^5 + 3x^4 - x^3 - x^2$
Section 7.3  Factoring Trinomials with a Leading Coefficient of 1

Example:
Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

a) \( x^2 - 5x - 24 \)

To factor an expression, the **first** check for a GCF of all of the terms.

For this expression, there is **not** a GCF for all terms.

There are three terms in the expression, so try factoring by “un-FOIL-ing”: find two binomials that multiply to this trinomial.

\( x^2 - 5x - 24 \)

Start with the first term in the trinomial, \( x^2 \), this is the product of the first two terms of the binomials. This means that the first term in each binomial is \( x \).

Next, since the last term in the trinomial is \(-24\), find two numbers that multiply to \(-24\). Possible choices are:

- \(-1 \times 24\)
- \(1 \times (-24)\)
- \(-2 \times 12\)
- \(2 \times (-12)\)
- \(-3 \times 8\)
- \(3 \times (-8)\)
- \(-4 \times 6\)
- \(4 \times (-6)\)

These two numbers are also multiplied by the first terms in the binomials (\( x \) and \( x \)). The sum of these “inner” and “outer” products must add up to \(-5x\), so choose the factors whose sum is \(-5\).

\( (x - 8)(x + 3) \)

This is the final answer. Notice that it is a product. The answer may also be written with the order of the binomials switched: \((x + 3)(x - 8)\)

Homework

1. In general, what does it mean to factor an expression?
2. What are the three types of factoring methods covered in Chapter 7 so far (Sections 7.1 – 7.3)?
3. When asked to factor an expression, how do you determine when to try factoring by “un-foiling”?
4. What factoring method should you always look for first, even if you eventually use another factoring method?
5. In your own words, describe how to determine if an expression has been factored completely.
6. What do you call an expression that cannot be factored using any factoring method?
7. In your own words, describe how to “un-foil” a trinomial.
8. Factor each expression. Notice that in parts a–d, the absolute values of the constants in the two binomials are always the same, but the signs are different depending both on the signs in the original trinomial and the middle coefficient of the original trinomial.
   a) \( x^2 + 9x + 20 \)
   b) \( x^2 - 9x + 20 \)
   c) \( x^2 - x - 20 \)
   d) \( x^2 + x - 20 \)
   e) In your own words, describe why the trinomial \( x^2 - 9x - 20 \) is prime (not factorable)?
   f) In your own words, describe why the trinomial \( x^2 - x + 20 \) is prime (not factorable)?
Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

9. \( x^2 + 15x + 50 \)
10. \( p^2 + 12p + 11 \)
11. \( y^2 - 10y + 21 \)
12. \( a^2 - 6a + 8 \)
13. \( m^2 - 3m - 40 \)
14. \( p^2 - 2p - 63 \)
15. \( x^2 + 12x - 28 \)
16. \( a^2 + 3a - 18 \)
17. \( y^2 + 7y + 4 \)
18. \( n^2 + 4n - 32 \)
19. \( x^2 - 5x - 6 \)
20. \( p^2 + 17p + 52 \)
21. \( y^2 - 7y - 30 \)
22. \( a^2 + 2a + 15 \)
23. \( c^2d^2 - 13cd + 42 \)
24. \( x^2y^2 + 2xy - 24 \)
25. \( p^2 - 7pq + 12q^2 \)
26. \( m^2n^2 - mn - 30 \)
27. \( x^2 + 17xy + 60y^2 \)
28. \( t^2 - 20tu + 3u^2 \)
29. \( x^2 + 2xy + y^2 \)
30. \( 2q^2 + 26q + 72 \) \( \text{Hint: GCF first! Watch for this in all problems.} \)
31. \( 5a^2 - 15a - 60 \)
32. \( 3n^2 - 21n + 30 \)
33. \( 4x^2y^2 - 32xy + 48 \)
34. \( p^2q + 5pq - 24q \)
35. \( 2c^3 - 14c^2d - 36cd^2 \)
36. \( x^3y^3 - 17x^2y^2 + 30xy \)
37. \( a^6 - 14a^3 + 33 \)
38. \( z^8 - 5z^4 - 14 \)
39. Factor each expression.
   a) $w^2 + 10w + 25$
   b) $y^2 + 14y + 49$
   c) $m^2 + 6m + 9$
   d) In your own words, describe the pattern for the above three trinomials and their corresponding factored expressions.
   e) Use this pattern to factor $x^2 + \frac{1}{2} x + \frac{1}{16}$.

40. Factor each expression.
   a) $p^2 - 8p + 16$
   b) $n^2 - 12n + 36$
   c) $a^2 - 16a + 64$
   d) In your own words, describe the pattern for the above three trinomials and their corresponding factored expressions.
   e) Use this pattern to factor $x^2 - x + \frac{1}{4}$.
Section 7.4  Factoring a Difference of Squares

7.4 — Difference of Squares Practice Worksheet

Example:
Factor. Make sure the expression is factored completely. If the expression is not factorable, state that it is prime.

a) \(9x^2 - 4y^2\)

To factor an expression, the first check for a GCF of all of the terms.

For this expression, there is not a GCF for all terms.

There are two terms in the expression, check to see if the expression is a DIFFERENCE of squares. In other words, both the first term and the last term must be perfect squares and the sign between MUST be a subtraction. There are other ways to factor two terms; they will be covered in the next math course.

If the expression is a DIFFERENCE of squares, try factoring by “un-FOIL-ing”: find two binomials that multiply to this binomial. Since there are only two terms (not three), the product of the two binomials follows a special pattern:

\[a^2 - b^2 = (a + b)(a - b)\]

\(9x^2 - 4y^2\)  
Start with the first term in the trinomial, \(9x^2\), this is a perfect square and must be the product of the first two terms of the binomials; therefore, the first term in each binomial is \(3x\).

The last term in the trinomial, \(4y^2\), is also a perfect square and must be the product of the last two terms of the binomials; therefore, the last term in each binomial (in absolute value) is \(2y\).

Notice that the “inner” and “outer” products add up to 0, since there is no middle term.

\((3x + 2y)(3x - 2y)\)  
This is the final answer. Notice that it is a product. The answer may also be written with the order of the binomials switched: \((3x - 2y)(3x + 2y)\). Remember, you may check any factoring problem by multiplying your answer.

Homework

1. In general, what does it mean to factor an expression?
2. What are the three types of factoring methods covered in Chapter 7 so far (Sections 7.1 – 7.4)?
3. When asked to factor an expression, how do you determine when you are factoring a difference of squares?
4. What factoring method should you always look for first, even if you eventually use another factoring method?
5. In your own words, describe how to determine if an expression has been factored completely.
6. What do you call an expression that cannot be factored using any factoring method?
7. In your own words, describe how to “un-foil” a difference of squares.
8. Is it possible to factor the expression \(x^2 + 9\) ? In your own words, explain why or why not.
9. Is it possible to factor the expression \(4x^2 + 16\) ? In your own words, explain why or why not.
10. If an expression has two terms and it is not a difference of squares, can it be factored? In your own words, explain why or why not.

Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

11. \(x^2 - 25\)
12. \(a^2 - 121\)
13.  $p^2 - 81$
14.  $y^2 - 1$
15.  $m^2 + 100$
16.  $k^2 - 196$
17.  $4c^2 - 9$
18.  $x^2 - y^2$
19.  $81w^2 - 1$
20.  $49d^2 - 16$
21.  $36 - n^2$
22.  $s^2t^2 - 64$
23.  $100w^2 - 169z^2$
24.  $y^4 - 144$
25.  $225 - x^6$
26.  $p^2 - \frac{1}{4}$
27.  $5a^2 - 80$  \textit{Hint: GCF first! Watch for this in all problems.}
28.  $3m^2 - 363$
29.  $x^3 - 196x$
30.  $v^2 - \frac{9}{25}$
31.  $2z^2 + 98$
32.  $25q^2 - 225$
33.  $16c^2 - 4$
34.  $144a^4 - 9a^2$
35.  $8y^2 - 8$
36.  $x^4 - 16$
37.  $12k^4 - 12$
### Section 7.5 Factoring Trinomials with a non-1 Leading Coefficient

#### 7.5 — Factoring Trinomials with a non-1 Leading Coefficient Worksheet

**Example:**
*Factor. Make sure the expression is factored completely. If the expression is not factorable, state that it is prime.*

a) \( 6x^2 + 17x + 5 \)

To factor an expression, the first check for a GCF of all of the terms.

For this expression, there is not a GCF for all terms.

There are three terms in the expression, so try factoring by “un-FOIL-ing”: find two binomials that multiply to this trinomial. Since the leading coefficient is not 1, more thought is required. If this is really challenging for you, ask your instructor about the “ac-grouping method”.

Start with the first term in the trinomial, \( 6x^2 \), this is the product of the first two terms of the binomials. This means that the first terms in each binomial are either: \( 2x \) and \( 3x \) OR \( 1x \) and \( 6x \). Notice that all terms are positive, so both signs in the binomials will be addition.

\[ (2x + \square)(3x + \square) \quad \text{OR} \quad (x + \square)(6x + \square) \]

Next, since the last term in the trinomial is 5, find two numbers that multiply to 5. Since all signs are positive, the only choices are: \( 1 \) and \( 5 \).

These two numbers are also multiplied by the first terms in the binomials (either: \( 2x \) and \( 3x \) OR \( 1x \) and \( 6x \)). The sum of these “inner” and “outer” products must add up to 17x.

Using 1 and 5 as the last terms with the above first terms, yields:

<table>
<thead>
<tr>
<th>Binomial Choices</th>
<th>“FOIL”</th>
<th>Simplified Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2x + 1)(3x + 5))</td>
<td>(6x^2 + 10x + 3x + 5)</td>
<td>(6x^2 + 13x + 5)</td>
</tr>
<tr>
<td>((2x + 5)(3x + 1))</td>
<td>(6x^2 + 2x + 15x + 5)</td>
<td>(6x^2 + 17x + 5)</td>
</tr>
<tr>
<td>((x + 1)(6x + 5))</td>
<td>(6x^2 + 5x + 6x + 5)</td>
<td>(6x^2 + 11x + 5)</td>
</tr>
<tr>
<td>((x + 5)(6x + 1))</td>
<td>(6x^2 + x + 30x + 5)</td>
<td>(6x^2 + 31x + 5)</td>
</tr>
</tbody>
</table>

Although all of the resulting trinomials have the same first and last terms, each has a different middle term. For this problem, choose the binomials that resulted in a middle term of 17x.

\[ (2x + 5)(3x + 1) \]

This is the final answer. Notice that it is a product. The answer may also be written with the order of the binomials switched: \((3x + 1)(2x + 5)\). Remember, you may check any factoring problem by multiplying your answer.

**Homework**

1. In general, what does it mean to factor an expression?
2. What are the three types of factoring methods covered in Chapter 7 so far (Sections 7.1 – 7.5)?
3. When asked to factor an expression, how do you determine when to “un-FOIL” or use the a-c grouping method?
4. What factoring method should you always look for first, even if you eventually use another factoring method?
5. In your own words, describe how to determine if an expression has been factored completely.
6. What do you call an expression that cannot be factored using any factoring method?
7. Choose one of the following:
   a) In your own words, describe how to “un-FOIL” a trinomial with a non-1 leading coefficient
   b) In your own words, describe how to factor a trinomial with a non-1 leading coefficient using the a-c grouping method.
8. Is it possible to factor the expression \(2x^2 + 3x + 1\) even though there are no factors of 1 that add up to 3? Why or why not?

9. Factor each expression. Notice: the absolute values of the constants in the two binomials are always the same, but the problems are different depending on signs, the value of the coefficient in the trinomial, and the placement of the constants in the binomials.

- a) \(2x^2 + 15x + 7\)
- b) \(2x^2 - 15x + 7\)
- c) \(2x^2 + 9x + 7\)
- d) \(2x^2 - 9x + 7\)
- e) \(2x^2 + 13x - 7\)
- f) \(2x^2 - 13x - 7\)
- g) \(2x^2 + 5x - 7\)
- h) \(2x^2 - 5x - 7\)

Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

10. \(2y^2 + 5y + 3\)
11. \(3m^2 + 16m + 5\)
12. \(5x^2 - 17x + 14\)
13. \(6a^2 + 35a + 11\)
14. \(6p^2 - 17p + 11\)
15. \(4d^2 - 13d - 12\)
16. \(4x^2 - 4x - 15\)
17. \(3k^2 + 2k + 8\)
18. \(8q^2 - 18q + 9\)
19. \(9c^2 + 48c + 64\)
20. \(2x^2 - x - 21\)
21. \(10y^2 + 159y - 16\)
22. \(9n^2 - 34n - 8\)
23. \(36a^2 - 60a + 25\)
24. \(21m^2 + 10m - 16\)
25. \(2x^2 + 13xy + 15y^2\)
26. \(9a^2 - 6ad + d^2\)
27. \(12p^2q^2 + 17pq + 5\)
28. \(15c^2d^2 - 8cd - 12\)
29. \(500n^2 + 300n + 45\)  
   Hint: GCF first! Watch for this in all problems.
30. \(20q^3 + 9q^2 - 18q\)
31. \(28k^3 - 54k^2 + 18k\)
32. \(36x^2 + 186xy + 240y^2\)
Section 7.6 General Factoring

7.6 — General Factoring Worksheet

Examples:
Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime.

a) \(4x^4 - 4\)

To factor an expression, the **first** check for a GCF of all of the terms.

For this expression, there is a GCF for all terms, so factor it out.

\(4x^4 - 4 \rightarrow 4(x^4 - 1)\)

Next, check to see if the remaining expression (without the GCF) can be factored further.

This expression has two terms and is a difference of squares.

\(4(x^4 - 1) \rightarrow 4(x^2 + 1)(x^2 - 1)\)

Now check to see if any of the factors can be factored further.

In this problem, \((x^2 + 1)\) cannot be factored further, but \((x^2 - 1)\) can be factored further.

\(4(x^2 + 1)(x^2 - 1) \rightarrow 4(x^2 + 1)(x + 1)(x - 1)\)

The final answer is: \(4(x^2 + 1)(x + 1)(x - 1)\). Factors may be written in any order.

b) \(15x^3 + 27x^2y - 6xy^2\)

To factor an expression, the **first** check for a GCF of all of the terms.

For this expression, there is a GCF for all terms, so factor it out.

\(15x^3 + 27x^2y - 6xy^2 \rightarrow 3x(5x^2 + 9xy - 2y^2)\)

Next, check to see if the remaining expression (without the GCF) can be factored further.

This expression has three terms, so try to “un-FOIL”.

\(3x(5x^2 + 9xy - 2y^2) \rightarrow 3x(5x - y)(x + 2y)\)

The final answer is: \(3x(5x - y)(x + 2y)\). Factors may be written in any order.

Homework
1. In general, what does it mean to factor an expression?
2. What are the all of the types of factoring methods covered in Chapter 7 so far (Sections 7.1 – 7.6)?
3. What factoring method should you always look for first, even if you eventually use another factoring method?
4. In your own words, describe how to determine if an expression has been factored completely.
5. What do you call an expression that cannot be factored using any factoring method?
Factor. Make sure each expression is factored completely. If the expression is not factorable, state that it is prime. Show all steps. Check ALL answers with answers provided for this workbook.

6. \( t^2 - 100 \)

7. \( 4x^2 + 12x \)

8. \( y^2 - y - 30 \)

9. \( p^3 + 3p^2 - 7p - 21 \)

10. \( 18m^2 + 3m - 10 \)

11. \( 3a^2 - 45a + 162 \)

12. \( z^2 + 9 \)

13. \( 5x^2 - 125 \)

14. \( 12y^3 - 50y^2 - 18y \)

15. \( 5n^4 + 5n^3 + 3n^2 + 3n \)

16. \( 675 - 3q^2 \)

17. \( t^2 - 144t \)

18. \( a^3 + 5a^2 - 4a - 20 \)

19. \( 4c^2 - 32cd + 60d^2 \)

20. \( 8x^4 - 8 \)

21. \( z^4 + 12z^2 - 13 \)

22. \( 64m^3 + 176m^2 + 121m \)

23. \( 81p^2 - 16q^2 \)

24. \( 14a^3c^3 - 29a^2c^2 + 12ac \)

25. \( 18y^5 + 90y^4 - 48y^3 - 240y^2 \)

26. \( 2n^2 + 300n + 1 \)

27. \( 10k^7 - 90k^6 + 80k^5 \)

28. \( 2wz^2 + z^2 - 72w - 36 \)

29. \( x^4 - y^4 \)

30. \( 30m^3 - 123m^2 - 135m \)

31. \( 2p^4 - 96p^2 - 98 \)

32. \( 160a^3 + 60a^2 - 40a - 15 \)

33. \( 18x^3y - 112x^2y^2 + 24xy^3 \)
Section 7.7  Solving Quadratic Equations by Factoring

Example:
Solve the equation. Give simplified answers.

a) \( x(2x + 3) = 2 \)

To solve a quadratic equation by factoring, first multiply/distribute and combine like-terms, in order to get the equation into standard form: \( ax^2 + bx + c = 0 \)

\[ x(2x + 3) = 2 \quad \rightarrow \quad 2x^2 + 3x = 2 \quad \rightarrow \quad 2x^2 + 3x - 2 = 0 \]

Once the equation is set equal to zero and all like-terms are combined, factor.

\[ 2x^2 + 3x - 2 = 0 \quad \rightarrow \quad (2x - 1)(x + 2) = 0 \]

Use the Zero Product Rule to find the values for \( x \) that satisfy the equation, by setting each factor equal to zero and solve.

\[ (2x - 1)(x + 2) = 0 \quad \rightarrow \quad 2x - 1 = 0 \quad \text{AND} \quad x + 2 = 0 \]
\[ 2x = 1 \quad \text{AND} \quad x = -2 \]
\[ x = \frac{1}{2} \]

The solutions are \( \frac{1}{2} \) and \( -2 \).

Homework

1. In your own words, describe a linear equation. How does one recognize that an equation is linear? Give two examples of linear equations.

2. In your own words, describe a quadratic equation. How does one recognize that an equation is quadratic? Give two examples of quadratic equations.

3. In your own words, describe how solve a quadratic equation by factoring.

Solve each equation. Show a check for every third problem. Give simplified answers.

4. \( x^2 + 4x - 60 = 0 \)

5. \( p^2 - 3p = 28 \)

6. \( (y - 3)(y - 12) = 0 \)

7. \( q^2 + 56 = 15q \)

8. \( 2n^2 + 7n + 5 = 0 \)

9. \( 3a^2 - 3a = 60 \)

10. \( 2x^2 - 4x = 0 \)

11. \( m^2 + 17m + 82 = 10 \)

12. \( 3k^2 - 15k = 40 - 8k \)

13. \( y^2 + 25 = 10y \)

14. \( a^2 - 8 = 1 \)

15. \( p^2 = 5p \)
16. \( n^2 - 9 = 12n - 44 \)
17. \( (x - 13)(x - 6) = 0 \)
18. \( 7q(q - 3) = 0 \)
19. \( 6k^2 + 100 = 56k - 20 \)
20. \( \frac{1}{2}(2m^2 + 20m) + 9 = 0 \)
21. \( \frac{1}{2}x^2 + \frac{5}{2}x = 7 \)
22. \( y^2 = \frac{27}{4}y + 10 \)
23. \( p^2 + 16p + 60 = 12 \)
24. \( (a - 11)^2 = 0 \)
25. \( (z + 3)^2 + 2 = 18 \)
26. \( k(k + 2) - 24 = 24 \)
27. \( 5x(2x + 9) = 15 - 2x \)
28. \( 4n(n - 1) = 7(1 - n) \)
29. \( (y - 4)^2 - 3 = 40 - 2y \)
30. \( (q - 7)(q - 2) = -6 \)
Section 7.8  General Application Problems Involving Quadratic Equations

7.8 — General Application Problems Involving Quadratic Equations Worksheet

Example:
For each problem below, define variables, write an equation, solve the equation, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate.

a) One leg of a right triangle is two inches more than twice the other leg. The hypotenuse is 13 inches. Find the lengths of the three sides of the triangle.

The “other” leg of the right triangle: \( x \)

“One” leg of the right triangle: \( 2x + 2 \)

The hypotenuse of the right triangle: 13

The Pythagorean Theorem is about the lengths of the sides of a right triangle. Use the appropriate geometric fact to write an equation.

\[ a^2 + b^2 = c^2 \]

the square of the length of the “other” leg + the square of the length of the “one” leg = the square of the hypotenuse

\[
x^2 + (2x + 2)^2 = 13^2
\]

\[
x^2 + (2x + 2)^2 = 169
\]

\[
x^2 + 4x^2 + 4x + 4 = 169
\]

\[
5x^2 + 8x + 4 = 169
\]

\[
5x^2 + 8x - 165 = 0
\]

\[
(5x + 33)(x - 5) = 0
\]

\[
5x + 33 = 0 \quad \text{OR} \quad x - 5 = 0
\]

\[
x = -\frac{33}{5}
\]

The value for \( x \) cannot be negative for this problem, since it represents the length of a leg of a triangle.

The “other” leg is 5 inches, the “one” leg is \( 2\left(\frac{33}{5}\right) + 2 = 12 \) inches, and the hypotenuse is 13 inches.

Homework

For each problem below, define variables, write an equation, solve the equation, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate.

1. Determine the value of \( x \).

2. Determine the value of \( x \).
3. Determine the value of \( x \).

\[ \begin{align*}
\text{26 inches} \\
\text{24 inches}
\end{align*} \]

4. One leg of a right triangle is two inches less than the other leg. The hypotenuse is 10 inches. Find the lengths of the three sides of the triangle.

5. A ladder is leaning against a house. If the base of the ladder is 5 feet from the house and the ladder reaches 12 feet high on the house, how long is the ladder?

6. The product of two consecutive positive even integers is 288. Find the two integers.

7. One integer is three less than twice the other integer. Their product is 35. Find the two integers.

8. One number is four more than another number. Their product is 96. Find the two numbers.
   Two pairs of numbers satisfy these conditions.

9. One number is two more than twice another number. Their product is 12. Find the two numbers.
   Two pairs of numbers satisfy these conditions.

10. The area of a rectangle is 88 square inches. Determine the length and width if the length is five inches less than twice the width.
Chapter 8 — Rational Expressions and Equations

Section 8.1 — Fraction Review
Section 8.2 — Simplifying Rational Expressions
Section 8.3 — Multiplication and Division of Rational Expressions
Section 8.4 — Addition and Subtraction of Rational Expressions
Section 8.5 — Rational Equations
Section 8.6 — General Application Problems Involving Rational Equations

Answers
Section 8.1  Fraction Review

Examples:
See Section 1.4 for examples.

Homework
1. In your own words, describe how to reduce or simplify a fraction.
2. In your own words, describe how to multiply two fractions.
3. In your own words, describe how to divide two fractions.
4. In your own words, describe how to add and/or subtract two fractions.

Simplify each fraction.
5. \(\frac{70}{154}\)
6. \(\frac{36x^5}{27x^3}\)
7. \(\frac{195}{273}\)

Perform each operation without using a calculator. If you have trouble with these problems, get help NOW! All answers must be simplified.
8. \(\frac{6}{15} \cdot \frac{14}{3}\)
9. \(\frac{6}{9} + \frac{5}{22}\)
10. \(\frac{7}{10} + \frac{8}{10}\)
11. \(\frac{3}{10} - \frac{1}{8}\)
12. \(\frac{5}{14} \cdot \frac{35}{13}\)
13. \(\frac{3}{8} + \frac{1}{10}\)
14. \(2 + \frac{1}{9}\)
15. \(\frac{10}{33} - 9\)
16. \(\frac{9}{14} - 5\)
17. \(\frac{7}{12} + \frac{11}{18}\)
18. \(\frac{5}{14} \cdot \frac{7}{10}\)
19. \(\frac{5}{14} + \frac{7}{10}\)
20. \(\frac{5}{14} \cdot \frac{7}{10}\)
Section 8.2  Simplifying Rational Expressions

Example:
Simplify.

\[ \frac{9d^2 - d^2}{5d - 15a} \]

To simplify a rational expression, first factor the numerator and denominator.

\[ \frac{(3a + d)(3a - d)}{5(d - 3a)} \]

The numerator is a difference of squares and the denominator has a GCF.

Cancel FACTORS common to numerator and denominator.

\[ \frac{(3a + d)(3a - d)^{-1}}{5(d - 3a)} \]

The factor \((3a - d)\) and the factor \((d - 3a)\) are opposites, so they cancel to \(-1\).

\[ \frac{-1(3a + d)}{5} \]

The answer may be written \(\frac{-3a + d}{5}\) or \(\frac{-3a + d}{-5}\) or \(\frac{3a + d}{5}\) or \(\frac{-3a + d}{5}\) or \(\frac{3a + d}{-5}\).

Homework

1. In your own words, define a rational expression?
2. When simplifying a rational expression, in your own words, describe how to determine when you can cancel “right away”? 
3. When simplifying a rational expression, if you cannot cancel “right away”, in your own words, describe the first step to take?
4. In your own words, define terms and factors in mathematics?
5. When simplifying a rational expression, in your own words, describe what you can cancel and what you NOT cancel?
6. In your own words, describe how to simplify a rational expression.
7. List all of the factoring “methods” covered in Chapter 7.
8. Simplify each expression, if possible. If you cannot simplify an expression, explain why not.

a) \( \frac{2x}{6x} \)

b) \( \frac{x + 2}{x + 6} \)

c) \( \frac{(x - 2) + (x + 3)}{x + 3} \)

d) \( \frac{(x - 2)(x + 3)}{x + 3} \)

e) \( \frac{3 - (x + 3)}{x + 3} \)
9. Simplify each expression, if possible. If you cannot simplify an expression, explain why not.
   a) \( \frac{-5}{5} \)
   b) \( \frac{x}{-x} \)
   c) \( \frac{x + 3}{x - 3} \)
   d) \( \frac{- (x + 3)}{x + 3} \)
   e) \( \frac{3 - x}{x - 3} \)

Simplify.
10. \( \frac{10xy}{22x^2y} \)
11. \( \frac{36p^5}{3p} \)
12. \( \frac{14mn^2}{28m^2n^2} \)
13. \( \frac{3(x + 6)}{15} \)
14. \( \frac{(2y - 1)(y + 5)}{y + 5} \)
15. \( \frac{8 + (k - 6)}{2} \)
16. \( \frac{(2a - 9) - (a + 5)}{a + 5} \)
17. \( \frac{(w - 2)(w - 8)}{w - 8} \)
18. \( \frac{x^2 + 12x + 32}{x + 4} \)
19. \( \frac{z^2 - 3z - 54}{z^2 - 11z + 18} \)
20. \( \frac{2m - 14}{m^2 - 4m - 21} \)
21. \( \frac{10p^2 + 50p}{8p^2 + 80p} \)
22. \( \frac{4 - a^2}{2a - 4} \)
23. \( \frac{2d^2 - 5d - 3}{d^2 + 5d - 24} \)
<table>
<thead>
<tr>
<th>Number</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.</td>
<td>( \frac{3n-15}{6n^2-48n+90} )</td>
</tr>
<tr>
<td>25.</td>
<td>( \frac{y^2+4y-12}{y^2-36} )</td>
</tr>
<tr>
<td>26.</td>
<td>( \frac{15x^2-16x-7}{3x^2-35x-12} )</td>
</tr>
<tr>
<td>27.</td>
<td>( \frac{6q^2-29q-5}{6q^2+49q+8} )</td>
</tr>
<tr>
<td>28.</td>
<td>( \frac{12m^2-27}{12m+18} )</td>
</tr>
<tr>
<td>29.</td>
<td>( \frac{2k+10}{4k^2-100} )</td>
</tr>
<tr>
<td>30.</td>
<td>( \frac{4z^2+28z+49}{12z^2+56z+49} )</td>
</tr>
<tr>
<td>31.</td>
<td>( \frac{4a^2-9d^2}{24d^2-16a} )</td>
</tr>
<tr>
<td>32.</td>
<td>( \frac{2p^3+10p^2-3p-15}{p^2-25} )</td>
</tr>
<tr>
<td>33.</td>
<td>( \frac{14m^2n^2+37mn+5}{98m^2n^2-2m} )</td>
</tr>
</tbody>
</table>
Section 8.3 Multiplication and Division of Rational Expressions

8.3 — Simplifying, Multiplying, and Dividing Review Worksheet

Examples:
Perform the indicated operation. All answers must be given in simplified form.

a) \( \frac{6x - 3}{4x^2 - 1} \cdot \frac{2x^2 - 13x - 7}{3x^2 - 6x - 105} \)

To multiply two rational expressions, first factor both numerators and both denominators.

\[
\frac{3(2x - 1)}{(2x + 1)(2x - 1)} \cdot \frac{(2x + 1)(x - 7)}{3(x^2 - 2x - 35)}
\]

Both the left numerator and the right denominator have GCFs. The left denominator is a difference of squares and the right numerator factors into two binomials.

\[
\frac{3(2x - 1)}{(2x + 1)(2x - 1)} \cdot \frac{(2x + 1)(x - 7)}{3(x + 5)(x - 7)}
\]

The right denominator factors further.

Cancel FACTORS common to a numerator and a denominator.

\[
\frac{1}{(x + 5)}
\]

b) \( \frac{x^2 - 2x - 15}{2x} \div \frac{x^2 + 10x + 21}{4x^2} \)

To divide two rational expressions, first change the problem to multiplying the dividend (first fraction) by the reciprocal of the divisor (second fraction).

\[
\frac{x^2 - 2x - 15}{2x} \cdot \frac{4x^2}{x^2 + 10x + 21}
\]

Now, factor both numerators and both denominators.

\[
\frac{(x - 5)(x + 3)}{2x} \cdot \frac{4x^2}{(x + 3)(x + 7)}
\]

Both the left numerator and the right denominator are trinomials that factor into two binomials. The left denominator and the right numerator are already in factored form.

Cancel FACTORS common to a numerator and a denominator.

\[
\frac{(x - 5)(x + 3)}{2x} \cdot \frac{2x^2}{(x + 3)(x + 7)} \Rightarrow \frac{2x(x - 5)}{x + 7}
\]

Homework

1. When multiplying two rational expressions, in your own words, describe how to determine when can you cancel “right away”?
2. When multiplying two rational expressions, if you cannot cancel “right away”, in your own words, describe the first step to take?

3. When dividing two rational expressions may you cancel “right away”?

4. In your own words, describe how to multiply two rational expressions.

5. List all of the factoring “methods” covered in Chapter 7.

6. In your own words, describe how to divide two rational expressions.

7. When “everything” cancels in a multiplication or division problem, what is the answer?

**Multiply. All answers must be given in simplified form.**

8. \[
\frac{18x^2y^5}{3x^2y} \cdot \frac{4x^2y}{6xy}
\]

9. \[
\frac{7mn^2}{10n^3} \cdot \frac{mn}{14m^2}
\]

10. \[
\frac{3p^2(p + 2)}{p + 2} \cdot \frac{1}{6p^2}
\]

11. \[
\frac{a^2 - 2a - 8}{a - 7} \cdot \frac{a + 3}{a^2 + 5a + 6}
\]

12. \[
\frac{5k^2 + 5k}{k^2 - 1} \cdot \frac{k^2 + 5k - 6}{5k^2 + 30k}
\]

13. \[
\frac{z^2 - 25}{2z^2 + 9z - 5} \cdot \frac{4z^2 - 1}{3z - 15}
\]

**Divide. All answers must be given in simplified form.**

14. \[
\frac{15x^2y}{35xy} \div \frac{6x^2y}{21xy^3}
\]

15. \[
\frac{24}{4ed} \div \left(18cd^2\right)
\]

16. \[
\frac{4(w + 8)}{6w} \div \frac{w + 8}{w^2}
\]

17. \[
\frac{n^2 - 49}{4} \div \frac{n^2 + n - 42}{2n - 12}
\]

18. \[
\frac{12a + 120}{a^2 + 11a + 10} \div \frac{3a - 3}{a^2 - 1}
\]

19. \[
\frac{3k - 6}{6k^2 - 24} \div \frac{1}{2k + 4}
\]

**Perform the indicated operation. All answers must be given in simplified form.**

20. \[
\frac{10m + 40}{m^2 - 16} \cdot \frac{m^2 - m - 12}{20m + 60}
\]

21. \[
\frac{13z^2}{6z + 30} \div \frac{26z^2}{4z}
\]

22. \[
\frac{(9y^2 - 25)}{3y^2 + 11y + 10} \div \frac{9y + 18}{9y + 18}
\]
23. \[ \frac{p^2 - q^2}{p^2 - pq} \div \frac{p^2}{p^3 + pq} \]

24. \[ \frac{a^2}{(a + c)^2} \div \frac{c^2}{a^2 - c^2} \]

25. \[ \frac{7n^2 + 16n - 15}{7n^2 + 65n - 50} \div \frac{4n^2 + 28n - 120}{2n^2 - 18} \]

26. \[ \frac{(4x + 3)^2}{4x^2 + 19x + 12} \div \frac{2x^2 + 7x - 4}{4x^2 - 50x + 24} \]

27. \[ \frac{64 - w^2}{6w^2 + 49w + 8} \div \frac{36w^2 - 1}{w^2 - 5w - 24} \]

28. \[ \frac{6m^2 + 25m + 24}{6m^2 + 30m} \div \frac{4m^2 - 9}{2m^2 + 7m - 15} \]

29. \[ \frac{15y^2 + 10y}{10y^3 + 40y^2} \div \frac{4 - 9y^2}{6y^3 + 20y^2 - 16y} \]

30. \[ \frac{10 - 6a}{3a^2 + 22a - 45} \div \frac{a^2 + 18a + 81}{4a - 36} \]

31. \[ \frac{6p^3 + 4p}{25q^2 - 9p^2} \div \frac{3p^3 + 3p^2q + 2p + 2q}{3p^3 - 2p^2q - 5pq^2} \]
Section 8.4  Addition and Subtraction of Rational Expressions

**Examples:**
*Perform the indicated operation. All answers must be given in simplified form.*

a) \[
\frac{9x^2 + 8x}{4x^2} - \frac{5x^2 + 6x}{4x^2}
\]

To subtract two rational expressions with like-denominators, combine the numerators, and keep the denominator the same.

\[
\frac{9x^2 + 8x - (5x^2 + 6x)}{4x^2}
\]

Remember to distribute the subtraction to all terms that are subtracted.

\[
\frac{9x^2 + 8x - 5x^2 - 6x}{4x^2}
\]

Combine like terms in the numerator.

\[
\frac{4x^2 + 2x}{4x^2}
\]

To simplify the answer, FACTOR and then cancel FACTORS common to a numerator and a denominator.

\[
\frac{2x(2x + 1)}{4x^2} \quad \rightarrow \quad \frac{2x(2x + 1)}{4x^2} \quad \rightarrow \quad \frac{2x + 1}{2x}
\]

b) \[
\frac{2x}{3x + 6} + \frac{7}{x^2 - 4}
\]

To add two rational expressions with different denominators, first rewrite each so that the denominators are the same.

\[
\frac{2x}{3(x + 2)} + \frac{7}{(x + 2)(x - 2)}
\]

To determine the least common denominator (LCD, factor each denominator. For this problem the LCD is \(3(x + 2)(x - 2)\).

\[
\frac{2x}{3(x + 2)} \cdot \frac{(x - 2)}{(x - 2)} + \frac{7}{(x + 2)(x - 2)} \cdot \frac{3}{3}
\]

The fraction on the left needs a factor of \((x - 2)\) in its denominator and the fraction on the right needs a factor of \(3\) in its denominator. Multiply each fraction’s numerator and denominator by the missing factor; thereby multiplying each fraction by \(1\). This changes the form of each fraction, but not its value.

\[
\frac{2x^2 - 4x}{3(x + 2)(x - 2)} + \frac{21}{3(x + 2)(x - 2)}
\]

Multiply the new factor into the numerator, but leave the denominator in factored form.

Combine the numerators, and keep the denominator the same.

\[
\frac{2x^2 - 4x + 21}{3(x + 2)(x - 2)}
\]

This is the answer unless the fraction can be simplified. To check for simplification, try to factor the numerator.

In this problem, the numerator does NOT factor, and there are no common factors to numerator and denominator; therefore the fraction is already simplified.
Homework

1. When adding or subtracting two rational expressions, can you cancel a numerator of one expression with the denominator of the other?

2. When adding or subtracting two rational expressions, do the expressions need to have a least common denominator?

3. When multiplying or dividing two rational expressions, do the expressions need to have a least common denominator?

4. In your own words, describe how to find the least common denominator of two rational expressions.

5. In your own words, describe how to add or subtract two rational expressions.

6. When “everything” cancels in the numerator of an addition or subtraction problem, what is the answer?

Perform the indicated operation. All answers must be given in simplified form.

7. \( \frac{2}{y} + \frac{7}{y} \)

8. \( \frac{x}{x+2} + \frac{3}{x+2} \)

9. \( \frac{a}{a+5} - \frac{4}{a+5} \)

10. \( \frac{5}{2z} + \frac{1}{2z} \)

11. \( \frac{11}{3p} - \frac{p+2}{3p} \)

12. \( \frac{d-1}{d^2-4} + \frac{3}{d^2-4} \)

13. \( \frac{c+2}{c^2-1} - \frac{c-3}{c^2-1} \)

14. \( \frac{2x+10}{x^2+11x+24} - \frac{x+7}{x^2+11x+24} \)

15. \( \frac{y^2+10y}{y^2+4y-12} + \frac{8y+1}{y^2+4y-12} \)

16. \( \frac{a^2+a-3}{a^2+4a-5} + \frac{a-12}{a^2+4a-5} \)

17. \( \frac{5q^2-5q-5}{q^2-17q+70} - \frac{4q^2+3q+15}{q^2-17q+70} \)

18. \( \frac{k^2+k}{4k^2-9} + \frac{k^2-3}{4k^2-9} \)

19. \( \frac{2c+5}{3c^2+16c+5} - \frac{c+7}{3c^2+16c+5} \)

20. \( \frac{d^2-9}{4d^2+7d+3} + \frac{d^2+7}{4d^2+7d+3} \)

21. \( \frac{8m^2+m}{6m^2} - \frac{5m^2+4m}{6m^2} \)
Perform the indicated operation. All answers must be given in simplified form.

22. \( \frac{2}{x-5} + \frac{3}{5-x} \)

23. \( \frac{y}{10-y} - \frac{4}{y-10} \)

24. \( \frac{8}{-q-2} + \frac{1}{q+2} \)

25. \( \frac{3}{2z} + \frac{5}{6z} \)

26. \( \frac{5}{x} + \frac{3}{x^2} \)

27. \( \frac{7}{10p} - \frac{1}{6p^2} \)

28. \( \frac{2k}{k+1} + \frac{3}{k} \)

29. \( \frac{1}{a} + \frac{3a}{a-2} \)

30. \( \frac{3}{n+5} + \frac{4}{n+1} \)

31. \( \frac{6}{d+2} - \frac{1}{2d} \)

32. \( \frac{5}{m^2+3m} - \frac{2}{m} \)

33. \( \frac{3}{c^2-1} + \frac{c}{c-1} \)

34. \( \frac{q}{q^2+5q} + \frac{3}{q} \)

35. \( \frac{7}{2z^2-6z} - \frac{11}{4z} \)

36. \( \frac{y^2}{6y^2-6y} + \frac{5}{3y} \)

37. \( \frac{5}{x^2-4} + \frac{8}{x^2-x-6} \)

38. \( \frac{1}{2k^2-11k-6} - \frac{4}{k^2-3k-18} \)

39. \( \frac{m}{m^2+5m-24} - \frac{2}{m^2-9} \)

40. \( \frac{3d}{d^2+d-2} + \frac{2}{d^2+3d+2} \)
41. \[ \frac{a + 6}{a^2 + 2a - 8} - \frac{4}{3a^2 + 12a} \]
42. \[ \frac{2n}{2n^2 + 3n - 9} - \frac{5}{2n^2 + 9n - 18} \]
43. \[ \frac{3p + 1}{p^2 + 2p} + \frac{1}{p^2 - 2p} \]
44. \[ \frac{c + 5}{2c^3 - 7c - 4} - \frac{2}{4c^2 - 1} \]
45. \[ \frac{z + 21}{13z^2 + 53z + 4} - \frac{z + 5}{13z^2 + 14z + 1} \]
46. \[ \frac{10x}{x^2 + 2x - 15} + \frac{6}{9 - x^2} \]
Section 8.5  Rational Equations

8.5 — Review of Rational Expressions and Equations Worksheet

Example:
Solve. If there is no solution, state so.

a) \( \frac{2x}{x+3} - \frac{x}{x+1} = \frac{12}{x^2 + 4x + 3} \)

To solve an equation containing rational expressions it’s often easiest to “get rid of the fractions”.

Determine the least common denominator (LCD), by factoring each denominator. For this problem the LCD is \((x+1)(x+3)\).

Before starting to solve the equation, it’s important to note what values make the expressions in the equation undefined.

For this problem: \( x \neq -3 \) and \( x \neq -1 \).

Multiply each term in the equation by this LCD. Since this is an equation to solve, it’s ok to multiply each fraction by a non-1 value, as long as it’s done consistently to each term in the equation.

\[
\frac{(x+1)(x+3)}{1} \cdot \frac{2x}{x+3} - \frac{(x+1)(x+3)}{1} \cdot \frac{x}{x+1} = \frac{(x+1)(x+3)}{1} \cdot \frac{12}{(x+1)(x+3)}
\]

Cancel factors common to numerator and denominator in each product. If this is done correctly, all denominators should cancel.

\[
\frac{(x+1)(x+3)}{1} \cdot \frac{2x}{x+3} - \frac{(x+1)(x+3)}{1} \cdot \frac{x}{x+1} = \frac{(x+1)(x+3)}{1} \cdot \frac{12}{(x+1)(x+3)}
\]

Solve the resulting equation.

\[
2x(x+1) - x(x+3) = 12
\]
\[
2x^2 + 2x - x^2 - 3x = 12
\]
\[
x^2 - x = 12
\]
\[
x^2 - x - 12 = 0
\]
\[
(x-4)(x+3) = 0
\]

or 
\[
x-4 = 0 \quad \text{or} \quad x+3 = 0
\]
\[
x = 4 \quad \text{or} \quad x = -3
\]

All solutions to rational equations need to be checked to see if any of them are extraneous. Basically, check to see if any solution makes the denominator undefined.

In this problem, the only true solution is \( x = 4 \), since \( x \neq -3 \).

Homework

1. In your own words, describe how to recognize a linear equation? In your own words, describe how to solve a linear equation?
2. In your own words, describe how to recognize a quadratic equation? As of now, what method have you learned for solving a quadratic equation?
3. What value can the denominator of a rational expression not be?
4. In your own words, describe how to determine for which values of a variable a rational expression is defined?
5. In your own words, describe how to solve an equation with rational expressions.

6. Is it important to check your answers after solving an equation with rational expressions? In your own words, explain why or why not.

Find the value(s) of the variable for which the expression is undefined, and then write a phrase that gives all values for which the expression is defined.

7. \[ \frac{3}{x + 2} \]

8. \[ \frac{7}{p - 4} \]

9. \[ \frac{y + 5}{y - 3} \]

10. \[ \frac{8}{m} \]

11. \[ \frac{10}{(a + 1)(a + 9)} \]

12. \[ \frac{n + 3}{(n + 3)(n + 8)} \]

13. \[ \frac{d^2 + 1}{d^2 - 4} \]

14. \[ \frac{12}{z^2 - 9z} \]

15. \[ \frac{c}{c^2 + 5c - 14} \]

16. \[ \frac{11}{k^2 + 4} \]

Solve. If there is no solution, state so.

17. \[ \frac{1}{3} + 2x = \frac{1}{9} \]

18. \[ \frac{1}{y} + 3 = \frac{2}{y} \]

19. \[ \frac{z}{4} - \frac{3}{5} = \frac{1}{2} \]

20. \[ 5 - \frac{19}{2m} = \frac{1}{4} \]

21. \[ \frac{1}{2} + \frac{1}{x} = \frac{1}{2} \]

22. \[ \frac{1}{2}(4 - p) = \frac{1}{3} \]

23. \[ \frac{a}{8} + \frac{1}{8} = \frac{3}{4} \]
24. \( \frac{n}{5} + \frac{7}{n} = \frac{12}{5} \)

25. \( \frac{3}{c+1} + \frac{6}{c} = \frac{12}{c} \)

26. \( \frac{8}{x+2} - 2 = \frac{8}{x+2} \)

27. \( \frac{k}{k+2} + \frac{4}{k} = 1 \)

28. \( \frac{4y+8}{y+4} - \frac{6}{y} = 3 \)

29. \( \frac{12}{x^2-9} + \frac{2}{x+3} = \frac{5}{x-3} \)

30. \( \frac{m}{m-2} + \frac{3}{m+2} = -\frac{6}{m^2-4} \)

31. \( \frac{4}{z+2} - \frac{2}{z^2-3z-10} = \frac{7}{z-5} \)

32. \( \frac{2n}{n+3} - \frac{n}{n+1} = \frac{12}{n^2+4n+3} \)

33. \( \frac{a+1}{a+4} + \frac{a}{a+2} = -\frac{1}{a^2+6a+8} \)

34. \( \frac{4k+12}{k+5} + \frac{1}{k} = \frac{k-3}{k+5} \)
Section 8.6  General Application Problems Involving Rational Equations

8.6 — General Application Problems Involving Rational Equations Worksheet

Example:
For the problem below, define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units.

a) Javier is vacationing near the Colorado River and plans to canoe in a section of the river where the current is 2 miles per hour. If it took Javier a total of 4 hours to travel 10 miles downstream and 2 miles upstream, determine the speed at which Javier’s canoe would travel in still water.

Canoe’s speed in still water: \( x \)

Define what the problem asks to find.

Fill in the distance/rate/time chart with the givens and variables.

<table>
<thead>
<tr>
<th>Directions</th>
<th>Rate = Distance \div Time</th>
<th>Time = Distance \div Rate</th>
<th>Distance = Rate \cdot Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream</td>
<td>( x + 2 )</td>
<td>( \frac{10}{x + 2} )</td>
<td>10</td>
</tr>
<tr>
<td>Upstream</td>
<td>( x - 2 )</td>
<td>( \frac{2}{x - 2} )</td>
<td>2</td>
</tr>
</tbody>
</table>

Unused fact/picture: 4 hours

The total time was 4 hours.

Use the appropriate statement from the problem to write an equation.

“time downstream” + “time upstream” = “total time”

\[
\frac{10}{x + 2} + \frac{2}{x - 2} = 4
\]

Solve the equation.

\[
\left( \frac{x + 2}{1} \right) \cdot \frac{10}{x + 2} + \left( \frac{x + 2}{1} \right) \cdot \frac{2}{x - 2} = \left( \frac{x + 2}{1} \right) \cdot 4
\]

Note: \( x \neq \pm 2 \)

\[
10(x - 2) + 2(x + 2) = 4(x + 2)(x - 2)
\]

\[
10x - 20 + 2x + 4 = 4(x^2 - 4)
\]

\[
12x - 16 = 4x^2 - 16
\]

\[
0 = 4x^2 - 12x
\]

\[
0 = 4x(x - 3)
\]

\[
x = 0 \quad \text{or} \quad x - 3 = 0
\]

\[
x = 0 \quad x = 3
\]

Since Javier actually travels upstream, his speed in still water cannot be 0. Javier’s rate in still water is 3 mph.
Homework

1. In your own words, define “distance” and give some examples of units for distance.

2. In your own words, define “time” and give some examples of units for time.

3. In your own words, define “rate” and give some examples of units for rate.

4. How can you find a distance given a rate and a time?

5. How can you find rate given a distance and a time?

6. How can you find a time given a distance and a rate?

7. In your own words, describe how to use a “chart” to organize information in a distance/rate/time application problem.

For each problem below, define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units when appropriate. You may want to use a chart to organize the information in the problem.

8. One integer is four less than another integer. The sum of their reciprocals is $\frac{3}{8}$. Determine the two integers. Note: This problem asks for integer answers only.

9. The difference between a number and twice its reciprocal is $\frac{17}{3}$. Find the number. Note: More than one number satisfies this condition.

10. Martita is planning to build a desk for her daughter. The top of the desk will be a rectangle, whose length is four inches less than $\frac{3}{4}$ of the width. If the area of the desktop is 640 square inches, what are the dimensions (width and length) of the desktop?

11. Sierra ran at a speed of 4 mph to her friend’s house to pick up her bike. She biked home, taking the same route she had run. If she biked at a speed of 16 mph, and her round trip time was 1 hour, how far is it from her house to her friend’s house? Give your answer accurate to 1 decimal place.

12. A biplane can travel 60 mph less than a jet plane. In the same time that the biplane travels 50 miles, the jet plane travels 100 miles. Find their speeds.

13. Hector is visiting his aunt’s cabin. To get there, he needs to take his row boat $\frac{3}{4}$ mile across a lake (still water) and travel down a river 3 miles. The current of the river is 2 mph, and he travels downstream. If his entire boating trip takes him 30 minutes, how fast can he row in still water?

14. Steffen and Pete decided to compete in a Muddy Buddy race—where two-person teams compete by running and biking through mud and obstacles. They were able to run at an average of 10 mph, and were able to bike at an average of 20 mph. If they biked four miles more than they ran and their total team time was $\frac{1}{2}$ hour, find the distance they ran and the distance they biked.

15. Lena is kayaking in the Merced River. She is able to kayak 5 mph in still water. If it took Lena a total of 4 hours to travel 12 miles downstream and 5 miles upstream, determine the speed of the river.

16. A ferry takes cars across a 2-mile stretch of the bay. The ferry—when empty—travels 2 mph faster than when it is full. If the ferry takes 3 minutes less to make the 2-mile trip when it is empty, determine the speed of the ferry when it is full and the speed of the ferry when it is empty.

17. Sara and Juan need to edit their group’s paper for their English class. Sara can edit the paper in 1.5 hours. Juan can edit the paper in 1 hour. How long would it take them, working together to edit the paper? Give your answer accurate to 1 decimal place or in minutes.

18. Tyrone and Eleanor decide to hang flyers for their school’s upcoming Social Justice Conference. When Tyrone hangs the flyers by himself, it takes him 45 minutes. When Tyrone and Eleanor work together to hang the flyers, it takes them 20 minutes. How long would it take Eleanor to hang the flyers if she worked alone?

19. Cameron and Emily are on a trip. If Cameron drove the entire trip, the trip would take about 10 hours. If Emily drove the entire trip, the trip would take about 8 hours. After Emily had been driving for 3 hours, Cameron takes over the driving. About how much longer will Cameron drive before they reach their final destination? Hint: This is a “work” problem. Give your answer accurate to 2 decimal places.
Chapter 9 – Radical Expressions and Equations

Section 9.1 – Introduction to Square Roots – Part I
Section 9.2 – Simplifying Radical Expressions
Section 9.3 – Addition, Subtraction, and Multiplication of Radical Expressions
Section 9.4 – Division of Radical Expressions
Section 9.5 – Simplifying Radical Expressions – Part II
Section 9.6 – Radical Equations

Answers
Section 9.1  Introduction to Square Roots

Example:
Simplify. If the expression is not a real number, state so.

a)  \[ \sqrt[4]{\frac{49}{4}} \]

To simplify a square root of a perfect square, use the fact that:

\[ \sqrt{a} = b \text{ if and only if } a = b^2. \]

If \( a \) is nonnegative, then \( \sqrt{a^2} = a \). If \( a \) is negative, then \( \sqrt{a^2} = -a \)

\[ -\sqrt{\frac{49}{4}} \Rightarrow -1 \cdot \left(\sqrt{\frac{7}{2}}\right)^2 \Rightarrow -1 \cdot \frac{7}{2} \Rightarrow -\frac{7}{2} \]

Homework

1. In your own words, define the square root of a number.
2. In your own words, define the square of a number.
3. Does the square root of a negative number have a real number value?
4. What is the expression under the square root sign called?

Simplify. If the expression is not a real number, state so.

5.  \( \sqrt{56} \)

6.  \( \sqrt{1} \)

7.  \( \sqrt{144} \)

8.  \( \sqrt{\frac{25}{4}} \)

9.  \( -\sqrt{100} \)

10.  \( \sqrt{6} \)

11.  \( \sqrt{-4} \)

12.  \( \sqrt{196} \)

13.  \( \sqrt{\frac{1}{36}} \)

14.  \( -\sqrt{81} \)

15.  \( -\sqrt{\frac{9}{64}} \)

16.  \( \sqrt{225} \)

17.  \( \sqrt{-169} \)

18.  \( -\sqrt{400} \)

19.  \( \sqrt{\frac{1}{100}} \)

20.  \( \sqrt{16} + \sqrt{4} \)

21.  \( \sqrt{25} - \sqrt{5} \)
22. Simplify each of the following.
   
   a) \( \sqrt{(4)^2} \)
   
   b) \( \sqrt{(16)^2} \)
   
   c) \( \sqrt{x^2} \) if \( x \) is a nonnegative value.
   
   d) \( \sqrt{(-4)^2} \)
   
   e) \( \sqrt{(-9)^2} \)
   
   f) \( \sqrt{x^2} \) if \( x \) is a negative value.
   
   g) \( (\sqrt{4})^2 \)
   
   h) \( (\sqrt{9})^2 \)
   
   i) \( (\sqrt{x})^2 \)
Section 9.2  Simplifying Radical Expressions — Part I

For all problems in the remainder of Chapter 9, assume that all variables are nonnegative.

Examples:
Simplify. Assume that all variables are nonnegative. If the expression is not a real number, state so.

a) \( \sqrt{1400x^{10}y^9} \)

One way to simplify a radical expression is to factor the radicand into primes.

\[ \sqrt{2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \cdot x^{10}y^9} \]

In this problem, there are a pair of 2s and a pair of 5s. Taking the square root of these “pairs”, will result in the following:

\[ 2 \cdot 5 \sqrt{7 \cdot x^{10}y^9} \]

This problem may also be viewed as \( \sqrt{1000 \cdot 14 \cdot x^{10}y^9} \).

Now work on the variables. To take the square root of a variable raised to a specific exponent, you may divide the EXPONENT by 2.

\[ 2 \cdot 5 \cdot x^5 \cdot y^4 \sqrt{2 \cdot 7 \cdot y} \]

Now, simplify by multiply the “outside” together and the “inside” together.

\[ 10x^5y^4 \sqrt{14y} \]

b) \( \sqrt{20a^2c} \sqrt{15a^{12}c^7} \)

The easiest way to simplify a product of two square roots is to write the expression as the square root of a single product.

\[ \sqrt{20a^2c \cdot 15a^{12}c^7} \]

Before taking the square root or multiplying the constants, find the prime factorization of each constant.

\[ \sqrt{2 \cdot 2 \cdot 5 \cdot 3 \cdot a^4 \cdot a^{12} \cdot c \cdot c^7} \]

Multiply the variables together by adding their exponents.

\[ \sqrt{2 \cdot 2 \cdot 5 \cdot 3 \cdot a^{16} \cdot c^8} \]

This problem may also be viewed as \( \sqrt{100 \cdot 3 \cdot a^{16} \cdot c^8} \).

\[ 2 \cdot 5 \cdot a^8 \cdot c^4 \sqrt{3} \]

Now, simplify by multiply the “outside” together.

\[ 10a^8c^4 \sqrt{3} \]

Homework

1. In your own words, describe how to simplify the square root of a non-perfect square.

2. When you “pull factors out from a square root”, what operation is between those factors and the factors that remain under the square root?

3. If the radicand in a square root expression has a variable raised to an exponent, what is the short cut rule for simplifying the square root for that variable?

4. In your own words, describe how simplifying each of the following expressions are different: \( \sqrt{16} \) and \( \sqrt{x^n} \).
5. **Determine whether each statement is true or false.**
   a) The square root of a positive perfect square is a rational number.
   b) A rational number is always real.
   c) A real number is always rational.
   d) An irrational number is not real.
   e) The square root of a positive non-perfect square is a real number.
   f) The square root of a positive non-perfect square is an irrational number.
   g) The square root of a negative perfect square is a real number.
   h) An irrational number is real.
   i) The square root of a negative non-perfect square is a real number.

6. **When multiplying two single-term square root expressions, what’s the “easiest” step to take first?**

7. **Assuming all variables are nonnegative, what is the simplified answer for a problem that “squares a square root” or “square roots a square”? In other words, what is the relationship between “squaring” and “square rooting”?**

   **Simplify. Assume that all variables are nonnegative. If the expression is not a real number, state so.**

8. \(\sqrt{18}\)
9. \(\sqrt{20}\)
10. \(\sqrt{45}\)
11. \(\sqrt{200}\)
12. \(\sqrt{-50}\)
13. \(\sqrt{162}\)
14. \(\sqrt{288}\)
15. \(\sqrt{112}\)
16. \(\sqrt{1575}\)
17. \(\sqrt{363}\)
18. \(\sqrt{432}\)
19. \(3\sqrt{44}\)
20. \(-5\sqrt{12}\)
21. \(10\sqrt{28}\)
22. \(7\sqrt{54}\)
23. \(\sqrt{x^7}\)
24. \(\sqrt{z^{11}}\)
25. \(\sqrt{k^9}\)
26. \(\sqrt{n^{100}}\)
27. \(\sqrt{p^{401}}\)
Simplify. Assume that all variables are nonnegative. Note that there is no addition or subtraction in any of the following problems.

40. \( \sqrt{5} \cdot \sqrt{5} \)
41. \( \sqrt{16} \cdot \sqrt{16} \)
42. \( (\sqrt{8})^2 \)
43. \( (\sqrt{11})^2 \)
44. \( \sqrt{2} \cdot \sqrt{6} \)
45. \( \sqrt{15} \cdot \sqrt{12} \)
46. \( \sqrt{18} \cdot \sqrt{42} \)
47. \( \sqrt{1} \cdot \sqrt{22} \)
48. \( \sqrt{20} \cdot \sqrt{45} \)
49. \( \sqrt{35} \cdot \sqrt{14} \)
50. \( \sqrt{63} \cdot \sqrt{28} \)
51. \( \sqrt{3x} \cdot \sqrt{3x^3} \)
52. \( \sqrt{5y^2} \cdot \sqrt{10y} \)
53. \( \sqrt{6a^3} \cdot \sqrt{15a^2} \)
54. \( \sqrt{27z^2} \cdot \sqrt{6z^4} \)
55. \( \sqrt{40m^3} \cdot \sqrt{15mn^3} \)
56. $\sqrt{8xy^4z} \cdot \sqrt{2xy^3}$
57. $\sqrt{10a^4c} \cdot \sqrt{35ac^5}$
58. $\sqrt{48p^5q} \cdot \sqrt{24p^5q}$
59. $(\sqrt{25w^6})^2$
60. $(\sqrt{11k^3})^2$
61. $(\sqrt{3a^9b})^2$
62. $(\sqrt{9x^8})^2 \cdot (\sqrt{4x^{10}})^2$
63. $(\sqrt{2y^3})^2 \cdot (\sqrt[3]{z})^2$
64. $(\sqrt[5]{m})^2 \cdot (\sqrt[3]{m})^2$
65. $(\sqrt[4]{cd})^2 \cdot (\sqrt[4]{cd})^2$
Section 9.3 — Addition, Subtraction, and Multiplication of Radical Expressions

Examples:

Simplify.

a) \( \sqrt{45} - 2\sqrt{45} + 7\sqrt{5} \)
   To add square roots, the radicands must be “like”. Before combining like-radical terms, simplify each term by taking the square root.
   \[ \sqrt{3 \cdot 3 \cdot 5} - 2 \cdot \sqrt{3 \cdot 3 \cdot 5} + 7 \cdot \sqrt{5} \]
   This problem may also be viewed as \( \sqrt{9 \cdot 5} - 2 \cdot \sqrt{9 \cdot 5} + 7 \cdot \sqrt{5} \).
   \[ 3\sqrt{5} - 2 \cdot 3\sqrt{5} + 7\sqrt{5} \]
   Remember, the factor that is “pulled” out is multiplied by what remains in the square root.
   \[ 3\sqrt{5} - 6\sqrt{5} + 7\sqrt{5} \]
   Combine like-radical terms. The radicand remains the same; only the constant in front of the radical changes.
   \[ 4\sqrt{5} \]

Multiply. All answers must be given in simplified form.

b) \( (8\sqrt{3} + \sqrt{6})(\sqrt{3} - 2\sqrt{6}) \)
   To multiply a two term square root expression by another expression, distribute.
   \[ (8\sqrt{3} + \sqrt{6})(\sqrt{3} - 2\sqrt{6}) \]
   In this problem, use the FOIL method.
   \[ 8\sqrt{3} \cdot \sqrt{3} - 8\sqrt{3} \cdot 2\sqrt{6} + \sqrt{6} \cdot \sqrt{3} - \sqrt{6} \cdot 2\sqrt{6} \]
   \[ 8\sqrt{3} \cdot \sqrt{3} - 8 \cdot 2\sqrt{3} \cdot 6 + \sqrt{6} \cdot \sqrt{3} - 2\sqrt{6} \cdot 6 \]
   You may leave the problem as written to simplify each square root or rewrite as:
   \[ 8\sqrt{9} - 16\sqrt{18} + \sqrt{18} - 2\sqrt{36} \]
   This may be written as:
   \[ 8 \cdot 3 - 16 \cdot 3\sqrt{2} + 3\sqrt{2} - 2 \cdot 6 \]
   Combine like-radical terms only.
   \[ 12 - 48\sqrt{2} + 3\sqrt{2} - 12 \]
   \[ 12 - 45\sqrt{2} \]

Homework

1. Can you add any two square root expressions? In your own words, explain why or why not.
2. In your own words, describe how to add two square root expressions.

Simplify.

3. \( 4\sqrt{2} + 5\sqrt{2} \)
4. \( 8\sqrt{11} - 3\sqrt{11} \)
5. \( 4\sqrt{10} + \sqrt{10} \)
6. \( \sqrt{8} + 5\sqrt{8} \)
7. \( 9\sqrt{14} - \sqrt{14} \)
8. \( 12\sqrt{5} - 3\sqrt{5} + \sqrt{5} \)
9. $\sqrt{12} - 4\sqrt{12} + 7\sqrt{2}$
10. $\sqrt{75} + \sqrt{12}$
11. $\sqrt{20} + \sqrt{45}$
12. $\sqrt{108} - \sqrt{3}$
13. $\sqrt{100} + \sqrt{44}$
14. $-2\sqrt{63} + \sqrt{175}$
15. $5\sqrt{192} - 3\sqrt{80}$
16. $2\sqrt{150} - \sqrt{6}$
17. $-\sqrt{121} + \sqrt{18}$
18. $\sqrt{225} - 4\sqrt{27}$
19. $2\sqrt{36} + \sqrt{275}$
20. $5\sqrt{162} + \sqrt{34} - 3\sqrt{8}$

Multiply. All answers must be given in simplified form.
21. $\sqrt{2} \left(5 + \sqrt{2}\right)$
22. $13 \left(\sqrt{7} + \sqrt{5}\right)$
23. $2 \left(\sqrt{10} - 6\right)$
24. $\sqrt{11} \left(9 - \sqrt{5}\right)$
25. $6 \left(3\sqrt{5} + 2\right)$
26. $4 \left(8\sqrt{3} - 12\right)$
27. $x \left(x + \sqrt{6}\right)$
28. $y \left(y + \sqrt{y}\right)$
29. $2a \left(3\sqrt{a} + a\right)$
30. $\sqrt{5} \left(3\sqrt{5} + 4\right)$
31. $\sqrt{14} \left(6 - 3\sqrt{14}\right)$
32. $5\sqrt{11} \left(8 + 3\sqrt{11}\right)$
33. $2\sqrt{7} \left(6 + 10\sqrt{5}\right)$
34. $\sqrt{10} \left(3\sqrt{2} + 4\sqrt{5}\right)$
35. $4\sqrt{5} \left(9\sqrt{14} - \sqrt{21}\right)$
36. $\sqrt{x} (8\sqrt{x} + 13)$
37. $3\sqrt{a} (3a + 3\sqrt{a})$
38. $4z (4\sqrt{z} + 4z)$
39. $\sqrt{5c} (5c + 5\sqrt{c})$
40. $2\sqrt{7n} (2\sqrt{7n} + 7n)$
41. $(6 + \sqrt{2}) (3 + \sqrt{5})$
42. $(8 - \sqrt{3}) (1 + \sqrt{7})$
43. $(4 - \sqrt{6}) (10 - \sqrt{3})$
44. $(11 + \sqrt{2}) (5 + \sqrt{2})$
45. $(1 - \sqrt{14}) (7 - \sqrt{14})$
46. $(10 + 3\sqrt{2}) (4 + 2\sqrt{5})$
47. $(12 - \sqrt{5}) (5 + 8\sqrt{3})$
48. $(8\sqrt{6} + \sqrt{2}) (\sqrt{6} + 7\sqrt{2})$
49. $(3\sqrt{2} - \sqrt{7}) (9\sqrt{2} - 4\sqrt{7})$
50. $(4 + \sqrt{10})^2$
51. $(1 - \sqrt{3})^2$
52. $(x + 2\sqrt{x}) (3x + \sqrt{x})$
53. $(y - \sqrt{z})^2$
54. $(n - 4\sqrt{3n}) (2n - \sqrt{3n})$
55. $(3 + 2\sqrt{5}) (3 - 2\sqrt{5})$
56. $(\sqrt{6} + 2\sqrt{7}) (\sqrt{6} - 2\sqrt{7})$
57. $(w - 8\sqrt{w}) (w + 8\sqrt{w})$
58. $(9\sqrt{a} - 4a) (9\sqrt{a} + 4a)$
59. $(3z + z\sqrt{10}) (3z - z\sqrt{10})$
60. In your own words, describe the similarities in the problems 55 through 59 above. In your own words, describe the pattern and what is common about all of the answers.
Section 9.4  Division of Radical Expressions

Examples:
Simplify. Assume all variables are nonnegative. You may need to rationalize the denominator in order to give a simplified answer.

a) \( \frac{\sqrt{240x^{12}y^{7}}}{\sqrt{140x^{3}y^{11}}} \)

To simplify the division of two square root expressions, the easiest first step is to write the radicands under one square root sign and then reduce or simplify the fraction.

\[
\frac{\sqrt{240x^{12}y^{7}}}{\sqrt{140x^{3}y^{11}}} \rightarrow \frac{\sqrt{12x^{9}}}{\sqrt{7y^{4}}}
\]

Next, take the square root of the numerator and the square root of the denominator.

\[
\frac{\sqrt{4 \cdot 3 \cdot x^{5}}}{\sqrt{7 \cdot y^{4}}} \rightarrow \frac{2x^{4}\sqrt{3x}}{y^{2}\sqrt{7}}
\]

If the above two steps result in an expression that does NOT have a radical in the denominator, the problem is simplified. If there is still a radical in the denominator, rationalize the denominator.

For this problem, there is still a radical in the denominator, so rationalize the denominator by multiplying the fraction by \( \frac{\sqrt{7}}{\sqrt{7}} \)

\[
\frac{2x^{4}\sqrt{3x}}{y^{2}\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} \rightarrow \frac{2x^{4}\sqrt{21x}}{7y^{2}}
\]

Rationalize the denominator.

b) \( \frac{2\sqrt{3}}{1-\sqrt{5}} \)

To rationalize the denominator of an expression that contains two terms in the denominator, multiply the numerator and denominator by the conjugate of the denominator.

\[
\frac{2\sqrt{3}}{1-\sqrt{5}} \cdot \frac{(1+\sqrt{5})}{(1+\sqrt{5})} \rightarrow \frac{2\sqrt{3}(1+\sqrt{5})}{1-5} \rightarrow \frac{2\sqrt{3}(1+\sqrt{5})}{-4}
\]

Notice how the denominator is multiplied (FOIL-ed) out, but the numerator is left in factored form.

In many cases, this will be the last step, but for this problem, the fraction can be simplified as it contains a factor of 3 in the numerator and denominator.

\[
\frac{2\sqrt{3}(1+\sqrt{5})}{-4} \rightarrow \frac{-\sqrt{3}(1+\sqrt{5})}{2} \quad \text{Your answer can also be written as } -\frac{\sqrt{3}+\sqrt{15}}{2}.
\]

Homework

1. When simplifying a square root whose radicand is a quotient, what is usually the “easiest” first step?
2. When looking at an expression that consists of a quotient of square roots (without an addition or subtraction), should you take the square root first or simplify the fraction first?
3. What are the three conditions that must be met in order for a square root expression to be considered “simplified”?

4. In your own words, describe, in general what it means to rationalize a denominator.

5. What are the two different types of expressions you will see in this class where you might need to rationalize the denominator? In your own words, describe how to rationalize the denominator for each type.

Simplify. Assume all variables are nonnegative.

6. \( \frac{\sqrt{25}}{9} \)

7. \( \frac{\sqrt{75}}{3} \)

8. \( \frac{\sqrt{16}}{4} \)

9. \( -\frac{\sqrt{18}}{2} \)

10. \( \frac{\sqrt{10}}{5} \)

11. \( \frac{\sqrt{24}}{3} \)

12. \( \frac{\sqrt{100}}{2} \)

13. \( \frac{\sqrt{18}}{72} \)

14. \( \frac{\sqrt{45}}{\sqrt{5}} \)

15. \( \frac{\sqrt{105}}{\sqrt{21}} \)

16. \( \frac{\sqrt{98}}{\sqrt{18}} \)

17. \( \frac{\sqrt{110}}{\sqrt{90}} \)

18. \( \frac{\sqrt{56}}{\sqrt{7}} \)

19. \( \frac{\sqrt{x^7}}{\sqrt{x^5}} \)

20. \( \frac{\sqrt{40m^8}}{125m^6} \)

21. \( \frac{\sqrt{28a^6c}}{63a^6c} \)
22. $\sqrt[12]{\frac{2y^4}{162y^{12}}}$
23. $\frac{\sqrt{78wz^{10}}}{\sqrt{24w^2z}}$
24. $\frac{\sqrt{75p^{11}q^{21}}}{\sqrt{27p^{13}q^5}}$
25. $\frac{\sqrt{80m^{25}n^9}}{\sqrt{16m^3n^4}}$
26. $\frac{\sqrt{3x^4y^6z}}{\sqrt{192x^6y^{12}z}}$
27. $\sqrt[12]{\frac{1815c^{16}d^{25}}{15c^9d}}$

Simplify. Assume all variables are nonnegative. You may need to rationalize the denominator in order to give a simplified answer.

28. $\frac{8}{\sqrt{3}}$
29. $\frac{\sqrt{7}}{\sqrt{2}}$
30. $\frac{\sqrt{6}}{\sqrt{10}}$
31. $\frac{\sqrt{12}}{\sqrt{15}}$
32. $\frac{\sqrt{14}}{\sqrt{7}}$
33. $\frac{\sqrt{7}}{\sqrt{14}}$
34. $\frac{\sqrt{5}}{\sqrt{y}}$
35. $\frac{\sqrt{4a^2}}{\sqrt{20a^3}}$
36. $\frac{\sqrt{8}}{\sqrt{2p}}$
37. $\frac{\sqrt{x^8}}{\sqrt{x^{11}}}$
38. $\frac{\sqrt{m^{13}}}{\sqrt{18m^{10}}}$
\[
\begin{align*}
39. & \quad \frac{\sqrt{52z^4}}{26z^2} \\
40. & \quad \frac{\sqrt{180c^{15}d^{10}}}{\sqrt{640c^2d^{16}}} \\
41. & \quad \frac{\sqrt{168x^{13}y^8}}{\sqrt{84x^{14}y^{10}}} \\
42. & \quad \frac{\sqrt{810pq^4}}{\sqrt{2430p^2q^6}} \\
43. & \quad \frac{\sqrt{33m^{17}n^3}}{\sqrt{88mn^6}} \\
44. & \quad \frac{\sqrt{63a^{20}c}}{\sqrt{35a^{11}c}} \\
45. & \quad \frac{\sqrt{120xy^3}}{\sqrt{180x^4y^8}} \\
\end{align*}
\]

Rationalize the denominator.

\[
\begin{align*}
46. & \quad \frac{5}{2+\sqrt{3}} \\
47. & \quad \frac{1}{3-\sqrt{7}} \\
48. & \quad \frac{\sqrt{2}}{1-\sqrt{6}} \\
49. & \quad \frac{8}{3+\sqrt{5}} \\
50. & \quad \frac{4\sqrt{10}}{3+\sqrt{2}} \\
51. & \quad \frac{6\sqrt{5}}{\sqrt{8}-\sqrt{5}} \\
52. & \quad \frac{x}{8+\sqrt{x}} \\
53. & \quad \frac{\sqrt{11}}{\sqrt{11}-\sqrt{y}} \\
54. & \quad \frac{2\sqrt{a}}{\sqrt{a+c}} \\
55. & \quad \frac{xy}{x-\sqrt{y}} \\
\end{align*}
\]
Section 9.5  Simplifying Radical Expressions — Part II

9.5 — Simplifying Radical Expressions — Part II Worksheet

Example:
Simplify.

a) \( \frac{-6 + \sqrt{36 - 24}}{4} \)

To simplify an expression like the one above, follow order of operation. The first step is to simplify the radicand.

\( \frac{-6 + \sqrt{12}}{4} \)

Next, simplify the square root if possible. Finally, if the numerator and denominator contain a common FACTOR, cancel it.

\( \frac{-6 + 2\sqrt{3}}{4} \rightarrow \frac{2(-3 + \sqrt{3})}{4} \rightarrow \frac{-3 + \sqrt{3}}{2} \)

Homework

1. In your own words, describe the order of operations in mathematics. What “level” is taking a square root?

2. In your own words, describe the steps for simplifying an expression of the type \( \frac{b + \sqrt{c-d}}{a} \).

Simplify.

3. \( \frac{-3 + \sqrt{9} - 4}{2} \)

4. \( \frac{5 - \sqrt{25} - 16}{8} \)

5. \( \frac{6 + \sqrt{36} + 4}{2} \)

6. \( \frac{-10 - \sqrt{100} - 16}{4} \)

7. \( \frac{-7 - \sqrt{49} + 1}{-2} \)

8. \( \frac{-8 + \sqrt{64} - 20}{10} \)

9. \( \frac{4 - \sqrt{16} + 24}{4} \)

10. \( \frac{-1 - \sqrt{1 + 120}}{12} \)
Section 9.6  Radical Equations

9.6 — Review of Radical Expressions and Equations Worksheet

Example:
Solve. If there is no solution, state so.

a)  $4 + \sqrt{x + 3} = x + 5$

To solve a radical equation first ISOLATE the radical expression.

$4 + \sqrt{x + 3} = x + 5$

Subtract 4 from both sides of the equation.

$\sqrt{x + 3} = x + 1$

To “undo” the square rooting, square both sides of the equation.

$(\sqrt{x + 3})^2 = (x + 1)^2$

Don’t forget to FOIL when necessary.

Continue to solve the remaining equation.

$x + 3 = (x + 1)(x + 1)$

$x + 3 = x^2 + 2x + 1$

Subtract $x$ and 3 from both sides of the equation.

$0 = x^2 + x - 2$

Since this is a quadratic equation, solve by factoring.

$0 = (x + 2)(x - 1)$

Since this is a quadratic equation, solve by factoring.

$x + 2 = 0 \quad \text{or} \quad x - 1 = 0$

$x = -2 \quad \text{or} \quad x = 1$

Check all solutions!

Check for $x = -2$:

$4 + \sqrt{(-2) + 3} \neq (-2) + 5$

$4 + \sqrt{3} \neq 3$

$4 + 1 \neq 3$

$5 \neq 3$

Check for $x = 1$:

$4 + \sqrt{1 + 3} \neq 1 + 5$

$4 + \sqrt{4} \neq 1 + 5$

$4 + 2 \neq 6$

$6 = 6$

Since $-2$ doesn’t work, the solution is only $x = 1$.

Homework

1. Multiply each of the following. If you know how to spot the difference between these problems and compute them correctly, you’ll have an easier time solving the equations in this section.

a)  $(2x)^2$

b)  $(2\sqrt{x})^2$

c)  $(x + 2)^2$

d)  $(\sqrt{x} + 2)^2$

2. In your own words, describe how to recognize a square root equation. What is the inverse of square rooting?
3. In your own words, describe how to solve a square root equation. Is it necessary to check your answers when solving a square root equation? In your own words, explain why or why not.

Solve. Check your answers. If there is no solution, state so.

4. \( \sqrt{x} = 8 \)
5. \( \sqrt{y} = 4 \)
6. \( 3 + \sqrt{a} = 12 \)
7. \( 22 = 12 + \sqrt{m} \)
8. \( \sqrt{2w - 1} = 3 \)
9. \( 1 = 4\sqrt{p} - 11 \)
10. \( 7 = 13 + \sqrt{3x} \)
11. \( \sqrt{n + 3} = 5 \)
12. \( 10 = \sqrt{q} - 6 \)
13. \( 17 + \sqrt{k - 5} = 23 \)
14. \( 13 - \sqrt{8w - 3} = 12 \)
15. \( 33 = 35 - \sqrt{d} + 7 \)
16. \( 45 + \sqrt{2y - 6} = 49 \)
17. \( \sqrt{3x - 8} = \sqrt{x} \)
18. \( \sqrt{2k - 5} - \sqrt{k - 1} = 0 \)
19. \( \sqrt{z - 1} = \sqrt{z + 5} \)
20. \( 2\sqrt{3a - 5} = \sqrt{2a} \)
21. \( \sqrt{7p + 38} = 3\sqrt{p + 4} \)
22. \( 4\sqrt{n + 5} = \sqrt{n + 35} \)
23. \( \sqrt{q - 3} = 3\sqrt{q} \)
24. \( \sqrt{x} = x - 12 \)
25. \( \sqrt{k + 8} = k + 6 \)
26. \( w = \sqrt{w + 20} \)
27. \( -7 + \sqrt{y + 13} = y \)
28. \( 1 + \sqrt{z + 1} = z \)
29. \( 2\sqrt{a} = a - 3 \)
30. \( 2\sqrt{7c + 8} = 3c \)
31. \( 2m = \sqrt{30m - 14} \)
32. \( 2 - \sqrt{4p + 5} = p \)
33. \( 6 - \sqrt{2x + 3} = x \)
Chapter 10 — Quadratic Equations

Section 10.1 — Factoring
Section 10.2 — The Square Root Property
Section 10.3 — Completing the Square
Section 10.4 — The Quadratic Formula
Section 10.5 — Application Problems Involving Quadratic Equations

Answers
Section 10.1  Factoring

Example:
Solve the equation. Give simplified answers.

a) \( x(2x + 3) = 2 \)

To solve a quadratic equation by factoring, first multiply/distribute and combine like-terms, in order to get the equation into standard form: \( ax^2 + bx + c = 0 \).

\[ x(2x + 3) = 2 \quad \rightarrow \quad 2x^2 + 3x = 2 \quad \rightarrow \quad 2x^2 + 3x - 2 = 0 \]

Once the equation is equal to zero and all like-terms are combined, factor.

\[ 2x^2 + 3x - 2 = 0 \quad \rightarrow \quad (2x - 1)(x + 2) = 0 \]

Use the Zero Product Rule to find the values for \( x \) that satisfy the equation, by setting each factor equal to zero and solving.

\[ (2x - 1)(x + 2) = 0 \quad \rightarrow \quad 2x - 1 = 0 \quad \text{AND} \quad x + 2 = 0 \]

\[ 2x = 1 \quad \quad \quad \quad \quad \quad \quad \quad x = -2 \]

\[ x = \frac{1}{2} \]

The solutions are \( \frac{1}{2} \) and \(-2\).

Homework

1. What are the types of factoring methods covered in Chapter 7?
2. In your own words, describe how solve a quadratic equation by factoring.

Solve each equation by factoring. Show a check for every third problem. Give simplified answers.

3. \( x^2 + 5x = 6 \)
4. \( 2y^2 = 8y \)
5. \( 3a^2 = 14a + 5 \)
6. \( w(w - 5) = 6(w + 2) \)
7. \( p^2 + 13p - 16 = 13p \)
8. \( (n+1)(n+4) = 28 \)
9. \( (z+1)(z-3) = 5z + 15 \)
10. \( 3x(x + 1) = x^2 - 1 \)
11. \( (x + 4)^2 = -2x \)
Section 10.2  The Square Root Property

Example:
Solve the equation using the **Square Root Property**. Give simplified answers. If the solution is not a real number, state this.

a) \((x-5)^2 - 8 = 4\)

To solve a quadratic equation using the Square Root Property, first get the equation into the form:
\[(\text{variable expression})^2 = \text{constant}\]

\((x-5)^2 - 8 = 4 \implies (x-5)^2 = 12\)

Now, take the square root of both sides of the equations; **don’t forget about the plus/minus sign**!

\[\sqrt{(x-5)^2} = \sqrt{12} \implies x-5 = \pm \sqrt{12}\]

Simplify the square root of the constant if possible.

\[x-5 = \pm \sqrt{4 \cdot 3} \implies x-5 = \pm 2\sqrt{3}\]

Isolate the variable.

\[x = 5 \pm 2\sqrt{3}\]

*If the terms are “like” then combine them. If not (as in this problem), you may leave the expression as it is.*

Homework

1. In your own words, describe the form of a quadratic equation that makes the Square Root Property the easiest method for solving that equation.

2. In your own words, describe how to use the Square Root Property to solve a quadratic equation.

Solve each equation using the **Square Root Property**. Show a check for every third problem. Give simplified answers. If the solution is not a real number, state this.

3. \(x^2 = 49\)

4. \(a^2 + 20 = 45\)

5. \(y^2 - 10 = 8\)

6. \(4z^2 = 9\)

7. \((n+5)^2 = 36\)

8. \((m-7)^2 = -2\)

9. \((c-1)^2 = 100\)

10. \((q+2)^2 = 63\)

11. \((w-3)^2 = 20\)

12. \((p-4)^2 - 8 = 1\)

13. \((x-6)^2 - 1 = 11\)
14. \((a+1)^2 + 13 = 7\)
15. \((y+10)^2 + 12 = 23\)
16. \((2x+1)^2 + 10 = 19\)
17. \((3m+2)^2 - 10 = 6\)
18. \((4w+7)^2 + 3 = 32\)
19. \((2d+5)^2 - 14 = 6\)

Determine whether to use factoring or the Square Root Property to solve each equation and then solve it using that method. Give exact, simplified answers. If the equation has no real solution, state this.
20. \((x+2)^2 = 9\)
21. \((y+2)^2 = 9\)

Rewrite each expression as a binomial squared. In other words, factor each expression. There is a pattern for the following problems; use it to help you factor problems 24 and 25.
22. \(a^2 + 8a + 16\)
23. \(n^2 - 10n + 25\)
24. \(c^2 + 5c + \frac{25}{4}\)
25. \(z^2 - z + \frac{1}{4}\)

Solve each equation. Note that some of the equations are quadratic equations, and some are square root equations. Give exact, simplified answers.
26. \(\sqrt{x+2} = 4\)
27. \((y+2)^2 = 4\)
28. \((2d+1)^2 - 6 = 5\)
29. \(9 - \sqrt{3x-2} = 1\)
30. \(\sqrt{w+36} = \sqrt{4w}\)
Section 10.3 Completing the Square

Example: Solve the equation by completing the square. Give exact, simplified answers. If the solution is not a real number, state this.

a) \( 2x^2 - 18 = 12x \)

To solve a quadratic equation by completing the square, first get the equation into the form:

\[ ax^2 + bx = c \]

\[ 2x^2 - 12x - 18 = 0 \quad \rightarrow \quad 2x^2 - 12x = 18 \]

Next, if the leading coefficient is not 1, divide the equation by its value.

\[ \frac{2x^2}{2} - \frac{12x}{2} = \frac{18}{2} \quad \rightarrow \quad x^2 - 6x = 9 \]

Now, find the value that completes the square, \( (\frac{b}{2})^2 \), for the variable side and add it to both sides of the equation.

\[ For \ this \ problem, \ b = -6, \ so \ the \ value \ to \ add \ to \ both \ sides \ is \ (\frac{-6}{2})^2 = (-3)^2 = 9. \]

\[ x^2 - 6x + 9 = 9 + 9 \quad \rightarrow \quad x^2 - 6x + 9 = 18 \]

Rewrite the trinomial into a perfect square binomial; in other words, factor it.

\[ (x-3)(x-3) = 18 \quad \rightarrow \quad (x-3)^2 = 18 \]

Now, take the square root of both sides of the equations; don’t forget about the plus/minus sign!

\[ \sqrt{(x-3)^2} = \sqrt{18} \quad \rightarrow \quad x - 3 = \pm \sqrt{18} \]

Simplify the square root of the constant if possible.

\[ x - 3 = \pm \sqrt{9 \cdot 2} \quad \rightarrow \quad x - 3 = \pm 3\sqrt{2} \]

Isolate the variable.

\[ x = 3 \pm 3\sqrt{2} \]

If the terms are “like” then combine them. If not (as in this problem), you may leave the expression as it is.

Homework

1. In your own words, describe how to use completing the square to solve a quadratic equation.

2. If given a choice about how to solve a quadratic equation, in your own words, describe how to determine which method you have learned so far (factoring, Square Root Property, or completing the square) works or is “easiest” for a particular equation.

Solve each equation by completing the square. Show a check for every third problem. Give exact, simplified answers. If the solution is not a real number, state this.

3. \( x^2 + 10x - 24 = 0 \)

4. \( p^2 - 8p + 15 = 0 \)

5. \( y^2 + 6y - 3 = 0 \)

6. \( d^2 + 12d + 30 = 0 \)
7. \( m^2 + 4m + 10 = 0 \)
8. \( w^2 - 2w = 9 \)
9. \( a^2 + 6a - 16 = 2a \)
10. \( 2z^2 - 28z - 22 = 0 \)
11. \( 3n^2 + 18 = 18n \)
12. \( c^2 + 5c + 4 = 0 \)
13. \( x^2 + x + 6 = 0 \)
14. \( q^2 + 7q = 1 \)
15. \( 2y^2 - 8 = 6y \)

Determine whether to use factoring, the Square Root Property, or completing the square to solve each equation and then solve it using that method. Give exact, simplified answers. If the equation has no real solution, state this.

16. \( a^2 + 11a - 26 = 0 \)
17. \( z^2 + 12z - 26 = 0 \)
18. \( 4m^2 = 24 \)
19. \( (w + 2)^2 = 81 \)
20. \( (d + 3)^2 = 20d \)
Section 10.4  The Quadratic Formula

Example:
Solve the equation by using the Quadratic Formula. Give exact, simplified answers. If the solution is not a real number, state this.

a) \((3x-1)(x+2)=10\)

To solve a quadratic equation, first get the equation into the form:
\[ax^2 + bx + c = 0\]

For this problem, the left side needs to be “FOIL”-ed.
\[3x^2 + 5x - 2 = 10 \implies 3x^2 + 5x - 12 = 0\]

Next, determine the values for \(a\), \(b\), and \(c\).

For this problem, \(a = 3\), \(b = 5\), and \(c = -12\).

Now, plug in those values for the corresponding variables in the Quadratic Formula:
\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
\[x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 3 \cdot (-12)}}{2 \cdot 3}\]

Simplify the expression, starting with the expression under the radical sign.
\[x = \frac{-5 \pm \sqrt{25 - 4 \cdot 3 \cdot (-12)}}{2 \cdot 3} \implies x = \frac{-5 \pm \sqrt{25 - 12 \cdot (-12)}}{2 \cdot 3} \implies x = \frac{-5 \pm \sqrt{25 + 144}}{2 \cdot 3} \implies x = \frac{-5 \pm \sqrt{169}}{2 \cdot 3} \implies x = \frac{-5 \pm 13}{6}\]

For this problem, the numerator can be combined as there are like-terms, so find the simplified forms of each solution. Since the solutions are rational, this problem could have been solved by factoring.

\[x = \frac{-5 + 13}{6} = \frac{8}{6} = \frac{4}{3}\] and \[x = \frac{-5 - 13}{6} = \frac{-18}{6} = -3\]

Homework
1. Write the Quadratic Formula.
2. How is the Quadratic Formula derived?
3. In your own words, describe how to use the Quadratic Formula to solve a quadratic equation.
4. If given a choice about how to solve a quadratic equation, in your own words describe how to choose which method—factoring, Square Root Property, completing the square, or the Quadratic Formula—for a particular equation and why.

Solve each equation by using the Quadratic Formula. Show a check for every third problem. Give exact, simplified answers. If the solution is not a real number, state this.

a) \(2x^2 + 7x + 3 = 0\)

b) \(a^2 - 9a + 18 = 0\)

c) \(z^2 - 5z - 2 = 0\)

d) \(2n^2 + 3n = 7\)
9. \( 4y^2 = 11 \)
10. \( 2c^2 + 13c = 0 \)
11. \( 3m^2 - 10m - 2 = 0 \)
12. \( p^2 - 3p = -8 \)
13. \( (x+3)(x-1) = 8 \)
14. \( (d-4)^2 = 5d \)
15. \( (z-5)^2 = 3z(z+2) \)
16. \( 0.1a^2 - 0.4a - 1.2 = 0 \)
17. \( \frac{1}{2} y^2 - 6y = \frac{3}{4} \)
18. \( 3q(q-4) = (q+5)(q-1) \)

**Solve each equation by using the Quadratic Formula.** Round your answer(s) to the nearest hundredth.

19. \( 3w^2 + 11w - 2 = 0 \)
20. \( 16n^2 + 20n - 7 = 0 \)
21. \( 1.98p^2 - 1.3p - 2 = 0 \)
Section 10.5  Application Problems Involving Quadratic Equations

10.5 — Application Problems Involving Quadratic Equations Worksheet

Example:
For the problem below, define variables, write an equation, solve the equation, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate.

a) A rocket is shot from the ground with an initial velocity of 44 feet per second. The distance \( d \) above the ground after \( t \) seconds is given by:

\[
d = 44t - 16t^2
\]

i. How many seconds after the start is the rocket at a height of 24 feet? Round your answer to 2 decimal places.

For this problem, \( d = 24 \), and you are asked to find the time \( t \). Using the given equation and plugging in the value given yields:

\[
d = 44t - 16t^2 \quad \Rightarrow \quad 24 = 44t - 16t^2
\]

Now solve for \( t \).

\[
0 = 44t - 16t^2 - 24 \quad \Rightarrow \quad 0 = -16t^2 + 44t - 24
\]

Divide every term by \(-4\) to make the coefficients easier to work with.

\[
\frac{t}{4} = \frac{-26 + 11\sqrt{3}}{4} \quad \Rightarrow \quad 0 = 4t^2 - 11t + 6
\]

This equation can be solved by factoring. If that isn’t the case, use the quadratic formula.

\[
0 = (4t - 3)(t - 2) \quad \Rightarrow \quad 4t - 3 = 0 \quad \text{or} \quad t - 2 = 0
\]

\[
4t = 3 \quad \text{or} \quad t = 2
\]

\[
t = \frac{3}{4}
\]

There are two positive time values for a height of 24 feet.

\( \frac{3}{4} \) or \( 0.75 \) of a second after the start, the rocket’s height is 24 feet; this happens when the rocket is on the way up.

\( 2 \) seconds after the start, the rocket’s height is 24 feet; this happens when the rocket is on the way down.

ii. When will the rocket hit the ground? Round your answer to 2 decimal places.

For this problem, “hitting the ground” corresponds to \( d = 0 \), and you are asked to find the time \( t \). Using the given equation and plugging in the value given yields:

\[
d = 44t - 16t^2 \quad \Rightarrow \quad 0 = 44t - 16t^2
\]

Now solve for \( t \).

\[
0 = -16t^2 + 44t
\]

Divide every term by \(-4\) to make the coefficients easier to work with.

\[
\frac{t}{4} = \frac{-26 + 11\sqrt{3}}{4} \quad \Rightarrow \quad 0 = 4t^2 - 11t
\]

This equation can be solved by factoring. If that isn’t the case, use the quadratic formula.

\[
0 = t(4t - 11) \quad \Rightarrow \quad t = 0 \quad \text{or} \quad 4t - 11 = 0
\]

\[
4t = 11
\]

\[
t = \frac{11}{4}
\]

\( t = 0 \) corresponds to the start of the rocket’s flight, so this is not the answer.

\( \frac{11}{4} \) or \( 2.75 \) seconds after the start, the rocket hits the ground.
Homework

For each problem below, define variables, write an equation, solve the equation, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate.

1. Determine the value of $x$. Round your answer to 2 decimal places.

2. Determine the value of $x$.

3. Determine the value of $x$. Round your answer to 2 decimal places.

4. The height of a certain box is 2 cm less than the width, and the length is 8 cm. Find the width and height of the box if the volume is 2,400 cm$^3$. Round your answers to 2 decimal places.

5. If you wish to anchor a 20-foot telephone pole with a wire cable that is at a distance of 4 feet from the base of the pole, how long will the wire cable be? Round your answers to 2 decimal places.

6. If you wish to lean a 10-foot ladder against a house so that the distance from the base of the ladder to the house is one-fourth of the distance the ladder reaches on the house, how far does the ladder reach on the house? Round your answers to 2 decimal places.

7. A rocket is shot from the ground with an initial velocity of 48 feet per second. The distance $d$ above the ground after $t$ seconds is given by:
   \[ d = 1 + 48t - 16t^2 \]
   a) What is the initial height of the rocket?
   b) What is the initial height of the rocket?
   c) What is the distance above the ground after 0.5 seconds?
   d) How many seconds after the start is the rocket at a height of 20 feet? Round your answers to 2 decimal places.
   e) When will the rocket hit the ground? Round your answers to 2 decimal places.

8. The height $h$ above the ground of a golf ball depends on the time $t$ it has been in flight. One golfer hits a tee shot that has a height approximately given by:
   \[ h = 80t - 17t^2 \]
   where $h$ is the height in feet and $t$ is the time in seconds.
   a) What is the initial height of the golf ball?
   b) What is the height after 1 second?
   c) How many seconds have elapsed when the golf ball is 50 feet in the air? Round your answers to 2 decimal places.
   d) How many seconds have elapsed when the golf ball finally hits the ground? Round your answers to 2 decimal places.
**Real Numbers—1.1**

**Set:** A list of elements listed within braces. For example, \( \{ 2, a, 6, -5, \frac{1}{2}, 674 \} \)

**Elements:** The “things” inside of a set.

**Empty set (null set):** A set that contains no elements. For example, \( \{ \} \).

**Natural numbers:** Natural numbers are sometimes called the “counting numbers”. \( \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots \} \).

**Whole numbers:** All of the natural numbers as well as 0. \( \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots \} \).

**Integers:** All of the natural numbers as well as their opposites and 0. \( \{ \ldots -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots \} \)

**Positive integers:** The set of positive integers is the same as the set of natural or “counting” numbers. Notice that 0 is not included in this set. \( \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots \} \).

**Negative integers:** The set of negative integers is the opposites of the positive integers. Notice that 0 is not included in this set. \( \{ \ldots -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1 \} \).

**The real number line:** A “line” that illustrates the relationship (value comparison) between all real numbers. This line is often marked by the integers, but doesn’t have to be.

**Rational numbers:** Any number that can be expressed as a quotient of integers where with the denominator of this quotient is not 0. Note: The number 0 IS a rational number, since it can be written as \( \frac{0}{1} \). Rational numbers are “nice”; in other words they include integers, fractions (of integers), and decimals that stop or have a pattern. There is no way to list all rational numbers, so here are a few examples:
\[
\left\{ 0, -1, 2, \frac{3}{4}, -13, 9, 15, 3.541, -6.3, -\frac{5}{3}, \frac{1}{14}, 10, \sqrt{4} \right\}
\]

**Irrational numbers:** Any number that can be represented on the real number line that is not rational. Irrational numbers are “not nice”; in other words they calculate out to be decimals that never end and do not have a pattern. There is no way to list all irrational numbers, so here are a few examples:
\[
\{ 7.325632884971, \pi, \sqrt{2}, \sqrt{\pi} \}
\]

**Real numbers:** All of the numbers that can be represented on a real number line. Real numbers include natural numbers, whole numbers, integers, rational numbers, and irrational numbers. All of the numbers that we will work with in this class are real numbers.

Determine whether each statement is true or false.

1. -1 is a negative integer.
2. \( \sqrt{3} \) is a real number.
3. \( \frac{3}{5} \) is an integer.
4. Every negative integer is a real number.
5. Every real number is an integer.
Given the set of numbers: \( \left\{ 1, -\sqrt{2}, 0, -2, -\frac{1}{3}, 17, 0.15, -3\frac{1}{7} \right\} \) list those that belong to the set of

a) natural numbers

b) whole numbers

c) integers

d) rational numbers

e) irrational numbers

f) real numbers
Order of Operations—1.4

There are four “levels of importance” for mathematical operations. The “level of importance” of an operation determines what is done first, second, third, fourth, etc., when computing a problem with more than one operation. Follow the following order when computing any problem with more than one operation.

1. **Grouping Symbols**
   Grouping symbols include parentheses ( ), brackets [ ], braces { }, and absolute-value symbols | |. “Fraction bars” are also considered grouping symbols. When any of these symbols appear in an expression, you must do the operations you can inside these symbols first. Always work from the inside out.

2. **Exponents**
   If there are any exponents in the expression, simplify this part of the expression next.

3. **Division or Multiplication**
   Division and multiplication are computed in order from left to right.

   *Sometimes* division is done before multiplying, such as in this problem:
   
   \[ 20 \div 2 \times 5 \]
   
   \[ 10 \times 5 \]
   
   \[ 50 \]

   because division appears first when reading the problem from the left side to the right side.

   *Other times* multiplication is done before division, such as in this problem:
   
   \[ 20 \times 2 \div 5 \]
   
   \[ 40 \div 5 \]
   
   \[ 8 \]

   because multiplication appears first when reading the problem from the left side to the right side.

4. **Subtraction or Addition**
   Subtraction and addition are computed in order from left to right. These two operations are always done last, unless they are inside of a set of grouping symbols.

**Example**

\[ 6 \div 3 + 4 \left[ 9 + 2 (3 - 1)^2 \right] \]

\[ 6 \div 3 + 4 \left[ 9 + 2 (2)^2 \right] \]

\[ 6 \div 3 + 4 \left[ 9 + 2 (4) \right] \]

\[ 6 \div 3 + 4 \left[ 9 + 8 \right] \]

\[ 6 \div 3 + 4 \left[ 17 \right] \]

\[ 2 + 4 \left[ 17 \right] \]

\[ 2 + 68 \]

\[ 70 \]
Phrases into Algebraic Expressions—1.6

Algebraic expression (or expression): any collection of numbers, letters (called variables), grouping symbols and operations. There is not an equal sign.

Algebraic equation (or equation): a statement that shows two algebraic expressions are equal. There is an equal sign.

Constant: a known value that does not change.

Variable: an unknown value that could change, usually represented by a letter.

Key Words:

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
<th>Equality</th>
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<td>+ Sum</td>
<td>- Difference</td>
<td>* Product</td>
<td>+ Quotient</td>
<td>= Equals</td>
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<td>Plus</td>
<td>Minus</td>
<td>Times</td>
<td>Divide</td>
<td>Is equal to</td>
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<td>Is/was</td>
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<td>Less than</td>
<td>Of</td>
<td>Divided by</td>
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<td>Decreased by</td>
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<td>(Perimeter)</td>
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**Candy Distribution Exercise**

In the Occupy movement which started in the fall of 2011, there has been a lot of reference to the 99%. To what are the “Occupiers” referring?

Occupy Wall Street protesters have been railing against the Top 1%, trying to raise anger and awareness of the growing economic gap between the rich and everybody else in America.

But just who are these fortunate folks at the top of the income ladder?

Well, there were just under 1.4 million households that qualified for entry. They earned nearly 17% of the nation's income and paid roughly 37% of its income tax.

Collectively, their adjusted gross income was $1.3 trillion. And while $343,927 was the minimum AGI to be included, on average, Top 1-percenters made $960,000.

—Tami Luhby, money.cnn.com, October 29, 2011

1. If the $x$ is the US population, write an expression that represents the amount of people who earned an average of $960,000.

2. If the $x$ is the US population, write an expression that represents the amount of people who earned less than $343,927.

3. If the $x$ is the amount (in US dollars) of total US income earned in 2010, write an expression that represents the amount earned by the Top 1% of income earners in the US.

4. If the $y$ is the amount (in US dollars) of US income tax received in 2010, write an expression that represents the amount paid by the Top 1% of income earners in the US.

5. Now, apply each expression above to this class. Let the students be the population of the US and use the treats as money.
Definitions from the World Economic Forum’s website (weforum.org) March 15, 2012:

The World Economic Forum is an independent international organization committed to improving the state of the world by engaging business, political, academic and other leaders of society to shape global, regional and industry agendas.

The typical Member Company is a global enterprise with more than US$ 5 billion in turnover, although this varies by industry and region.

In addition, these enterprises rank among the top companies within their industry and/or country…and play a leading role in shaping the future of their industry and/or region.

An excerpt from The Elite's Pure Greed by Derrick Z. Jackson, published February 8, 2002 in the Boston Globe:

The 3,000 participants at the World Economic Forum, which drifted through the hallways of the Waldorf, dropped $100 million on New York hotels, ballrooms, and restaurants, according to the New York City tourism board.

That comes out to $33,333.33 per person. In five days in New York, each participant of the World Economic Forum spent on average what the average person makes in America in a year. four times what the average person makes in Mexico in a year, 14 times what the average person makes in India in a year, 22 times what the average person makes in Bangladesh, and 74 times what the average person makes in a year in Sierra Leone, according to United Nations figures.

6. If the \( x \) is the average amount (in US dollars) made by the average person in the US in a year, write an expression that represents the amount spent by a participant at the World Economic Forum in 5 days.

7. If the \( x \) is the average amount (in US dollars) made by the average person in Mexico in a year, write an expression that represents the amount spent by a participant at the World Economic Forum in 5 days.

8. If the \( x \) is the average amount (in US dollars) made by the average person in Sierra Leone in a year, write an expression that represents the amount spent by a participant at the World Economic Forum in 5 days.

9. According to a report published last December by the Organization for Economic Cooperation and Development, the association of free market democracies, the share of after-tax household income that went to the top 1 percent of earners in the United States…doubled in less than three decades to…percent in 2007 from \([x]\) percent in 1979.

—Eric Pfanner, nytimes.com, January 24, 2012

If the \( x \) is the share of after-tax household income that went to the top 1 percent of US earners in 1979, write an expression that is the share of after-tax household income that went to the top 1 percent of US earners in 2007.

For each problem below, define a variable and then use that variable to write an algebraic expression for the requested quantity.

10. According to Gapminder, the average income in Shanghai, for example, is about 10 times the level in Guizhou, a less developed Chinese province.

—Eric Pfanner, nytimes.com, January 24, 2012

What is the average income in Shanghai?

11. The difference in the infant mortality rate is even greater, with such deaths occurring…one-twelfth as often in Shanghai [than in Guizhou].

—Eric Pfanner, nytimes.com, January 24, 2012

What is the approximate infant mortality rate in Shanghai?

12. Santa Cruz County’s per capita taxable sales were 18% less than California in 2009.

—Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

What were Santa Cruz County’s per capita taxable sales in 2009?
13. The hourly self-sufficiency wage for a single adult in Santa Cruz County increased 33% between 2003 and 2011.
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

   What was the hourly self-sufficiency wage for a single adult in Santa Cruz County in 2011?

14. In 2011, the median family income in Santa Cruz County was...$21,600 more than the U.S. median in 2011.
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

   What was the median family income in Santa Cruz County in 2011?

15. Due to an increased cost of living, less income, unemployment and the recession, most survey respondents (69%) felt they were worse off financially this year than last year. When asked their top reason for why they did not feel economically better off, Latinos said it was due to “less income” and Caucasians said it was due to the cost of living. [The number of Latino respondents was 85 less than half of the number of Caucasian respondents.]
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

   What was the number of Latino respondents?

16. More than half of Caucasian [Santa Cruz County] CAP respondents reported saving money for the future through...retirement compared to...21% [of Latinos] who saved through retirement, a statistically significant difference between Latinos and Caucasians. [The number of Caucasian respondents who reported saving through retirement was 23 more than six times the number of Latino respondents who reported saving through retirement.]
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011

   What was the number of Caucasian respondents who reported saving through retirement?

17. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the weekly earnings (in US$) of a person with a Bachelor’s degree were $150 more than twice the weekly earnings of a person with less than a high school diploma.
   What were the weekly earnings of a person with a Bachelor’s degree? (Write an expression. Do not give a dollar amount.)

18. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the unemployment rate (%) of a person with less than a high school diploma was 0.9 more than twice a person with an Associate’s degree.
   What is the unemployment rate of a person less than a high school diploma? (Write an expression. Do not give a percentage.)

---

**Education pays:**

<table>
<thead>
<tr>
<th>Unemployment rate in 2010 (%)</th>
<th>Doctoral degree</th>
<th>Professional degree</th>
<th>Master’s degree</th>
<th>Bachelor’s degree</th>
<th>Associate degree</th>
<th>Some college, no degree</th>
<th>High school diploma</th>
<th>Less than a high school diploma</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9</td>
<td>1,550</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>2.4</td>
<td>1,610</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.0</td>
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<td>9.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average: 8.2%

Median weekly earnings in 2010 ($)

<table>
<thead>
<tr>
<th></th>
<th>$782</th>
</tr>
</thead>
<tbody>
<tr>
<td>High school diploma</td>
<td>626</td>
</tr>
<tr>
<td>Less than a high school diploma</td>
<td>444</td>
</tr>
<tr>
<td>Some college, no degree</td>
<td>712</td>
</tr>
<tr>
<td>Associate degree</td>
<td>767</td>
</tr>
<tr>
<td>Bachelor’s degree</td>
<td>1,038</td>
</tr>
<tr>
<td>Master’s degree</td>
<td>1,272</td>
</tr>
<tr>
<td>Professional degree</td>
<td>1,610</td>
</tr>
<tr>
<td>Doctoral degree</td>
<td>1,550</td>
</tr>
</tbody>
</table>

19. Express each phrase as an algebraic expression:
   a) A 6% sales tax on $c$ dollars
   b) The cost, $c$, increased by a 6% sales tax
   c) The cost, $c$, reduced by 25%

20. Two numbers are consecutive integers. What are the numbers?

21. Two numbers are even consecutive integers. What are the numbers?

22. Two numbers are odd consecutive integers. What are the numbers?

23. An 80-foot tree is cut into two pieces. These pieces are not necessarily the same size. How long are the pieces?

24. Dante bought a new guitar. It was 15% off the regular price; he also had to pay a 9% sales tax. What was the total cost of the guitar? Tax is applied after the discount.

25. The length of a new bookshelf is to be 3 feet more than twice the width. What is the length of the bookshelf?

26. In an isosceles triangle, two of the angles measure the same. The measurement of one of those “same” angles is 15° less than three times the measurement of the “solo” angle. What is the measurement of one of those “same” angles?
Linear Equations in One-Variable—2.3

Solving an equation means finding the value of the variable that makes the equation true, in other words, that makes the left side equal the right side. Solutions—answers—look like this:

\[ x = \text{"a constant"} \quad \text{or} \quad \text{"a constant"} = x. \]

Steps for Solving Linear Equations

1. If there are parentheses in the equation, use the distributive property to remove them.

2. If there are fractions in the equation, multiply each term by the \( \frac{\text{L C D}}{1} \). Simplify each product by canceling the denominator, and multiplying the numerators.

3. Combine like-terms—if there are any—on the left side of the equal sign. Combine like-terms—if there are any—on the right side of the equal sign.

4. Decide which side of the equation is the “variable term side” and which side of the equation is the “constant side”.

5. Use the Addition Property of Equality—add or subtract—to get all variable terms on the “variable term side”. Combine like-terms where possible.
   — If the variable terms “disappear” after doing this, determine whether the statement left is true or false. If it is true, the solution is ALL REAL NUMBERS. If it is false, there is NO SOLUTION.

6. Use the Addition Property of Equality—add or subtract—to get all constants on the “constant side”. Combine like-terms where possible.

7. Use the Multiplication Property of Equality to “get rid of” the coefficient of the variable, if it isn’t 1. In most cases, this means divide both sides of the equation by the coefficient—the constant in front—of the variable.

8. Check the solution—answer—by plugging it into the original equation. If the left side is equal to the right side after simplifying, then the solution is correct.

Example

<table>
<thead>
<tr>
<th>Constant side</th>
<th>Variable side</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2x + 2 )</td>
<td>( 5x - 7 )</td>
<td>( 2(x + 3) - 4 = 5x - 7 )</td>
</tr>
<tr>
<td>( 2x + 6 - 4 )</td>
<td>( 5x - 7 )</td>
<td>( 2(3 + 3) - 4 = 5(3) - 7 )</td>
</tr>
<tr>
<td>( 2x + 2 )</td>
<td>( 5x - 7 )</td>
<td>( 2(6) - 4 = 5(3) - 7 )</td>
</tr>
<tr>
<td>( 0 + 2 )</td>
<td>( 3x - 7 )</td>
<td>( 12 - 4 = 5(3) - 7 )</td>
</tr>
<tr>
<td>( 2 )</td>
<td>( 3x - 7 )</td>
<td>( 8 = 5(3) - 7 )</td>
</tr>
<tr>
<td>( + 7 )</td>
<td>( + 7 )</td>
<td>( 8 = 15 - 7 )</td>
</tr>
<tr>
<td>( 9 )</td>
<td>( 3x )</td>
<td>( 8 = 8 )</td>
</tr>
<tr>
<td>( \frac{9}{3} )</td>
<td>( \frac{3}{3} )</td>
<td>( \frac{3}{3} = x )</td>
</tr>
</tbody>
</table>
Equations and Expressions Worksheet—2.4

1. **Solve.**
   
   \[6x - (3x + 8) = 16\]

2. **Simplify.**
   
   \[6x - (3x + 8) + 16\]

3. **Solve.**
   
   \[4(x - 7) - (x + 1) = 15 - x\]

4. **Solve.**
   
   \[x - 5 = 5 - x\]

5. **Simplify.**
   
   \[4(x - 7) - (x + 1) + 15 - x\]
6. **Simplify.**

\[3y + 2(y - 1) - 4(y + 2) - (y + 5)\]

7. **Solve.**

\[3y + 2(y - 1) = 4(y + 2) - (y + 5)\]
Ratios and Proportions—3.1

Express as a ratio.

1. Math majors don’t always get much respect on college campuses, but fat post-grad wallets should be enough to give them a boost. The top 15 highest-earning college degrees all have one thing in common: math skills. Only three of the 15 top paying degrees were outside the field of engineering, but they each still require math skills. It’s a supply and demand issue, [Ed Koc, director of research at National Association of Colleges and Employers] added. So few grads offer math skills, and those who can are rewarded.

Source: Most Lucrative College Degrees, Julianne Pepitone, CNNMoney.com, July 24, 2009

Solve.

2. **2 African-American students for every 4 Asian/Pacific Islander/Filipino students.** If there are 6 Asian/Pacific Islander/Filipino students in a class, **how many African-American students** would you expect there to be?

Source: Cabrillo College Factbook 2011

3. **approximately 47 male students for every 53 female students.** *How many students are in our class? How many does the ratio lead us to believe should be male? How many does the ratio lead us to believe should be female? Is this ratio accurate for this class? Round your answers to the nearest person.*

Source: Cabrillo College Factbook 2011

4. In 2010, at Cabrillo College, **29 students out of 100 students were Latino.** If there are 40 students in a class, **how many Latino students** would you expect there to be? Round your answer to the nearest person.

Source: Cabrillo College Factbook 2011

What percentage of the student population at Cabrillo College is Latino? How does this compare to the fact that according to the Santa Cruz Chamber of Commerce, Santa Cruz County has a population that is 34.8% Latino?

5. In 2010, at Cabrillo College, **for every 100 faculty members, approximately 12 were Latino.** If there are 20 fulltime mathematics faculty members at Cabrillo College, **how many would you expect to be Latino? Round your answer to the nearest person.**

*How does your answer compare to the fact that three of the 20 fulltime math instructors are Latino? How does the ratio of Latino students to total students compare to Latino faculty to total faculty?*

Source: Cabrillo College Factbook 2011

6. **much greater percentage of the community college student population was Latino than in the University of California (UC) or California State University (CSU) systems.** Yet, Latinos were still underrepresented in community college, compared with their share of the state population.

Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

In 2003, **for every 100 California residents, approximately 35 are Latino.** If there are approximately 2.5 million community college students, **how many would you expect to be Latino? Round your answer to the nearest person.**

The actual 2003 ratio of Latino community college students to the total number of community college students was 29:100.

7. In 2003, **for every 100 California residents, approximately 35 are Latino.** In a group of **50 CSU students,** **how many would you expect to be Latino? Round your answer to the nearest person.**

The actual 2003 ratio of Latino CSU students to the total number of CSU students was close to 6:25 (or 12:50).

Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

8. In 2003, **for every 100 California residents, approximately 6 are African-American.** In a group of **50 CSU students,** **how many would you expect to be African-American? Round your answer to the nearest person.**

The actual 2003 ratio of African-American CSU students to the total number of CSU students was 6:100.

Source: California’s Community College Students, Ria Sengupta and Christopher Jepsen, Public Policy Institute of California, November 2006

9. Convert **22,704 feet to miles.** *(5280 feet = 1 mile)*
10. Bobby’s house is assessed at $625,000. If the property tax in Santa Cruz County is $8.235 per $1000 of assessed value, how much property tax would he owe?

11. If a 40-pound bag of fertilizer covers 5000 square feet, how many pounds of fertilizer will Jeanne need if she plans to cover an area of 26,000 square feet?

12. A recipe for pie dough calls for 4 tablespoons of olive oil for every 1½ cups of flour. If you use 5 cups of flour, how much olive oil would you need to follow the recipe? Write your answer as a mixed number.

13. The SAC East building at Cabrillo College is approximately 40 feet tall. In a photo, it is 2 inches tall. How tall in real life is a tree in front of the building that is \( \frac{3}{8} \) inch in the photo? Write your answer as mixed number.

14. **World Wealth/Poverty and Population Distribution Exercise**

   strate the distribution of wealth and population throughout the world.

   Percentages are given in the chart for reference. You have a choice of whether or not to use them for this exercise; however, please note that any percentage can be changed into a ratio. For example: 19% is 19:100; 73% is 73:100; 35% is 35:100 (or 7:20); 120% is 120:100 (or 6:5).

   How many total students are in our class?

   How many “treats” total are there?

   corresponding number of treats for your group.

<table>
<thead>
<tr>
<th>Area</th>
<th>Population (in millions)</th>
<th>% of World Population</th>
<th>Number of Students</th>
<th>GDP (PPP) (in billions of dollars)</th>
<th>% of World GDP</th>
<th>“Treats”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Africa</td>
<td>1,036.4</td>
<td>14.7%</td>
<td>3,005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asia</td>
<td>4,234.7</td>
<td>4.9%</td>
<td>30,871</td>
<td></td>
<td>40.3%</td>
<td></td>
</tr>
<tr>
<td>Canada, Greenland, &amp; U.S.</td>
<td>347.3</td>
<td>4.9%</td>
<td>15,992</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caribbean, Central America, &amp; Mexico</td>
<td>195.8</td>
<td>2.8%</td>
<td>2182</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>674.6</td>
<td>9.6%</td>
<td>17,397</td>
<td>22.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Middle East</td>
<td>134.1</td>
<td>1.9%</td>
<td>1,859</td>
<td>2.4%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oceania</td>
<td>34.6</td>
<td>0.5%</td>
<td>1,028</td>
<td>1.3%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South America</td>
<td>399.8</td>
<td>5.7%</td>
<td>4,336</td>
<td>5.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7,057.6</td>
<td>100%</td>
<td>76,670</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[The] gross domestic product (GDP) or value of all final goods and services produced within a nation in a given year. A nation’s GDP at purchasing power parity (PPP) exchange rates is the sum value of all goods and services produced in the country valued at prices prevailing in the United States. This is the measure most economists prefer when looking at per-capita welfare and when comparing living conditions or use of resources across countries. The measure is difficult to compute, as a US dollar value has to be assigned to all goods and services in the country regardless of whether these goods and services have a direct equivalent in the United States (for example, the value of an ox-cart or non-US military equipment); as a result, PPP estimates for some countries are based on a small and sometimes different set of goods and services. In addition, many countries do not formally participate in the World Bank’s PPP project that calculates these measures, so the resulting GDP estimates for these countries may lack precision. For many developing countries, PPP-based GDP measures are multiples of the official exchange rate (OER) measure. The difference between the OER- and PPP-denominated GDP values for most of the wealthy industrialized countries are generally much smaller.

Sources: CIA World Factbook (indexmundi.org)
Formulas—3.2

Use the formula to find the value of the variable indicated. When necessary, round your answer to the nearest hundredth.

1. \( P = 2l + 2w \), find \( P \) when \( l = 2 \) and \( w = 5 \).

2. \( A = \frac{1}{2}bh \), find \( A \) when \( b = 8 \) and \( h = 3 \).

3. \( y = mx + b \), find \( x \) when \( y = 11 \), \( m = 2 \) and \( b = 4 \).

4. \( S = R - rR \), find \( R \) when \( S = 92 \) and \( r = 0.08 \).

Simple Interest Formula

\[
i = \text{interest} = \text{principle} \times \text{rate} \times \text{time}
\]

5. Sean invested $2000 in an account earning 3% simple interest. How much interest will he earn after 3 years?

6. Bonnie lent her brother $4000 for a period of 2 years. At the end of the 2 years, her brother repaid the $4000 plus $640 interest. What simple interest rate did her brother pay?

One-Dimensional Measurements (in, ft, yd, m, cm, km, miles, etc.)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Sketch</th>
<th>Perimeter/Circumference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td><img src="image" alt="Square Sketch" /></td>
<td></td>
</tr>
<tr>
<td>Rectangle</td>
<td><img src="image" alt="Rectangle Sketch" /></td>
<td>( P = l + l + w + w ) or ( P = 2l + 2w )</td>
</tr>
<tr>
<td>Parallelogram</td>
<td><img src="image" alt="Parallelogram Sketch" /></td>
<td></td>
</tr>
<tr>
<td>Trapezoid</td>
<td><img src="image" alt="Trapezoid Sketch" /></td>
<td>( P = a + b + c + d )</td>
</tr>
<tr>
<td>Triangle</td>
<td><img src="image" alt="Triangle Sketch" /></td>
<td>( P = a + b + c )</td>
</tr>
<tr>
<td>Circle</td>
<td><img src="image" alt="Circle Sketch" /></td>
<td>( C = 2\pi r )</td>
</tr>
</tbody>
</table>
**Sheltered from the Storm Exercise**

7. The following is a diagram of a floor plan, with the average floor space of an American home in 1975. The entire number of square meters has been encapsulated in this single floor. **Each square represents four square meters.**

![Diagram of a floor plan](image)

a) *How many square meters* was the average American home in 1975?

b) By the year 2000, the average American home was 38% larger. Use a black marker and *increase the floor design by 38%.*

c) The average American home in the year 2000 was 26 times the living space of the average person in Africa. *Use a red marker and indicate the living space of the average African* within the American floor layout for the year 2000.


8. Petra has a rectangular lot that measures **100 feet by 60 feet.** If Petra wants to fence in her lot, *how much fencing* will she need?

---

**Two-Dimensional Measurements** *(in², ft², yd², m², cm², km², square miles, etc.)*

<table>
<thead>
<tr>
<th>Figure</th>
<th>Sketch</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td><img src="image" alt="Square" /></td>
<td>( A = lw )</td>
</tr>
<tr>
<td>Rectangle</td>
<td><img src="image" alt="Rectangle" /></td>
<td>( A = lh )</td>
</tr>
<tr>
<td>Parallelogram</td>
<td><img src="image" alt="Parallelogram" /></td>
<td>( A = \frac{1}{2} h (b + d) )</td>
</tr>
<tr>
<td>Trapezoid</td>
<td><img src="image" alt="Trapezoid" /></td>
<td>( A = \frac{1}{2} bh )</td>
</tr>
<tr>
<td>Triangle</td>
<td><img src="image" alt="Triangle" /></td>
<td>( A = \pi r^2 )</td>
</tr>
</tbody>
</table>

9. The screen of the Texas Instruments rectangular display screen (or window) is **2.5 inches by 1.5 inches.** Find the area of the window.
10. The top of a *round* living room table has a **radius of 3 feet**. Find the **area** of the tabletop.

11. Marge made a sign to display at a baseball game. The sign was in the shape of a *trapezoid*. Its **bases are 4 feet and 3 feet**, and its **height is 2 feet**. Find the **area** of the sign.

**Three-Dimensional Measurements** 
(*in.*, *ft.*, *yd.*, *m.*, *cm.*, *km.*, cubic miles, etc.)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Sketch</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular solid (box)</td>
<td><img src="rectangular-solid.png" alt="" /></td>
<td>[ V = l \times w \times h ]</td>
</tr>
<tr>
<td>Right circular cylinder</td>
<td><img src="cylinder.png" alt="" /></td>
<td>[ V = \pi r^2h ]</td>
</tr>
<tr>
<td>Right circular cone</td>
<td><img src="cone.png" alt="" /></td>
<td>[ V = \frac{1}{3} \pi r^2h ]</td>
</tr>
<tr>
<td>Sphere (ball)</td>
<td><img src="sphere.png" alt="" /></td>
<td>[ V = \frac{4}{3} \pi r^3 ]</td>
</tr>
</tbody>
</table>

12. Alfonso has an empty oil drum that he uses for storage. The oil drum is **4 feet high** and has a **diameter of 24 inches**. Find the **volume** of the drum in cubic feet.
Laundry Soap Marketing Exercise

13. Suppose you own a laundry soap company. There are ways to increase your market share without advertising. The original scoop design for your laundry detergent is shaped in a rectangular solid with a base that is 5 cm by 5 cm and a height that is 7 cm. Assume that the market research is correct and that the majority of consumers use 85% of the full scoop. Calculate the volume of laundry detergent used with each scoop.

14. A new scoop design is proposed. The designer says it will increase the market share because it looks about the same size but holds more product. To prevent consumers from thinking that the new scoop is a different volume (and by extension, that they should modify how much detergent they pick up with the scoop) the designer says to use a cylindrical shape with a diameter of 5 cm and a height that’s the same as the first scoop. Assume that consumers still fill this scoop 85%. Calculate the volume of the detergent used with each scoop. Use π = 3.14 to approximate your answer to three decimal places.

15. Each full box of detergent is 2,500 cm$^3$. Calculate how many scoops it would take to finish the full box for each of the two designs. Is the new design more or less profitable than the original design?

16. Calculate the percentage that the smaller scoop is compared to the bigger scoop. If one box of detergent is $5.99, of which $1.78 is profit, use the percentage that you calculated for the scoop size to figure out the gain or loss of profit per box using the new design.

17. A third design is proposed. The designer argues that the diameter of the cylindrical scoop should actually be equal to the distance from one corner of the square base (on the original design) to the opposite corner. The height remains 7 cm. What is the volume of the new scoop?

18. If consumers don’t notice the difference in the size of this new design, and still fill the scoop 85% full each time, how many cubic centimeters will the new scoop use?

19. Calculate how many scoops it would take to finish the full box of the new design. Is the new design more or less profitable than the original design and the second design? Explain.

Geometry Application Problems—3.4

1. Cesar is planning to build a rectangular dining room table. Cesar wants the perimeter of the table to be 240 inches. He also wants the length of the table to be 44 inches longer than the width. Find the dimensions of the table that Cesar wants to build.

Define the following:

Perimeter:

Area:

Isosceles Triangle:

Complementary Angles:

Supplementary Angles:

What is the sum of the measures of all three angles of a triangle?

What is the sum of the measures of the angles of a quadrilateral?

2. One angle of a triangle is 20º larger than the smallest angle, and the third angle is 6 times as large as the smallest angle. Find the measure of the three angles.

3. Maricela has a corner lot that is an isosceles triangle. Two angles of her triangular lot are the same and the third angle is 30º greater than the measure of one of the others. Find the measure of the three angles.
4. A horse’s water trough has ends that are **trapezoids**. The measures of the two top angles are the same. **Each bottom angle measures 15º less than twice the measure of one of the top angles.** Find the measure of each angle.

![Diagram of a trapezoid]

5. Star has a goat farm. She plans to separate the goats by fencing in three equal areas, shown below. The **length of the fenced-in area is to be 30 feet greater than the width** and the **total amount of fencing available is 660 feet**. Find the length and width of the fenced-in area.

![Diagram of three equal areas]

6. Angles A and B are **complementary** angles, and **angle B is 14º less than angle A**. Find the measures of angle A and B.

7. A bookcase is to have four shelves as shown. The **height of the bookcase is to be 2 feet more than the width**, and only **20 feet of lumber is available**. What should be the width and height of the bookcase?

![Diagram of a bookcase with four shelves]
General Applications Problems of Linear Equations—3.5

For each problem below, define a variable, write an equation, solve the equation, and answer the question. Give your answer in simplest form with the correct units when appropriate.

1. According to Gapminder, the average income in Shanghai, for example, is about 10 times the level in Guizhou, a less developed Chinese province.
   —Eric Pfanner, nytimes.com, January 24, 2012
   In 2007, if the average income (Purchasing Power Parity US$) in Shanghai is $25,500 (source: International Monetary Fund), what is the average income (PPP US$) in Guizhou?

2. Santa Cruz County’s per capita taxable sales were 18% less than California in 2009.
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   If Santa Cruz County’s per capita taxable sales in 2009 were $9,755, what were California’s per capita taxable sales in 2009?

3. The hourly self-sufficiency wage for a single adult in Santa Cruz County increased 33% [between 2003 and 2011].
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   If the hourly self-sufficiency wage for a single adult in Santa Cruz County in 2011 was $15.28, what was the hourly self-sufficiency wage for a single adult in Santa Cruz County in 2003? Round your answer to the nearest cent.

4. [In 2011, the] median family income in Santa Cruz County was...$21,600 more than the U.S. median in 2011.
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   In 2011, if the sum of the median family income in Santa Cruz County and the median family income in the U.S. is $150,000, what was the median family income in the U.S. and what was the median family income in Santa Cruz County?

5. Due to an increased cost of living, less income, unemployment and the recession, most survey respondents (69%) felt they were worse off financially this year than last year. When asked their top reason for why they did not feel economically better off, Latinos said it was due to “less income” and Caucasians said it was due to the cost of living. [The number of Latino respondents was 85 less than half of the number of Caucasian respondents. Fifty-two respondents did not identify as either Caucasian or Latino.]
   —Santa Cruz County Community Assessment Project, Year 17, appliedsurveyresearch.org, 2011
   In 2011, if the total number of respondents was 702, how many identified as Caucasian and how many identified as Latino?
   According to the Santa Cruz Chamber of Commerce, Santa Cruz County has a population that is 56.6% white and 34.8% Latino, does the survey accurately reflect our population?

6. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the weekly earnings (in US$) of a person with a Bachelor’s degree were $150 more than twice the weekly earnings of a person with less than a high school diploma.
   If the difference of the weekly earnings of a person with a Bachelor’s degree and the weekly earnings of a person with less than a high school diploma was $549, what were the weekly earnings of a person with less than a high school diploma and what were the earnings of a person with a Bachelor’s degree?

7. According to the Bureau of Labor Statistics’ 2010 Current Population Survey, the unemployment rate (%) of a person with less than a high school diploma was 0.9 more than twice a person with an Associate’s degree.
   If the sum of the unemployment rate of a person with less than a high school diploma and the unemployment rate of a person with an Associate’s degree was 21.9, what is the unemployment rate of a person with an Associate’s degree and what is the unemployment rate of a person with less than a high school diploma?

8. Two numbers are consecutive integers. If the sum of the numbers is 171, what are the numbers?

9. Two numbers are odd consecutive integers. If the sum of the numbers is 140, what are the numbers?

10. An 80-foot tree is cut into two pieces. If one piece is 5 feet less than twice the other, how long are the pieces?

11. Dante bought a new guitar. It was 15% off the regular price; he also had to pay a 9% sales tax. If he paid a total of $625, what was the pre-sale cost of the guitar without tax? Tax is applied after the discount. Round your answer to the nearest cent.
Distance/Rate/Time Application Problems—3.6

Use the distance/rate/time formula to answer each question.

1. In India, 11 kilometers of roads used to be constructed every year. Today, **10 kilometers are being built every day** after the creation of the National Highways Authority.  
   —weforum.org, June 12, 2011  
   At this rate, **how many days did it take to build the highway from Mumbai to Pune (a distance of 93 kilometers)?**

2. **If you travel in a car at 40 mph for half an hour, how far have you traveled?**

3. **It took Judy 20 minutes to bike 1.5 miles to Cabrillo. How fast did she bike?**

For each problem below, define a variable, write an equation, solve the equation, and answer the question. You may use a chart to organize the information in the problem. Give your answer in simplest form with the correct units when appropriate.

4. Two highway paving crews are **20 miles** apart working toward each other. One crew paves **0.4 mile** of road per day more than the other crew, and the two crews meet after **10 days. Find the rate at which each crew paves the road.**

   **Find:**
   - the rate of crew #1:
   - the rate of crew #2:

   **Given:**
   - time each crew works:
   - total distance:

   Unused fact/picture:

   **Equation that comes from the unused fact/picture:**
   the total distance is 20 miles, or
   “crew #1’s distance” + “crew #2’s distance” = 20 miles

   **Solve the equation:**

   **Answer the question:**
5. A family travels in two canoes to a campground. The kids (in one canoe) paddle 4 miles per hour, and the parents (in another canoe) paddle at 2 miles per hour. In how many hours will the kids and the parents be 5 miles apart?

Find:

time until kids and parents are 5 miles apart:

Given:

kids’ speed:
parents’ speed:
total distance:

<table>
<thead>
<tr>
<th>Directions</th>
<th>Rate</th>
<th>Time</th>
<th>Distance = Rate · Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kids</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parents</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Unused fact/picture:

Equation that comes from the unused fact/picture:

the difference between the kids’ distance and the parents’ distance = 5 miles, or

“the kids’ distance” – “the parents’ distance” = 5 miles

Solve the equation:

Answer the question:

6. Alice and Bonnie start running at the same time from the same point and run in the same direction. Alice runs at 6 mph while Bonnie runs at a slower pace. After 2 hours, they are 4 miles apart. Determine the speed at which Bonnie is running.

7. A group of marathon runners began running west on the Katy Trail at 4.6 mph. After a certain distance, they turned around and ran east on the same path at 4 mph back to where they started. If the runners took 0.25 hour longer running east than running west, determine the time it took them to run east. Give your answer accurate to 2 decimal places.

8. Two stunt planes plan to fly towards each other, with one passing just below the other. The orange plane’s rate is 15 mph less than the rate of the purple plane. If they start out 2.5 miles apart, and it takes 2 minutes for them to meet (pass), what is the purple plane’s rate and what is the orange plane’s rate?
1. Plot the following ordered pairs. Label each point with its corresponding letter. Since you are not graphing an equation, do not connect the points.

   A \((-4, 6)\)
   B \((0, 8)\)
   C \((2, 1)\)
   D \((-3, 0)\)
   E \((-1, -7)\)
Graphing Linear Equations in Two-Variables—4.2

Determine which of the following ordered pairs satisfy each equation. Plot all points that satisfy the equation on the same axes and draw a straight line through the points. (This line shows a picture (graph) of all of the solutions to the equation.)

1. \( y - x = 3 \)
   a) \((-2, 5)\)
   b) \((-1, 2)\)
   c) \((0, -4)\)
   d) \((1, 4)\)

2. \( y = \frac{1}{2}x + 1 \)
   a) \((-2, 1)\)
   b) \((0, -2)\)
   c) \((-2, 0)\)
   d) \((-5, \frac{3}{2})\)
Graph each equation by plotting at least three points.

3. \( y = \frac{1}{4} x + 1 \)

4. \( y = -\frac{1}{2} x \)

5. \( x - 3y = 6 \)
6. \( x = -2y \)

**Horizontal Lines**

Six points are graphed below. State the coordinate pair (ordered pair) of each point.

Note: In the all of the points above, the y-values are 2 and the x-values are different. If you drew a line through all of these points, all of the points on this horizontal line would have a y-value of 2 (but the x-value could be any real number); therefore the relationship between the x-values and y-values of this line is that y must be 2. This can be described by the equation \( y = 2 \). In other words, the equation for the horizontal line with a y-intercept of \((0, 2)\) is \( y = 2 \). Notice that there is no x variable in the equation.

All horizontal lines are represented by the equation \( y = \) "a constant", where the constant is the y-value of the y-intercept.

*Graph each equation by plotting at least three points.*

7. \( y = -4 \)

8. \( 2y = 5 \)
**Vertical Lines**

Seven points are graphed below. State the coordinate pair (ordered pair) of each point.

Note: In the all of the points above, the \(x\)-values are 1 and the \(y\)-values are different. If you drew a line through all of these points, all of the points on this \(vertical\) line would have an \(x\)-value of 1 (but the \(y\)-value could be any real number); therefore the relationship between the \(x\)-values and \(y\)-values of this line is that \(x\) must be 1. This can be described by the equation \(x = 1\). In other words, the equation for the \(vertical\) line with an \(x\)-intercept of \((1, 0)\) is \(x = 1\). Notice that there is no \(y\)-variable in the equation.

All \(vertical\) lines are represent by the equation, \(x = "a\ \text{c o n s t a n t}"\), where the constant is the \(x\)-value of the \(x\)-intercept.

---

**Graph each equation by plotting at least three points.**

9. \(x = -\frac{3}{2}\)  
10. \(3x = -6\)
Intercepts and Slope — 4.3

Intercepts

Below is a graph of \( 2y = -x + 4 \)

State the \( x \)-intercept:

State the \( y \)-intercept:

The \( x \)-intercept of an equation is the point where the graph crosses the \( x \)-axis. The \( x \)-intercept’s \( y \)-value is always 0. In other words, the \( x \)-intercept is always a point of the form \(( ?, 0)\). To find the \( x \)-intercept from an equation, plug in 0 for \( y \) and then find the \( x \)-value that goes with \( y = 0 \). Remember, for the \( x \)-intercept, you are looking for an \( x \)-value.

The \( y \)-intercept of an equation is the point where the graph crosses the \( y \)-axis. The \( y \)-intercept’s \( x \)-value is always 0. In other words, the \( y \)-intercept is always a point of the form \( (0, ?) \). To find the \( y \)-intercept from an equation, plug in 0 for \( x \) and then find the \( y \)-value that goes with \( x = 0 \). Remember, for the \( y \)-intercept, you are looking for a \( y \)-value.

Find the \( x \)-intercept, the \( y \)-intercept, and a different third “check” point, then graph the line.
1. \( 4x - 2y = -12 \)

Find the \( x \)-intercept:

Find the \( y \)-intercept:

Check point:
**Slope**

All linear equations in two variables have a constant rate of change or slope. The slope of a line is a ratio of the vertical change to the horizontal change between two selected points on a line. In other words, slope is \( \frac{\text{vertical change}}{\text{horizontal change}} \) or \( \frac{\text{rise}}{\text{run}} \).

\[
\begin{align*}
\text{rise} & \quad \text{or} \quad \frac{\text{the change in the } y\text{-values}}{\text{the change in the } x\text{-values}} \\
\text{run} & \quad \text{or} \quad \frac{\text{the change up or down}}{\text{the change left or right}}.
\end{align*}
\]

2. **Find the slope of the line graphed.**

   By counting the units of rise—vertical (y) change—and the units of run—horizontal (x) change—between any two points:

3. **Find the slope of the line graphed.**

   By counting the units of rise—vertical (y) change—and the units of run—horizontal (x) change—between any two points:
The slope of a line can also be found by using the slope formula. If two points on the line are labeled the first ordered pair: \((x_1, y_1)\) and the second ordered pair: \((x_2, y_2)\), then the slope of the line can be found by the following formula:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

It does not matter which ordered pair is labeled the first ordered pair and which is labeled the second ordered pair.

4. **Find the slope of the line graphed.**

By counting the units of rise—vertical (y) change—and the units of run—horizontal (x) change—between any two points:

By using two points on the line and the slope formula:

5. **Find the slope of the line graphed.**

By counting the units of rise—vertical (y) change—and the units of run—horizontal (x) change—between any two points:

By using two points on the line and the slope formula:

**Graphing a linear equation means drawing a line that shows all of the points that satisfy the equation.** There is more than one way to draw this line.

1. One way is to **find any two points** that work and then another check point.
2. Another way is to specifically **find the x-intercept and the y-intercept** and then another check point.
3. The third way is to **plot one point and then use the slope** to find two other points.
For problems 6 through 8, graph the line with the given slope that goes through the given point.

6. Through \((0, -2)\) with \(m = \frac{2}{3}\)

7. Through \((-3, 5)\) with \(m = -2\)

8. Through \((-3, 0)\) with \(m = -\frac{2}{5}\)
**Slopes of Horizontal and Vertical Lines**

*Find the slope of the line through the given points.*

9. \((0, -2)\) and \((3, -2)\)  

A graph of this line:

**Slope:**

**Y-intercept:**

**Equation of the line:**

![Graph of line through points (0, -2) and (3, -2)](image)

The slope of any **horizontal line** is 0. So, if the equation is \(y = \text{“a constant”}\), then the slope, \(m\), is 0.

*Find the slope of the line through the given points.*

10. \((3, -1)\) and \((3, 2)\)  

A graph of this line:

**Slope:**

**X-intercept:**

**Equation of the line:**

![Graph of line through points (3, -1) and (3, 2)](image)

The slope of any **vertical line** is undefined. So, if the equation is \(x = \text{“a constant”}\), then the slope, \(m\), is undefined.
11. The graphs of two lines are shown below. Find the slope of each line.

Parallel Lines
Any non-vertical lines that have the same slope, but different y-intercepts are parallel. Any two distinct vertical lines are parallel to each other. For example, line 1 that has a slope $m_1 = \frac{3}{5}$ and a y-intercept of $(0, 2)$ is parallel to line 2 that has a slope of $m_2 = \frac{3}{5}$ and a y-intercept of $(0, -6)$. Slope indicates how a line “leans”, and the y-intercept indicates “where” a line is located.

12. The graphs of two lines are shown below. Find the slope of each line.

Perpendicular Lines
Two lines whose slopes are opposite reciprocals are perpendicular to each other. For example, line 1 that has a slope $m_1 = -\frac{1}{3}$ is perpendicular to line 2 that has a slope of $m_2 = 3$. The y-intercept of these lines is irrelevant to the fact that they are perpendicular. In the case of a vertical line and a horizontal line: any vertical line is perpendicular to any horizontal line.
For problems 13 through 16, the slopes of two distinct lines are listed. Use the slopes to determine whether each pair of lines is parallel, perpendicular, or neither parallel or perpendicular.

13. Line 1: \( m_1 = 3 \), Line 2: \( m_2 = 3 \)

14. Line 1: \( m_1 = 2 \), Line 2: \( m_2 = -2 \)

15. Line 1: \( m_1 = -\frac{1}{3} \), Line 2: \( m_2 = -3 \)

16. Line 1: \( m_1 = 5 \), Line 2: \( m_2 = -\frac{1}{5} \)

Use the pair of points given for each line to find the slope of that line and then determine if the pair of lines is parallel, perpendicular, or neither parallel or perpendicular.

17. Line 1: \((-\frac{1}{2}, 1)\) and \((\frac{3}{4}, 2)\) Line 2: \((7, 3)\) and \((4, -3)\)

18. Line 1: \((0, 6)\) and \((1, 4)\) Line 2: \((-4, 5)\) and \((0, 13)\)

### 4.1–4.3 Review Problems

19. Graph \(-3y = 6\)

20. Graph \(y = \frac{3}{5}x\)

21. Graph the line that goes through \((1, -2)\) and has a slope, \(m = -3\)

22. Find the x-intercept and the y-intercept. \(-5x + 20 = 2y\)

23. Find the slope of the line between two points given. \((-4, 6)\) and \((1, 5)\)
Equations of Lines—4.4

For each graph, solve each equation for y (if necessary), then write down the slope and y-intercept.

1. \( y = \frac{1}{2} x - 4 \)

2. \( y + 2x = 3 \)

Equation solved for y:

\[ m: \text{ y-intercept:} \]

\[ m: \text{ y-intercept:} \]

**Slope-Intercept Form**

If an equation of a line is solved for \( y \), and therefore in the form \( y = mx + b \), the slope of the line is the coefficient of \( x \) or \( m \), and the y-value of the y-intercept is \( b \). Not all equations of lines are written in this form; however you can solve any equation for \( y \) and get it in this form.

\( y = mx + b \) is called the slope-intercept form of an equation of a line because the slope and the y-value of the y-intercept is readily seen.

Determine the slope and the y-intercept of the line represented by the given equation.

3. \( y = 7x + 3 \)

4. \( 3x + y = \frac{1}{2} \)

For problems 5 through 8, determine the slope and y-intercept of the line represented by each equation. Graph the line using the slope and y-intercept.

5. \( y = -2x + 3 \)

6. \( -x + 4y = -8 \)

Equation solved for y:

\[ m: \text{ y-intercept:} \]

\[ m: \text{ y-intercept:} \]
7. \( y = -\frac{1}{3}x \)

Equation solved for \( y \):

<table>
<thead>
<tr>
<th>( m )</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. \( y = 6 - x \)

Equation solved for \( y \):

<table>
<thead>
<tr>
<th>( m )</th>
<th>( y )-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Parallel and Perpendicular Lines**

The slopes of two parallel lines are the same, and the slopes of two perpendicular lines are opposite reciprocals.

The slope-intercept form of an equation of a line, \( y = mx + b \), is very useful for determining the slope of the line, \( m \), and the \( y \)-intercept of the line, \((0, b)\). This information may be used to graph the line, or to figure out if two lines are parallel, perpendicular or neither.

Without graphing, determine whether each pair of lines is parallel, perpendicular or neither.

9. \( 2x + 3y = 8 \)

\( y = -\frac{2}{3}x + 5 \)

Equation 1 solved for \( y \):

\( m_1: \)

Equation 2 solved for \( y \):

\( m_2: \)

10. \( 3x + 7y = 8 \)

\( 7x + 3y = 8 \)

Equation 1 solved for \( y \):

\( m_1: \)

Equation 2 solved for \( y \):

\( m_2: \)
11. \[8x + 2y = 8\]
\[x - 9 = 4y\]

Equation 1 solved for \(y\):

\[m_1:\]

Equation 2 solved for \(y\):

\[m_2:\]

Point Slope Form

If an equation of a line is in the form \(y - y_1 = m(x - x_1)\), the slope of the line is \(m\), and \((x_1, y_1)\) is a point on the line. Any point (one at a time) can be used for \((x_1, y_1)\). Notice that there are minus signs “built into the formula”. Usually equations that are given are not written in this form; however, this form is very useful for writing an equation of a line.

For each equation, write down the slope and the point on the line that can be read directly from the equation.

12. \[5x - 6y = 10\]
\[-6x + 5y = 20\]

Equation 1 solved for \(y\):

\[m_1:\]

Equation 2 solved for \(y\):

\[m_2:\]

13. \[y - 5 = \frac{1}{2}(x - 3)\]

\[m:\]

\((x_1, y_1) = \]

14. \[y - 1 = 3\left(x - \frac{3}{4}\right)\]

\[m:\]

\((x_1, y_1) = \]

15. \[y + 1 = 6(x + 4)\]

\[m:\]

\((x_1, y_1) = \]
Writing Equations of Lines
Understanding the slope-intercept form of an equation of a line, \( y = mx + b \), and the point-slope form of an equation of a line \( y - y_1 = m(x - x_1) \), not only allows a line to be easily graphed from an equation, it also gives a way to write an equation of a "slanted" line given either the graph or other information.

If the \( y \)-intercept is given, use slope-intercept form:
1. Write down slope-intercept form: \( y = mx + b \)
2. Find the \( m \)-value, which is the slope of the line. Make sure to take the rise over the run, or the change in \( y \) over the change in \( x \)!
   — The slope may be stated in the problem.
   — You may have to find the slope graphically.
   — You may have to find it by using two points that are given in the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \).
   — You may have to use the fact that parallel lines have the same slope.
   — You may have to use the fact that perpendicular lines have slopes that are the negative reciprocal of each other.
3. Find the \( b \)-value, which is the \( y \)-value of the \( y \)-intercept \((0, b)\) of the line.
   — The \( y \)-intercept \((0, b)\) may be stated in the problem.
   — You may have to find it graphically; it is the point where the line crosses the \( y \)-axis.
   — You may have to use a point-slope equation first (see below) then solve that equation for \( y \).
4. Plug in the \( m \)-value and the \( b \)-value into \( y = mx + b \). Don’t forget the \( x \)!

If the \( y \)-intercept is NOT given, use point-slope form first:
1. Write down point-slope form: \( y - y_1 = m(x - x_1) \)
2. Find the \( m \)-value, which is the slope of the line.
   — The slope may be stated in the problem.
   — You may have to find the slope graphically.
   — You may have to find it by using two points that are given in the formula \( m = \frac{y_2 - y_1}{x_2 - x_1} \).
   — You may have to use the fact that parallel lines have the same slope.
   — You may have to use the fact that perpendicular lines have slopes that are the negative reciprocal of each other.
3. Use a point given in the problem as the point \((x_1, y_1)\).
4. Plug in the \( m \)-value and the point \((x_1, y_1)\) into \( y - y_1 = m(x - x_1) \). Watch out for negatives and subtraction signs!
Equations of Horizontal and Vertical Lines

If the line is horizontal, use \( y = \text{"the y-value of the y-intercept"} \).

If the line is vertical, use \( x = \text{"the x-value of the x-intercept"} \)

Determine the equation of each line.

16. 
\[ m: \quad y\text{-intercept:} \]
Equation:

17. 
\[ m: \quad y\text{-intercept:} \]
Equation:

18. 
\[ m: \quad y\text{-intercept:} \quad x\text{-intercept:} \]
Equation:

19. 
\[ m: \quad y\text{-intercept:} \quad x\text{-intercept:} \]
Equation:
Write the equation of the line, with the given properties, in slope-intercept form.

20. Slope = 2, through \((0, 11)\)

21. Slope = \(-\frac{1}{4}\) through \((8, 5)\)

22. Slope = \(\frac{1}{3}\) through \((-6, 0)\)

23. Through \((2, 7)\) and \((0, 5)\).

24. Through \((-3, 2)\) and \((1, -4)\).

25. Parallel to \(-2y = 6x + 8\) and through \((1, 3)\)

26. Perpendicular to \(y = -\frac{1}{2}x + 8\) and through \((3, -7)\)
Review of Equations of Lines—4.4

Write the equation of each line, with the given properties, in slope-intercept form.

1. Slope = 3 through \((0, -2)\).

2. Through \((0, 11)\) and \((-3, 5)\).

3. Parallel to \(x + y = 7\) and through \((0, 2)\).

4. Through \((1, -3)\) and \((2, -3)\).

5. Slope = 3 through \((-1, 8)\).

6. Through \((3, -2)\) and \((8, 13)\).

7. Parallel to \(y + 1 = \frac{1}{3}(x - 6)\) and through \((-2, 4)\).
Application Problems of Linear Equations in Two-Variables—4.5

1. The graph below shows the stopping distance on wet pavement for a midsize car. State the slope with the correct units.

![Graph showing stopping distance on wet pavement](image)

Correlated or Causal Exercise

2. A new study by U.S. researchers looked at the amount of time preschoolers spend in front of the television versus their ability to focus when they reach school age.

<table>
<thead>
<tr>
<th># of hours per day watching TV in preschool years</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase in chance of attention problem in school</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
</tr>
</tbody>
</table>

a) Define two variables and then write a linear equation that describes the pattern in the table above.

b) Of 1,350 young children, the researcher found that one-year olds watched an average of 2.2 hours of television per day. Use your equation to find their average increase in attention problems by school age.

c) By age three, the researcher found that children watched an average of 3.6 hours of television per day. Use your equation to find their average increase in attention problems by school age.

d) How many hours of television per day would a preschooler have to average in order to have an average of a 50% increase in the chance of having attention problems in school?

e) Speculate as to why watching television when you are very young could have significant impacts on your ability to focus in school in later years.

A Little Goes a Long Way Exercise
More than half a million people in Bhopal, India were exposed to lethal gasses from the Union Carbide pesticide factory in December of 1984. To this day, 30 people a month continue to die as a result of the incident, and survivors have been plagued with chronic illnesses, aborted pregnancies, and a host of genetic defects.

3. A sample way taken of 865 pregnant women living one km from the Bhopal plant. Of the 486 live births, 68 babies did not survive their first month. How does this rate compare with the 2.6 to 3% rate of infant mortality before the accident?

4. A study is released on the toxicity levels at different distances from a chemical spill.

<table>
<thead>
<tr>
<th>Distance from spill</th>
<th>Toxicity level (mg/L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 meters</td>
<td>5.34</td>
</tr>
<tr>
<td>100 meters</td>
<td>5.27</td>
</tr>
<tr>
<td>200 meters</td>
<td>5.20</td>
</tr>
<tr>
<td>300 meters</td>
<td>5.13</td>
</tr>
</tbody>
</table>

   a) Define two variables and then write a linear equation that describes the pattern in the table above.

   b) This particular chemical is considered highly toxic at levels of **4.5 mg/L** or more. How far is that from the spill site?

5. If the density of the population in this region is **25,000 per square kilometer**, use the distance you found in part b above to determine how many people fall within the highly toxic zone? You’ll need to think about the shape of the zone in order to answer this question.

6. Below is a graph of Chemico’s accumulated profits. Chemico is a fictitious company.

   ![Graph of Chemico's accumulated profits]

   a) Define two variables and then write a linear equation that describes graph above.

   b) What will this company’s accumulated profits be in year 25?
7. Chemico is successfully taken to court and prosecuted on average once every year, for the environmental damage caused by toxins leaked from the plant, spills in transport of materials, illegal dumping, and use of prohibited substances. The algebraic equation that describes the accumulated cost of the fines is as follows:

\[ F = x \, , \]

where \( F \) is the amount of accumulated fines in millions of dollars and \( x \) is the number of years.

Assuming that the company pays the fines, what is the equation that defines the company’s net accumulated profits?

8. The company could pay to replace is ageing machinery with newer, more environmentally friendly systems. They could increase the number and quality of filters on their smoke stacks and improve emergency warning technology. Staff could be trained on a regular basis. The company could put money into research and development in order to find less toxic materials for production.

A study is done to determine the annual cost of these measures. The algebraic equation that describes the pattern is as follows:

\[ C = 4x \, , \]

where \( C \) is the amount of accumulated cost to implement the above improvements in millions of dollars and \( x \) is the number of years. Assuming that the company doesn’t pay the fines, what is the equation that defines the company’s net accumulated profits?

9. Explain why it’s better for the company to continue to destroy the environment and take its chance in court rather than implement the improvements to the system. Does one option yield the company more money? If so, how much more?

10. An environmental lobby group pushes the government to changes the laws to allow for stiffer fines. They suggest that the way fines should be handed out is more effective if it follows this pattern:

\[ E = x^2 + x \, , \]

where \( E \) is the amount of accumulated costs of these stricter fines in millions of dollars and \( x \) is the number of years.

a) Is this equation linear or non-linear? Why does this matter?

b) In which year do the accumulated fines equal the accumulated profits?

c) Based on the new system of fines, how long do you think it would take the company to implement new environmental designs and systems? Explain.

11. The graph below shows the relationship between Fahrenheit temperature and Celsius temperatures

![Graph showing Fahrenheit vs. Celsius temperatures](image)

a) Determine the slope of the line.

b) Determine the equation of the line.

c) Use the equation you obtained in part b) to find the Fahrenheit temperature when the Celsius temperature is 20°.

d) Use the equation your equation to determine the Celsius temperature when the Fahrenheit temperature is 14°.
Solving Systems of Linear Equations by Graphing—5.1

Solve each system of linear equations by graphing. If the system has one solution, find it. If the system has no solution, state so. If the system has an infinite number of solutions, state what these solutions are.

1. \[y = 2x + 5\]
\[x + y = -7\]

Description of the graph:
Characteristics of lines:
Type of system:
Answer:

2. \[4x + 2y = 8\]
\[3x - y = 1\]

Description of the graph:
Characteristics of lines:
Type of system:
Answer:
3. \[ y + 4x = -1 \]
   \[ 8x + 2y = 6 \]

\[ \begin{array}{c}
\text{Description of the graph:} \\
\text{Characteristics of lines:} \\
\text{Type of system:} \\
\text{Answer:} \\
\end{array} \]

4. \[ -6x + 4 = 2y \]
   \[ 2 - y = 3x \]

\[ \begin{array}{c}
\text{Description of the graph:} \\
\text{Characteristics of lines:} \\
\text{Type of system:} \\
\text{Answer:} \\
\end{array} \]
Summary of Systems
There are three types of systems of linear equations in two variables:

1. **Independent systems** have two lines that cross at a single point. This single point is the **one and only solution** to the system. These two lines have different slopes.

2. **Inconsistent systems** have two parallel lines that never cross. There is **no solution** to this type of system. These two lines have the same slope, but different y-intercepts.

3. **Dependent systems** have two equations that represent the same line. **All of the points on this line** are solutions to the system; this is an infinite number of solutions. These two equations have the same slope and the same y-intercept; hence, they represent the same line.

Express each equation in slope-intercept form. **Without graphing**, state whether the system of equations has exactly one solution, no solution, or an infinite number of solutions.

5. \[ x + y = 6 \\
   x - y = 6 \]

6. \[ y = \frac{1}{2} x + 4 \\
   2y = x + 8 \]

7. \[ x - y = 3 \\
   \frac{1}{2} x - 3y = -6 \]

8. \[ x - y = 2 \\
   2x - 2y = -2 \]
Application Problems of Systems of Linear Equations—5.4

For each problem below, define variables, write two equations, solve the system, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate. You may use a chart to organize your information, but it isn’t necessary.

1. Natalie owns a coffee shop. In her shop there are many varieties of coffee. One, an Ethiopian coffee sells for $7 per pound, and a second, a Colombian coffee, sells for $4 per pound. She’s found that some of her customers like a blend of the Ethiopian coffee and Colombian coffee. How much of each type of coffee should she mix to get 12 pounds of a mixture that sells for $6 per pound? Note: the price per pound of the mix is given, NOT the total value.

   Find:
   
   Number of pounds of Ethiopian coffee:
   Number of pounds of Colombian coffee:
   
   Given:
   
   Price per pound of Ethiopian coffee:
   Price per pound of Colombian coffee:
   Price per pound of mixture of both coffees:
   Number of pounds in the mix of both coffees:

<table>
<thead>
<tr>
<th>Title</th>
<th>Price per pound</th>
<th>Individual Amount</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>French Roast</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Colombian</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mix</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Equation about the individual amounts:
   the number of pounds of Ethiopian coffee + the number of pounds of Colombian coffee = the number of pounds in the mix of both coffees

   Equation about total value:
   the total value of Ethiopian coffee + the total value of Colombian coffee = the total value in the mix of both coffees

   Solve the system:

   Answer the question:

Questions to answer while working on a “Mixture” application from 5.4

—Is this a “Mixture” problem?
   If it is a “Mixture” problem, you might want to set up a chart like the following:

<table>
<thead>
<tr>
<th>Title</th>
<th>Percentage or Price per unit</th>
<th>Individual Amount</th>
<th>Total Content or Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thing A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thing B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mix of A and B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   —What is given in the problem? Are any percentages or prices per unit given? Are any individual amounts given? Is a total amount given? Is a total value or content given?
       Fill in one box at a time. Start with the “givens”. Pay attention to the units.
   —What am I trying to find?
       After you have filled in the “givens”, use two different variables to define what you are trying to find and put these in the appropriate boxes. Write down what each variable represents, so you don’t forget! Pay attention to the units.
   —Fill in the remaining boxes by using the expressions that are already in the other boxes. Usually this means multiplying the expression in the second column by the expression in the third column to get the expression in the last column. Pay attention to the units.
   —Do the units in each of the columns match up?
       If the units in each of the columns match up, you can write an equation using the information about the individual amounts and another equation using the information about the total value.

   Over →
2. A feed store owner wishes to make his own store-brand mixture of bird seed by mixing sunflower seed that cost $0.50 per pound with a premixed assorted seed that costs $0.15 per pound. How many pounds of each will he have to use to make a 50-pound mixture that will cost $14.50? Note: the price per pound of the mix is NOT given, only the total value of the mix is given.

Find:

<table>
<thead>
<tr>
<th>Number of pounds of sunflower seed:</th>
<th>Number of pounds of premixed assorted seed:</th>
</tr>
</thead>
</table>

Given:

<table>
<thead>
<tr>
<th>Price per pound of sunflower seed:</th>
<th>Price per pound of premixed assorted seed:</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Number of pounds in the mix of both seed:</th>
<th>Total value of mixture of both seed:</th>
</tr>
</thead>
</table>

Equation about the individual amounts:

the number of pounds of sunflower seed + the number of pounds of premixed assorted seed = the number of pounds in the mix of both seed

Equation about total value:

the total value of sunflower seed + the total value of premixed assorted seed = the total value in the mix of both seed

Solve the system:

Answer the question

3. Alejandro invested $7000, part at 8% simple interest and the rest at 5% simple interest for a period of 1 year. If he received a total annual interest of $476 from both investments, how much did he invest at each rate? Note: The percentage rate of the mix investment is NOT given, only the total value of the mixed investment is given.

4. Micheaela, a pharmacist, has a 60% solution of the drug sodium iodite. She also has a 25% solution of the same drug. She gets a prescription calling for a 40% solution of the drug. How much of each solution should she mix to make 0.5 liter of the 40% solution?
Polynomials—6.3

Definitions

A term is a constant or a product of a constant and a variable or a product of variables that are raised to powers. Terms are separated by addition.

A monomial is a single term that only has whole number powers.

The degree of a monomial is the sum of the exponents of the variables.

The coefficient of a term is the constant part of a term.

A polynomial is a monomial or a sum of monomials.

The leading term of a polynomial is the term of the highest degree.

The leading term’s coefficient is the leading coefficient of the polynomial.

The leading term’s degree is the degree of the polynomial.

A polynomial of degree 0 or 1 is called linear.

A polynomial of degree 2 is called quadratic.

A polynomial of degree 3 is called cubic.

A polynomial with two terms is called a binomial.

A polynomial with three terms is called a trinomial.

Polynomials are usually arranged in descending order, so that the exponents of a certain variable decrease from left to right.

1. For each polynomial given, find the degree of each term, the degree of the polynomial, the leading term, and the leading coefficient. If the polynomial has a specific name—monomial, binomial, or trinomial—give that name.

   a) \( x^3 - 2x + 7 \)

   
<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
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</thead>
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</table>

   b) \( -2u^2 + 3v^5 - u^3 v^4 - 7 \)

   
<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
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</table>


2. Arrange each polynomial in descending order. Give the degree of each polynomial.
   a) \[ 1 - x^3 + 5x \]

   b) \[ a^3 - 7 + 11a^4 + a^9 - 5a^2 \]
Review of 6.1—6.5

Work all problems on another sheet of paper. Copy problem, show your work and circle your answer.

1. Simplify. \( z^4z \)
2. Simplify. \( (-5x^3)^2 \)
3. Simplify. \( -(5x^3)^2 \)
4. Simplify. \( 2x(-5x^3)^2 \)
5. Simplify. \( \left( \frac{20x^3}{16x^8} \right)^2 \)
6. Simplify. \( \left( \frac{-64xy^6}{32xy^{-3}} \right)^4 \)
7. Add. \( (x^2y + x - y) + (2x^2y + 2x - 6y + 3) \)
8. Subtract \( (x^2 + 2x - 8) \) from \( (3x^2 + 2x - 5) \)
9. Multiply. \( (2x + 3)^2 \)
10. Multiply. \( 5x^2(2x + 3) \)
11. Multiply. \( (2a + 1)(a^2 - 4a + 7) \)
12. Multiply. \( (x + 5)(x + 7) \)
13. Multiply. \( (a + 3)(a + 8) \)
14. Multiply. \( (y - 3)(y - 6) \)
15. Multiply. \((y + 1)(y - 12)\)

16. Multiply. \((x - 10)(x + 5)\)

17. Multiply. \((x + 6)(x - 6)\)

18. Multiply. \((a - 1)(a + 1)\)

19. Multiply. \((z + 7)(z - 7)\)

20. Multiply. \((2a + 5)(2a - 5)\)

21. Multiply. \((a + 4)^2\)

22. Multiply. \((3x - 8)^2\)

23. Fill in the appropriate terms in the boxes below to make the binomials multiply to the given result.

\[
\left( \square + 3 \right) \left( x + \square \right) \quad x^2 + 5x + 6
\]

24. Fill in the appropriate terms in the boxes below to make the binomials multiply to the given result.

\[
\left( \square + 4 \right) \left( \square + 2 \right) \quad y^2 + 6y + 8
\]

25. Fill in the appropriate terms in the boxes below to make the binomials multiply to the given result.

\[
\left( \square + 5 \right) \left( \square + \square \right) \quad x^2 + 8x + 15
\]

26. Fill in the appropriate terms in the boxes below to make the binomials multiply to the given result.

\[
\left( x + \square \right) \left( \square + \square \right) \quad x^2 + 7x + 12
\]
GCF and Grouping—7.1 & 7.2

Factor each expression completely. If a problem cannot be factored, state this. CHECK YOUR ANSWERS (by multiplication)!

1. \(x^2y + 2xy^2 - 15xy\)

2. \(4p^3 - 24p - 28\)

3. \(24x^7y^2 - 12x^2y^5 + 30x^3y^4\)

4. \(2a^2 + 7\)

5. \(w^3 - 15w\)

6. \(x^2(y + 5) - 2(y + 5)\)

7. \(x^2(2x + 7) + 2x + 7\)

8. \(x^2y + 2x^2 - 4y - 8\)

9. \(st^2 + st + 5t^2 + 5t\)

10. \(pq^2 + 2pq + 3p^2 + 3q\)
Review of Chapter 6

Perform the indicated operation then simplify.

11. \[ \frac{x^4 y^2}{x^7 y^{-7}} \]

12. \[ \frac{2^7}{2^4} \]

13. \[ \left( \frac{15a^{-8} c^{10}}{12a^5 c^4} \right)^5 \]

14. \( (st)^2 \)

15. \( (s + t)^2 \)

16. \( 4s - s \)

17. \[ 5xy^2 \left( 2x^4 y^3 \right)^2 \]

18. \[ 5xy^2 \left( 2x^4 + y^3 \right) \]

19. \[ \frac{5xy^2 - 15xy + 20y}{-5x} \]

20. \[ \frac{3x^2 - 4x + 20}{x + 2} \]
Difference of Squares—7.4

Factor each expression completely. If a problem cannot be factored, state that it is prime. CHECK YOUR ANSWERS (by multiplication)!

1. \( a^2 - 36 \)

2. \( y^2 + 3y - 40 \)

3. \( x^2 - 1 \)

4. \( 8y^3 - 18y \)

5. \( 2x^3 + 20x^2 + 48x \)

6. \( 5x^2y + 15x^5y^2 \)

7. \( 8a^3b - 12a^2b^2 + 28a^2b \)
8. $x^2 + 17xy - 18y^2$

9. $a^2 - 4ab - 5ab + 20b^2$

10. $x^2 - 9y^2$

11. $c^2d - 14cd^2 - 32d^3$

12. $p^2 - 16p$

13. $100x^2y^2 - 64$

14. $49 - y^2$

15. $x^2 - 20x + 84$
# Factoring Trinomials with a non-1 Leading Coefficient — 7.5

Fill in the missing terms or signs for each factor. CHECK YOUR ANSWERS!

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $x^2 + 14x - 32$</td>
<td>$(16)(x - )$</td>
</tr>
<tr>
<td>2. $x^2 + 9xy + 14y^2$</td>
<td>$(x + )( + 7y)$</td>
</tr>
<tr>
<td>3. $2x^2 + 9x + 7$</td>
<td>$( + 1)(2x + )$</td>
</tr>
<tr>
<td>4. $5x^2 + 33x - 14$</td>
<td>$( + 7)(5x )$</td>
</tr>
<tr>
<td>5. $6x^2 + 19x + 15$</td>
<td>$(3x + 5)( )$</td>
</tr>
<tr>
<td>6. $4x^2 - 27x + 18$</td>
<td>$(4x - )( - )$</td>
</tr>
<tr>
<td>7. $6x^2 - 7xy - 5y^2$</td>
<td>$(2x + )( - 5y)$</td>
</tr>
<tr>
<td>8. $8x^2 - 18x + 9$</td>
<td>$(4x - )(2x )$</td>
</tr>
<tr>
<td>9. $3x^2 + 7x + 2$</td>
<td>$( )( )$</td>
</tr>
<tr>
<td>10. $2x^2 - 3x + 1$</td>
<td>$( )( )$</td>
</tr>
</tbody>
</table>
General Application Problems of Quadratic Equations—7.8

Pythagorean Theorem

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs.

$$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$$

If $a$ and $b$ represent the legs, and $c$ represents the hypotenuse, then $a^2 + b^2 = c^2$.

For each problem below, define variables, write an equation, solve the equation, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate.

1. Determine the value of $x$.

![Diagram of a right triangle with sides labeled $x$, 15 inches, and 12 inches.]

2. One leg of a right triangle is two inches more than twice the other leg. The hypotenuse is 13 inches. Find the lengths of the three sides of the triangle.

3. The product of two consecutive positive integers is 342. Find the two integers.

4. The area of a rectangle is 84 square inches. Determine the length and width if the length is two inches less than twice the width.
Simplifying, Multiplying, and Dividing Rational Expressions—8.2 & 8.3

For problems 1 through 4, reduce the rational expression, if possible. If the expression cannot be reduced, state this.

1. \[\frac{10a^2b^6}{-5a^4b^4}\]

2. \[\frac{6x^2 - 7x - 5}{4x^2 - 1}\]

3. \[\frac{5 - x}{x^2 - 25}\]

4. \[\frac{x - 3}{x + 3}\]

For problem 5, determine the values for which the expression is defined.

5. \[\frac{3}{x^2 - 7x + 10}\]

For problems 6 through 9, perform the indicated operation. Be sure your answers are in lowest terms.

6. \[\frac{10r^3s}{6mr} \cdot \frac{9m^4}{4mrs}\]

7. \[\frac{18x^3y^2}{7z^4} \div \left(14x^2y^2z^2\right)\]

8. \[\frac{y^2 - 4y}{y^4} \cdot \frac{y^3}{y^2 - 3y - 4}\]

9. \[\frac{z^2 + 6z}{(z + 6)^2} \div \frac{z^2 + z}{z^2 + 7z + 6}\]
Rational Expressions and Equations—Chapter 8

For problems 1 and 2, reduce the rational expression, if possible. If the expression cannot be reduced, state this.

1. \[
\frac{4 - 2a}{6a^2 - 24}
\]

2. \[
\frac{y^2 - 8y + 16}{y^2 - 10y + 24}
\]

For problems 3 through 6, perform the indicated operation. Reduce your answers, if possible.

3. \[
\frac{m^2 + 3m - 10}{5m^2} \div \frac{m^2 + 10m + 25}{m^2 + 5m}
\]

4. \[
\frac{x^2 + 2x}{x^2 - x - 2} \cdot \frac{2 - x}{x}
\]

5. \[
\frac{8x + 3}{5x + 10} - \frac{5x - 3}{5x + 10}
\]

6. \[
\frac{12}{x^2 + x - 2} - \frac{4}{x^2 - x}
\]

For problems 7 and 8, solve each equation. If there is no solution to the equation, state this.

7. \[
\frac{8}{x - 2} + 3 = \frac{x + 6}{x - 2}
\]

8. \[
\frac{2}{x - 2} + \frac{x^2}{x^2 - 6x + 8} = \frac{3}{x - 4}
\]
Application Problems Involving Rational Equations—8.6

Number Problems
1. One number is 3 times larger than another. The sum of their reciprocals is \( \frac{4}{3} \). Determine the two numbers.

Find: one number: another number: Unused fact:

Equation that comes from the unused fact: 
the reciprocal of “one number” + the reciprocal of “another number” = \( \frac{4}{3} \)

Solve the equation:

Answer the question:

2. Forrest is interested in purchasing a carpet whose area is 60 square feet. Determine the length and the width if the width is 5 feet less than \( \frac{3}{5} \) of the length.

Find: length: width: Unused fact:

Equation that comes from the unused fact: 
length times width = area

Solve the equation:

Answer the question:

Distance/Rate/Time Problems
3. Iriana’s Lexus travels 30 mph faster than Gina’s Harley. In the same time that Gina travels 75 miles, Iriana travels 120 miles. Find their speeds.

Find: Gina’s speed: Iriana’s speed: Given: Gina’s distance: Iriana’s distance:

<table>
<thead>
<tr>
<th>Names</th>
<th>Rate = Distance ÷ Time</th>
<th>Time = Distance ÷ Rate</th>
<th>Distance = Rate • Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gina</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iriana</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Unused fact:

Equation that comes from the unused fact: 
Gina’s time = Iriana’s time

Solve the equation:

Answer the question:
4. During a two-person relay race, Emmett rides his bike 5 miles per hour faster than Sasha. Each of them rides for 10 miles. Their team time is 44 minutes. Determine each of their speeds.

Find:  
   Sasha’s speed:  
   Emmett’s speed:  

<table>
<thead>
<tr>
<th>Names</th>
<th>Rate = Distance ÷ Time</th>
<th>Time = Distance ÷ Rate</th>
<th>Distance = Rate · Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sasha</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emmett</td>
<td></td>
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</tbody>
</table>

Unused fact:

Equation that comes from the unused fact: Sasha’s time + Emmett’s time = total time

Solve the equation:

Answer the question:

5. Ava takes two trains to commute to work. First she rides a commuter train that travels at 30 miles per hour. She then gets on an express train travels at 50 miles per hour. If she rides the commuter train for 20 miles less than the express train and the total time of her trip is 40 minutes, find the distance she rides each train.

Find:  
   commuter train’s distance:  
   express train’s distance:  

<table>
<thead>
<tr>
<th>Names</th>
<th>Rate = Distance ÷ Time</th>
<th>Time = Distance ÷ Rate</th>
<th>Distance = Rate · Time</th>
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<tbody>
<tr>
<td>Commuter</td>
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<tr>
<td>Train</td>
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<td>Express</td>
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<tr>
<td>Train</td>
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Unused fact:

Equation that comes from the unused fact: commuter train’s time + express train’s time = total time

Solve the equation:

Answer the question:

6. Javier is vacationing near the Colorado River and plans to canoe in a section of the river where the current is 2 miles per hour. If it took Javier a total of 4 hours to travel 10 miles downstream and 2 miles upstream, determine the speed at which Javier’s canoe would travel in still water.

Find:  
   canoe’s speed in still water:  
   Other quantities: speed downstream:  
   speed upstream:  

<table>
<thead>
<tr>
<th>Directions</th>
<th>Rate = Distance ÷ Time</th>
<th>Time = Distance ÷ Rate</th>
<th>Distance = Rate · Time</th>
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<tbody>
<tr>
<td>Downstream</td>
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<tr>
<td>Upstream</td>
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Unused fact:

Equation that comes from the unused fact: time downstream + time upstream = total time

Solve the equation:

Answer the question:
**Work Problems**

7. Alma and Stephanie work as volunteers at a town’s recycling depot. Alma can sort a day’s accumulation of recyclables in 4 hours, while Stephanie takes 6 hours to do the same job. How long would it take them, working together to sort the recyclables?

<table>
<thead>
<tr>
<th>Find:</th>
<th>Time worked together:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given:</td>
<td>Alma’s rate for one job:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Rate = [\frac{1}{\text{amount of time to do one job}}]</th>
<th>Actual Time Worked</th>
<th>Amount of Work = Rate · Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alma</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stephanie</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Work equation: the amount of work done by Alma + the amount of work done by Stephanie = 1 job

Solve the equation:

Answer the question:

8. Zion and Nick own a cleaning service. When Zion cleans the Moose Club by himself, it takes him 7 hours. When Zion and Nick work together to clean the same club, it takes them 4 hours. How long does it take Nick by himself to clean the Moose Club?

<table>
<thead>
<tr>
<th>Find:</th>
<th>Time it takes Nick to complete 1 job by himself:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given:</td>
<td>Time worked together:</td>
</tr>
<tr>
<td>Other quantities:</td>
<td>Nick’s rate for one job:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Rate = [\frac{1}{\text{amount of time to do one job}}]</th>
<th>Actual Time Worked</th>
<th>Amount of Work = Rate · Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nick</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Work equation: the amount of work done by Zion + the amount of work done by Nick = 1 job

Solve the equation:

Answer the question:
Simplifying Radical Expressions — Part II — 9.5

Simplify each of the following.

1. \( \frac{6 + 4\sqrt{3}}{2} \)

2. \( \frac{10 - \sqrt{16}}{2} \)

3. \( \frac{3 + \sqrt{29}}{3} \)

4. \( \frac{6 + \sqrt{12}}{8} \)

5. \( \frac{8 + \sqrt{48}}{4} \)

6. \( \frac{10 - \sqrt{24}}{12} \)

7. \( \frac{2 + \sqrt{8}}{2} \)

8. \( \frac{3 - \sqrt{45}}{3} \)
Review of Radical Expressions and Equations — 9.6

For problems 1 through 9, simplify, if possible. If there is no real number equivalent to the expression, state this. Do not write a decimal approximation.

1. \( \sqrt{x^6 y^4} \)

2. \( \sqrt{36x^2} \)

3. \( \sqrt{108r^9} \)

4. \( \sqrt{12yz^2} \cdot \sqrt{6y^3} \)

5. \( \sqrt[3]{48x^3} \)

6. \( \sqrt[3]{21t^3} \)

7. \( 4\sqrt{12} + \sqrt{27} - \sqrt{12} \)

8. \( (\sqrt{7} - 5)(\sqrt{7} - 2) \)

9. \( \frac{\sqrt{7}}{\sqrt{7} - \sqrt{5}} \)

For problem 10, solve the equation. If the equation has no solution, state this.

10. \( \sqrt{x} + 18 = x - 2 \)

Extra practice on a distance/rate/time problem:

11. Ronnie walks over to a friend’s house at the rate of 6 km/h and jogs home at the rate of 14 km/h. If the total time, walking and jogging, is 3 hours, how far is it to the friend’s house?
Application Problems Involving Quadratic Equations—10.5

1. The length of a certain box is 2 cm longer than the height, and the width is 15 cm. Find the length and height of the box if the volume is 1,200 cm³. Round your answers to 2 decimal places.

Pythagorean Theorem
For any right triangle, the sum of the square of the legs of the triangle equals the square of the hypotenuse. In other words,

\[ a^2 + b^2 = c^2 \]

2. If you wish to anchor a telephone pole with a 50-foot wire cable that is at a distance equal to one-third the height of the pole from the base of the pole, how tall will the telephone pole be? Round your answer to 2 decimal places.
3. A rocket is shot from the ground with an initial velocity of 44 feet per second. The distance \(d\) above the ground after \(t\) seconds is given by:

\[ d = 44t - 16t^2 \]

a) How many seconds after the start is the rocket at a height of 24 feet? Round your answer to 2 decimal places.

b) When will the rocket hit the ground? Round your answer to 2 decimal places.

4. C3PO has installed an ejection seat in his land rover for quick escapes. R2D2 is in the air after accidentally bumping the ejection switch. The seat throws R2 upward as described by:

\[ y = 4 + 100x - 16x^2 \]

where \(x\) is the number of seconds after the ejection button is hit and \(y\) is the number of feet up after \(x\) seconds.

a) How far above the ground is R2D2 before the button was bumped? What does this mean?

b) How long will it take for R2D2 to reach a height of 10 feet? Round your answer to 2 decimal places.

c) How long will it take R2D2 to land? Round your answer to 2 decimal places.
Chapter 1 – Answers

Section 1.1
Section 1.2
Section 1.3
Section 1.4
Section 1.5
Section 1.6
Section 1.1  Real Numbers

1.  a–f) These answers should be in your own words.
2.  a) Integer, rational, real  
    b) Irrational, real  
    c) Whole, integer, rational, real  
    d) Rational, real  
    e) Irrational, real  
    f) Rational, real  
    g) Natural, whole, integer, rational, real  
    h) Rational, real  
    i) \( \sqrt{9} = 3 \); natural, whole, integer, rational, real  
    j) Irrational, real  
    k) Rational, real  
3.  a) Natural numbers : \{ 17, 53 \}  
    b) Whole numbers : \{ 0, 17, 53 \}  
    c) Integers : \{ –56, 0, 17, 53 \}  
    d) Rational numbers : \{ –0.3, 17, –56, 4 \frac{1}{2}, 0, 53, 9\frac{4}{10} \}  
    e) Irrational numbers : \{ \sqrt{7} \}  
    f) Real numbers : \{ –0.3, 17, –56, 4 \frac{1}{2}, \sqrt{7}, 0, 53, 9\frac{4}{10} \}  
4.  Note that \( \sqrt{4} = 2 \)  
    a) Natural numbers : \{ \sqrt{4}, 11 \}  
    b) Whole numbers : \{ \sqrt{4}, 11 \}  
    c) Integers : \{ \sqrt{4}, 11 \}  
    d) Rational numbers : \{ \sqrt{4}, 5 \frac{1}{2}, –0.25, 11, \frac{1}{6} \}  
    e) Irrational numbers : \{ \sqrt{10}, \pi \}  
    f) Real numbers : \{ \sqrt{4}, 5 \frac{1}{2}, –0.25, 11, \frac{1}{6}, \sqrt{10}, \pi \}  
5.  True  
6.  False  
7.  True  
8.  True  
9.  True  
10. False  
11. a–d) These answers should be in your own words.
Section 1.2  Integers

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>−21</td>
</tr>
<tr>
<td>2.</td>
<td>−17</td>
</tr>
<tr>
<td>3.</td>
<td>4</td>
</tr>
<tr>
<td>4.</td>
<td>5</td>
</tr>
<tr>
<td>5.</td>
<td>25</td>
</tr>
<tr>
<td>6.</td>
<td>−16</td>
</tr>
<tr>
<td>7.</td>
<td>−14</td>
</tr>
<tr>
<td>8.</td>
<td>−15</td>
</tr>
<tr>
<td>9.</td>
<td>9</td>
</tr>
<tr>
<td>10.</td>
<td>30</td>
</tr>
<tr>
<td>11.</td>
<td>0</td>
</tr>
<tr>
<td>12.</td>
<td>−30</td>
</tr>
<tr>
<td>13.</td>
<td>−240</td>
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<tr>
<td>14.</td>
<td>−100</td>
</tr>
<tr>
<td>15.</td>
<td>−25</td>
</tr>
<tr>
<td>16.</td>
<td>−360</td>
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<tr>
<td>17.</td>
<td>20</td>
</tr>
<tr>
<td>18.</td>
<td>56</td>
</tr>
<tr>
<td>19.</td>
<td>−40</td>
</tr>
<tr>
<td>20.</td>
<td>0</td>
</tr>
<tr>
<td>21.</td>
<td>−16</td>
</tr>
</tbody>
</table>
Section 1.3 Fractions

1. \( \frac{1}{1} \)
2. \( \frac{4}{23} \)
3. \( \frac{3}{3} \)
4. \( 0 \)
5. \( 0 \)
6. \( \frac{23}{12} \) or \( 1 \frac{11}{12} \)
7. \( \frac{5}{6} \)
8. \( -\frac{2}{13} \)
9. \( 3 \)
10. \( \frac{7}{3} \) or \( 2 \frac{1}{3} \)
11. \( -\frac{1}{20} \)
12. 
   \(-\frac{7}{8} - \left( -\frac{9}{10} \right) \)
   \(-\frac{7}{8} + \left( +\frac{9}{10} \right) \) LCD is 40.
   \(-\frac{7}{8} \cdot \frac{5}{5} + \frac{9}{10} \cdot \frac{4}{4} \) LCD is 40
   \(-\frac{35}{40} + \frac{36}{40} \)
   \(-\frac{1}{40} \)
13. \( \frac{1}{3} \)
14. \( -\frac{5}{42} \)
15. 
   \(-1\frac{1}{6} + \left( -\frac{21}{5} \right) \)
   \(-\frac{7}{6} + \left( -\frac{21}{5} \right) \)
   \(-\frac{7}{6} \cdot \left( -\frac{5}{21} \right) \)
   \(-\frac{35}{42} \cdot \left( -\frac{5}{21} \right) \)
   \(-\frac{5}{18} \)
16. \( \frac{33}{10} \) or \( 3 \frac{3}{10} \)
17. \( 26 \frac{1}{3} \) or \( 105 \frac{5}{4} \)
18. \( 28 \)
19. \( \frac{35}{12} \) or \( 2 \frac{11}{12} \)
20. \( \frac{21}{20} \) or \( 1 \frac{1}{20} \)
Section 1.4  Exponents and Order of Operations

1. a–d) These answers should be in your own words.
2. a) 9 
    b) 9 
    c) -27 
    d) -9 
    e) -27 
3. a) -16 
    b) -64 
    c) -64 
    d) 16 
    e) 16 
4. 45 
5. -1 
6. 5 
7. -3 
8. 6 
9. \(2^3\) or \(3^2\) 
10. 2 
11. -50 
12. -26 
13. \(2 - 8(16 - 12)\) 
    \(= 2 - 8(4)\) 
    \(= 2 - 32\) 
    \(= -30\) 
    \(= -30 \div \frac{1}{4}\) 
    \(= -30 \cdot \frac{4}{1}\) 
    \(= -30 \cdot 4\) 
    \(= -120\) 
14. \(\frac{1}{9}\) 
15. \(-\frac{16}{47}\)
Section 1.5  Simplifying Expressions

1.  a–d) These answers should be in your own words.
2.  a)  \(4m - 2 + 3m\) : 3 terms
   b)  \(2(7p - 3) + 11\) : 2 terms;  \(14p + 6 + 11\) : 3 terms
   c)  \(5x + 13 - 4(x + 1)\) : 3 terms;  \(5x + 13 - 4x - 4\) : 4 terms
   d)  \(-3(w - y) + 10(w - 6)\) : 2 terms;  \(-3w + 3y + 10w - 60\) : 4 terms
3.  \(7x\)
4.  \(9y\)
5.  \(15x\)
6.  \(7m - 2\)
7.  \(14p + 5\)
8.  \(2x - \frac{3}{2}\)
9.  \(15 - (y + 3)\)
   \(15 - y - 3\)
   \(15 - 3 - y\)
   \(12 - y\) or \(-y + 12\)
10.  \(44 - 4w\)
11.  \(9y + 6\)
12.  \(x + 9\)
13.  \(-6x + 91\)
14.  \(-15w + 25\)
15.  \(-7\)
16.  \(\frac{19}{12}y - 1\)
17.  \(9x + 12\)
18.  \(2k + 20\)
19.  \(\frac{1}{2}(6y + 4) + 3 - 7(y + 5)\)
   \(3y + 2 + 3 - 7y - 35\)
   \(3y - 7y + 2 + 3 - 35\)
   \(-4y - 30\)
20.  \(1.24x\)
21.  \(0.85p\)
22.  \(2x\)
23.  \(-6a - 21\)
24.  \(0\)
25.  \(7w + 3y - 60\)
26.  \(18xy - 20\)
27.  \(-3n + 2\)
28.  \(63x + 24y - 15\)
29.  \(-20a + 84\)
30. \[
\frac{3}{4}(6-x) - \frac{5}{6}(3x+1)
\]
\[
\frac{3}{4} \cdot 6 - \frac{3}{4} \cdot x - \frac{5}{6} \cdot 3x - \frac{5}{6} \cdot 1
\]
\[
\frac{3}{4} \cdot 1 - \frac{3}{4} \cdot x - \frac{5}{6} \cdot \frac{x}{1} - \frac{5}{6} \cdot 1
\]
\[
\frac{9}{4} - \frac{3}{4}x - \frac{5}{2}x - \frac{5}{6}
\]
\[
- \frac{3}{4}x - \frac{5}{2}x + \frac{9}{2} - \frac{5}{6}
\]
\[
- \frac{3}{4}x - \frac{5}{2} \cdot \frac{3}{2}x + \frac{9}{2} \cdot \frac{3}{3} - \frac{5}{6}
\]
\[
- \frac{3}{4}x - \frac{10}{4}x + \frac{22}{6} - \frac{5}{6}
\]
\[
- \frac{13}{4}x + \frac{22}{6}
\]
\[
- \frac{13}{4}x + \frac{11}{3}
\]

31. \[
\frac{5}{2}a - 2e + 71
\]
Section 1.6  Phrases into Algebraic Expressions

a) Division
b) Addition
c) Subtraction
d) Addition
e) Subtraction
f) Multiplication
g) Subtraction

2.  
a) 12\div 3  
b) 2\div 16  
c) 11+8  
d) 2x  
e) 0.40x  
f) 15\div 11  

3.  
first number: x  
next consecutive number: x+1

4.  
the average amount the average person in India makes in a year: x  
the average amount spent by a participant at the World Economic Forum in 5 days: 14x

5.  
the median family income in California in 2011: x  
the median family income in Santa Cruz County in 2011: x+15,400

6.  
Santa Cruz County’s taxable sales in 2008: x  
Santa Cruz County’s taxable sales in 2009: x\div 0.13x

7.  
regular shoe price: x  
sale shoe price: x\div 0.20x

8.  
“other” number: x  
“one” number: 2x\div 3

9.  
the average income of the poorest 10 percent of the population: x  
the average income of the richest 10 percent of the population: 9x

10.  
Santa Cruz County’s travel spending in 2008: x  
Santa Cruz County’s travel spending in 2009: x\div 40,000,000

11.  
cost of jacket without tax: x  
cost of jacket with tax: x\div 0.08x

12.  
the unemployment rate of a person a Bachelor’s degree: x  
the unemployment rate of a person a high school diploma: 2x\div 0.5

13.  
the number of jobs in Santa Cruz County in 2000: x  
the number of jobs in Santa Cruz County in 2010: x\div 0.26x

14.  
the number of Latino respondents who reported moving at least once in the last year: x  
the number of Caucasian respondents who reported moving at least once in the last year: x\div 10

15.  
first number: x  
next consecutive odd number: x+2

16.  
the 2007 Santa Cruz County median home sales price: x  
the 2010 Santa Cruz County median home sales price: x\div 225,000

17.  
the average rent for a three-bedroom dwelling in Santa Cruz County in 2008: x  
the average rent for a three-bedroom dwelling in Santa Cruz County in 2011: x\div 0.16x

18.  
Martita’s amount of money: x  
Nicole’s amount of money: 75\div x

19.  
the number of respondents who earned $65,500/year or more and felt they had job opportunities in Santa Cruz County: x  
the number of respondents who earned less than $35,000/year and felt they had job opportunities in Santa Cruz County: x\div 23

20.  
the weekly earnings of a person with a high school diploma: x  
the weekly earnings of a person with a Master’s degree: 2x\div 20

21.  
“other” number: x  
“one” number: 1\div 2x\div 14
Chapter 2 — Answers

Section 2.1
Section 2.2
Section 2.3
Section 2.4
Section 2.5
Section 2.1  Additive Property of Equality

1. This answer should be in your own words.
2. The inverse operation of addition is subtraction.
3. The inverse operation of subtraction is addition.
4. a) \( x = 5 \)
   b) \( y = 81 \)
   c) \( x - 3 \)
   d) \( k = 25 \)
   e) \( -5x + 21 \)
5. \( y = 5 \)
6. \( p = 29 \)
7. \( p = -21 \)
8. \( z = -6 \)
9. \( m = 1 \)
10. \( n = -45 \)
11. \( x = 0 \)
12. \( z = 136 \)
13. \( w = 26 \)
14. \( x = -28 \)
15. \( m = 4.3 \)
16. \( x = 18 \)
17. \( c = 44 \)
18. \( p = -64 \)
19. \( y = -43 \)
20. \( x = -83 \)
21. \( x = -8 \)
22. \( m = -44 \)
23. \( x = -8 \)
Section 2.2   Multiplicative Property of Equality

1. The inverse operation of multiplication is division.
2. The inverse operation of division is multiplication.
3. 
   a) \( x = 6 \)
   b) \( 3x + 18 \)
   c) \( p = -5 \)
   d) \( -y - 5 \)
   e) \( x = 7 \)
4. \( m = 3 \)
5. \( p = -2 \)
6. \( r = -30 \)
7. \( x = -6 \)
8. \( y = \frac{7}{5} \)
9. \( w = -4 \)
10. \( p = -27 \)
11. \( x = \frac{1}{4} \)
12. \( a = 4 \)
13. \( z = -\frac{2}{3} \)
14. \( y = -20 \)
15. \( x = 12 \)
16. \( w = 50 \)
17. \( m = -\frac{7}{4} \)
18. \( c = -\frac{5}{4} \)
19. \( x = -14 \)
20. \( w = -\frac{10}{3} \)
21. \( x = -\frac{1}{2} \)
22. \( a = -28 \)
23. \( p = 4 \)
24. \( n = \frac{3}{8} \)
25. \( y = 32 \)
26. \( z = \frac{1}{2} \)
27. \( x = \frac{3}{2} \)
28. \( k = -50 \)
Section 2.3  Linear Equations in One-Variable

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. a) \( x = 8 \)
   b) \( 5x + 40 \)
   c) \( -3y - 12 \)
   d) \( x = 5 \)
5. \( x = 8 \)
6. \( y = 5 \)
7. \( p = -\frac{1}{2} \)
8. \( w = -\frac{13}{4} \)
9. \( x = -\frac{1}{2} \)
10. \( x = \frac{3}{5} \)
11. \( x = 11 \)
12. \( x = \frac{39}{14} \)
13. \( x = -11 \)
14. \( x = 10 \)
15. \( a = 7 \)
16. \( x = 3 \)
17. \( x = -5 \)
18. \( x = 5 \)
19. \( x = \frac{14}{3} \)
20. \( x = -1 \)
21. \( x = -2 \)
22. \( y = -\frac{1}{21} \)
23. \( x = 4 \)
24. \( x = 10 \)
25. all real numbers
26. \( x = -8 \)
27. no solution
28. \( p = -4 \)
29. \( x = \frac{1}{2} \)
30. \( x = -2 \)
31. all real numbers
32. \( m = \frac{23}{4} \)
33. no solution
34. \( x = \frac{17}{8} \)
35. \( x = -\frac{23}{19} \)
36. all real numbers
37. \( n = 11 \)
38. \( x = -\frac{14}{5} \)
39. all real numbers
40. \( z = 8 \)
41. \( x = 3 \)
42. all real numbers
43. \( x = 0 \)
Section 2.4  Linear Equations in One-Variable with Fractions

1. This answer should be in your own words.
2.  
   a) \(-\frac{3}{4} = \frac{5}{6}w - 2\) : 3 terms
   
   b) \(4 + \frac{2}{9}(x + 1) = \frac{2}{3}\) : 3 terms; \(4 + \frac{5}{9}x + \frac{2}{3} = \frac{2}{3}\) : 4 terms
   
   c) \(3 = \frac{1}{11} - \left(\frac{1}{22}x + 4\right)\) : 3 terms; \(3 = \frac{1}{11} - \frac{1}{22}x - 4\) : 4 terms

3. \(x = \frac{13}{6}\)
4. \(y = \frac{10}{7}\)
5. \(p = \frac{1}{42}\)
6. \(w = \frac{25}{24}\) or \(1\frac{1}{24}\)
7. \(x = 2\)
8. \(x = -\frac{37}{9}\) or \(-4\frac{1}{9}\)
9. \(x = -16\)
10. \(x = -152\)
11. \(x = \frac{8}{9}\)
12. \(y = -\frac{4}{3}\)
13. \(x = \frac{39}{29}\) or \(1\frac{10}{29}\)
14. \(x = -\frac{23}{4}\) or \(-5\frac{3}{4}\)
15. \(m = -\frac{92}{117}\)
16. \(x = \frac{2}{9}\)
17. \(x = \frac{3}{162}\)
18. \(x = \frac{1}{2}\)
19. no solution
20. \(z = -\frac{20}{3}\) or \(-6\frac{2}{3}\)
21. \(x = \frac{2}{7}\)
22. all real numbers
23. \(n = -\frac{349}{320}\) or \(-1\frac{229}{320}\)
24. \(x = -\frac{529}{177}\) or \(-2\frac{175}{177}\)
25. all real numbers
26. no solution
27. \(p = \frac{245}{33}\) or \(7\frac{7}{34}\)
28. \(x = -\frac{11}{9}\) or \(-1\frac{2}{9}\)
29. \(y = \frac{13}{18}\)
30. \(x = \frac{173}{6}\) or \(28\frac{5}{6}\)
31. \(x = \frac{67}{30}\) or \(2\frac{7}{30}\)
32. \(w = -6\)
33. \(x = -15\)
34. \(x = -\frac{13}{21}\)
35. all real numbers
36. \(x = \frac{46}{17}\) or \(2\frac{12}{17}\)
37. \( x = \frac{46}{17} \) or \( 2 \frac{12}{17} \)

38. all real numbers

39. \( x = -\frac{20}{3} \) or \(-6 \frac{2}{3}\)

40. \( x = -\frac{232}{7} \) or \(-33 \frac{1}{7}\)
Section 2.5  Linear Inequalities in One-Variable

1. a-c) These answers should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. This answer should be in your own words.
5. This answer should be in your own words.
6. This answer should be in your own words.
7. \( x > -4 \)
8. \( x < -\frac{1}{2} \)
9. \( x \geq 10 \)
10. \( x \leq -8 \)
11. \( 5 \geq x \)
12. \( -2 < x \)
13. \( -3 > x \)
14. \( x > 0 \)
15. \( x \leq 0 \)
16. \( x > 4 \)
17. \( x \leq -3 \)
18. \( x > 8 \)
19. \( p \geq -8 \)
20. \( x \leq \frac{5}{6} \)
21. \( x \leq -6 \)
22. \( a \leq \frac{23}{4} \)
23. \( x > \frac{46}{7} \)
24. \( y \leq -1 \)
25. \( x < -\frac{7}{5} \)
26. \( x \geq -5 \)
27. \( w \leq -\frac{22}{27} \)
28. \( x > \frac{11}{2} \)
29. \( x \leq -\frac{7}{2} \)
30. \( z \leq -5 \)
31. \( x < \frac{19}{2} \)
32. \( x > \frac{1}{3} \)
33. \( x \leq -\frac{168}{25} \)
34. \( x \geq -\frac{14}{3} \)
35. \( q \geq 5 \)
36. \( x \leq -1 \)
37. all real numbers
38. no solution
39. \( x \geq 0 \)
40. all real numbers
41. no solution
42. no solution
43. no solution
44. \( y \leq -\frac{6}{5} \)
45. all real numbers
46. all real numbers
47. \( x < -6 \)
48. \( x > -7 \)
49. \( x \leq -\frac{50}{43} \)
50. \( w \leq 51 \)
51. \( x \leq -\frac{5}{2} \)
Chapter 3 — Answers

Section 3.1
Section 3.2
Section 3.3
Section 3.4
Section 3.5
Section 3.6
Section 3.1 Ratios and Proportions

1. This answer should be in your own words.
2. This answer should be in your own words.
3. A ratio has the same units for both quantities. In a rate the quantities have different units.
4. This answer should be in your own words.
5. You may cross-multiply to solve a linear equation if there is only one term on each side of the equation.
6. This answer should be in your own words.
7. a) \( \frac{274}{33} \)
   b) \( \frac{\$10}{\text{unit}} \)
   c) \( \frac{\$36}{\text{unit}} \)
8. 114 Caucasian
9. about 16 students
10. 12 Latino students
11. 150,000 students
12. 3 students
13. 4 students
14. 1¼ cups of onion
15. \( \frac{3}{5} \) tablespoons of sugar
16. 19.2 feet
17. 28 miles
18. 0.42 grams
Section 3.2  Formulas

1. \( P = 12 \)
2. \( P = 12 \)
3. \( c = 10 \)
4. \( r = 0.03 \)
5. \( A = 42 \)
6. \( l = 8 \)
7. \( m = 24 \)
8. \( V = 2 \)
9. \( t = 5 \)
10. \( F = 120 \)
11. \( P = \frac{1}{5} \)
12. \( V = 60 \)
13. \( V = \frac{1}{8} \)
14. \( p = 5 \)
15. \( H = 116 \)
16. \( t = 3.75 \)
17. \( g = \frac{10}{3} \)
18. \( V_0 = 31.2 \)
19. \( n_t = 5 \)
20. Using the \( \pi \) button on your calculator: \( A = 28.27 \); using \( \pi \approx 3.14 \): \( A = 28.26 \)
21. \( y = 11 \)
22. \( y = 6 \)
23. \( y = -7 \)
24. \( y = -12 \)
25. \( y = 2 \)
26. \( x = 5 \)
27. \( x = 4 \)
28. \( x = \frac{5}{3} \)
29. \( x = -6 \)
30. \( x = -1 \)
31. \$160 \)
32. \( 3.75\% \)
33. \( 24 \) square feet or \( 24 \) ft\(^2\)
34. \( 30 \) feet
35. \( 6.1875 \) in\(^2\)
36. \( 3.85 \) in\(^3\)
37. \( 4 \) years
38. exact: \( 2.25\pi \) in\(^2\); approximate: \( 7.069 \) in\(^2\)
39. area: \( 24 \) ft\(^2\); volume: \( 36 \) ft\(^3\) ?
40. exact: \( \frac{32}{3}\pi \) in\(^3\); approximate: \( 33.51 \) in\(^3\)
Section 3.3  Literal Equations

1. This answer should be in your own words.
2. \( w = \frac{A}{I} \)
3. \( s = \frac{p}{a} \)
4. \( a = P - b - c \)
5. \( r = \frac{L}{m} \)
6. \( h = \frac{2A}{b} \)
7. \( I = \frac{P - 2w}{2} \)
8. \( m = Dv \)
9. \( h = \frac{V}{lw} \)
10. \( r = \frac{d}{T} \)
11. \( a = \frac{E}{m} \)
12. \( a = \frac{f}{P} \)
13. \( V = IR \)
14. \( C = \frac{RT}{V} \)
15. \( U = H - pV \)
16. \( S = \frac{G - H}{T} \); or \( S = \frac{H - G}{T} \); or \( S = \frac{H}{T} - \frac{G}{T} \).
17. \( a = \frac{V - V_0}{T} \); or \( a = \frac{V}{T} - \frac{V_0}{T} \)
18. \( g = \frac{p}{H_p} \)
19. \( V_0 = V - 0.6t_c \)
20. \( n_2 = N + n_1 \)
21. \( y = -2x + 4 \)
22. \( y = -3x + 15 \)
23. \( y = x - 7 \)
24. \( y = 4x + 2 \)
25. \( y = -\frac{3}{4}x + \frac{5}{4} \)
26. \( y = 6 \)
27. \( y = 2x - 3 \)
28. \( y = 3x + \frac{4}{5} \)
29. \( y = \frac{1}{2}x \)
30. \( y = 5x - 4 \)
Section 3.4  Geometry Application Problems

1. You may find the perimeter of any polygon by adding up the lengths of all sides.
2. \( A = lw \)
3. \( A = \frac{1}{2}bh \)
4. \( A = \pi r^2 \)
5. 90°
6. 180°
7. 180°
8. An isosceles triangle has two sides of the same length, as well as two angles that measure the same.
9. 360°
10. \( l = 27 \text{ in} \), \( w = 51 \text{ in} \)
11. 27.5°, 42.5°, 110°
12. 43.5°, 63.5°, 73°
13. 11 feet, 11 feet, 8 feet
14. 47°, 47°, 86°
15. 94°, 94°, 86°, 86°
16. \( w = 20 \text{ ft} \), \( l = 30 \text{ ft} \)
17. \( A = 156° \), \( B = 24° \)
18. \( A = 35° \), \( B = 55° \)
19. \( w = 6 \text{ ft} \), \( h = 4 \text{ ft} \)
Section 3.5  General Application Problems of Linear Equations in One Variable

1. 33 respondents identified as Latino; 258 respondents identified as Caucasian
2. 158; 160
3. $70,400 was the median family income in California, $85,800 was the median family income in Santa Cruz County
4. $50
5. $3,032,723
6. the average income of the poorest 10 percent of the population is $4640; the average income of the richest 10 percent of the population is $41,760
7. the average income of the poorest 10 percent of Americans was $12,000; the average income of the richest 10 percent of Americans was $138,000
8. $33.24
9. the unemployment rate of a person with a Bachelor’s degree was 5.4%; the unemployment rate of a person with a high school diploma was 10.3%
10. 53 respondents identified as Latino; 43 respondents identified as Caucasian
11. 189; 191
12. the Santa Cruz County median home sales price in 2007 was $572,00
13. Nicole: $21; Martita: $54
14. the weekly earnings of a person with a high school diploma were $626; the earnings of a person with a Master’s degree were $1272
15. 155 respondents earned $65,500 per year or more; 132 respondents earned less than $35,000 per year
16. 8; 18
17. $4.29 per gallon
18. 53 hours
19. In about 3.9 years, or January 2014
20. 80 overage minutes
21. $2344 per month in 2008; $2719 per month in 2012
Section 3.6  Distance, Rate, Time Application Problems

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. To find a distance, multiply rate and time.
5. This answer should be in your own words.
6. \( \frac{2}{3} \) mile per minute; 40 miles per hour
7. 4 miles
8. 5 \( \frac{1}{2} \) hours
9. \( \frac{1}{8} \) hour or 7.5 minutes
10. \( \frac{1}{2} \) or 0.5 hour
11. southbound car: 55.5 mph; northbound car: 74.5 mph
12. 1.75 hours
13. Greta: 2.5 mph; John: 25 mph
14. 0.1 hour or 6 minutes
15. 68.5 meters per minute; extra: 1.43 minutes
Section 4.4 Equations of Lines

1. The **slope-intercept form** of a linear equation is \( y = mx + b \).
2. The **point-slope form** of a linear equation is \( y - y_1 = m(x - x_1) \).
3. This answer should be in your own words.
4. This answer should be in your own words.
5. The slope of a horizontal line is 0. The general form of the equation of a horizontal line is \( y = a \) constant.
6. The slope of a vertical line is undefined. The general form of the equation of a vertical line is \( x = a \) constant.
7. If you plan to use slope-intercept form of a linear equation to write the equation of a line, you must find the slope and the y-intercept of that line before you write the equation.
8. If you plan to use point-slope form of a linear equation to write the equation of a line, you must find the slope and a point on that line before you write the equation.
9. slope: \( -2 \); y-intercept: \( (0, -7) \)
10. slope: 1; y-intercept: \( (0, -3) \)
11. slope: 5; y-intercept: \( (0, \frac{1}{2}) \)
12. slope: \(-3\); y-intercept: \( (0, 11) \)
13. slope: \(-5\); y-intercept: \( (0, 15) \)
14. slope: \(-\frac{1}{2}\); y-intercept: \( (0, \frac{1}{2}) \)
15. slope: 0; y-intercept: \( (0, -3) \)
16. slope: 7; y-intercept: \( (0, 6) \)
17. slope: \(-3\); y-intercept: \( (0, -14) \)
18. x-intercept: \( (6, 0) \); y-intercept: \( (0, 3) \); slope: \(-\frac{1}{2}\); equation: \( y = -\frac{1}{2}x + 3 \)
19. x-intercept: \( (3, 0) \); y-intercept: \( (0, -5) \); slope: \( \frac{5}{3} \); equation: \( y = \frac{5}{3}x - 5 \)
20. x-intercept: \( \text{none} \); y-intercept: \( (0, 5) \); slope: 0; equation: \( y = 5 \)
21. x-intercept: \( (-\frac{5}{2}, 0) \); y-intercept: \( (0, 5) \); slope: \(-2\); equation: \( y = -2x + 5 \)
22. x-intercept: \( \left(\frac{9}{2}, 0\right) \); y-intercept: \( \text{none} \); slope: undefined; equation: \( x = \frac{9}{2} \)
23. x-intercept: \( (60, 0) \); y-intercept: \( (0, 40) \); slope: \(-\frac{2}{3}\); equation: \( y = -\frac{2}{3}x + 40 \)
24. \( y = 2x - 4 \)
25. \( y = \frac{1}{2}x + 1 \)

26. \( y = -x + 3 \)

27. \( y = -\frac{2}{3}x \)

28. \( y = 3 \)
29. \(8 = -2x\)

30. \(3x + y = 2\)

31. \(y = x\)

32. \(2x - 3y = 2\)
33. \[ y = \frac{5}{2} \]

34. \[ 40x + 30y = 120 \]

35. \[ x - 5y = 2 \]

36. \[ 2x + 2y = -1 \]
37. \( x = -4y + 2 \)

38. \( 8 - 3x = y \)

39. \( 9 - 3y = 5x \)

40. \( \frac{1}{2} x - y = \frac{2}{3} \)
41. \[ 1 + \frac{3}{2} y = \frac{6}{3} x \]

42. \[ y = \frac{3}{4} x + 6 \]
43. \[ y = -3x + \frac{1}{2} \]
44. \[ y = x - 3 \]
45. \[ y = 2 \]
46. \[ x = -4 \]
47. \[ y = 4x - 9 \]
48. \[ y = -\frac{1}{2} x + 2 \]
49. \[ y = -\frac{3}{2} x + 9 \]
50. \[ y = 3x - 2 \]
51. \[ y = \frac{1}{3} x - 4 \]
52. \[ y = x + 5 \]
53. \[ y = 2 \]
54. \[ y = -\frac{1}{2} x + \frac{15}{2} \]
55. \[ y = 4x + 1 \]
56. \[ x = -13 \]
57. \[ y = 5x - 4 \]
58. \[ y = -3x + 1 \]
59. \[ y = 3x + 1 \]
60. \[ y = x + 9 \]
61. \[ y = -\frac{1}{2} x - 10 \]
62. \[ y = 2x - 17 \]
63. \[ y = -\frac{1}{2} x - 13 \]
64. \[ y = \frac{1}{2} x - 11 \]
65. \[ y = -\frac{1}{2} x - 13 \]
66. \[ y = -\frac{1}{2} x + \frac{1}{2} \]
67. \[ y = \frac{3}{2} x + 11 \]
68. \[ y = x - 1 \]
Section 4.5  Application Problems of Linear Equations in Two-Variables

1. 5 feet per second
2. 1 dollar per 2 tickets
3. decreasing 4 miles per 5 hours, or 0.8 mile per hour
4. 2 adults per 5 children
5. a) $y = 0.5x$; b) 90 tickets
6. a) $C = \frac{1}{4}m + 20$; b) $27$; c) 36 miles
7. a) $C = 4T - 160$; b) $180$ chirps per minute; c) $60\text{°F}$
8. a) $N = -\frac{25}{3}p + 175$; b) 100 necklaces; c) $900$; d) $6$ each; e) $750$; f) 75 necklaces for $12$ each
9. a) $T = -\frac{1}{400}a + 95$; b) $75\text{°F}$; c) 2,000 feet
Chapter 5 — Answers

Section 5.1
Section 5.2
Section 5.3
Section 5.4
Section 5.1  Solving Systems of Linear Equations by Graphing

1. This answer should be in your own words.
2. This answer should be in your own words.
3. The three different types of systems of linear equations in two variables are consistent, inconsistent, and dependent.
4. When asked to solve a system of linear equations in two variables, the three appropriate types of answers are a specific point of intersection, “no solution”, or “all points on the line (with the specific line specified)”.
5. A **consistent** system of linear equations in two variables has one solution.

6. An **inconsistent** system of linear equations in two variables has no solution.

7. A **dependent** system of linear equations in two variables has infinitely many solutions (all points on that line).

8. This answer should be in your own words.
9. This answer should be in your own words.
10. This answer should be in your own words.
11. \( (2, 6) \) is not a solution to the system.
12. \( (-3, 4) \) is a solution to the system.
13. \( \left( \frac{1}{2}, 5 \right) \) is a solution to the system.
14. \( (0, 8) \) is **not** a solution to the system.
15. \( (-2, 4) \)
16. no solution
17. \( (3, -5) \)
18. all points on the line \( y = -\frac{1}{2}x + 3 \)
19. no solution
20. solution: (1, 1)

21. solution: (2, 4)

22. solution: (−3, 2)

23. no solution

24. solution: (4, 3)
25. \( \text{solution: } (-4, -3) \)

26. \( \text{solution: } (3, -4) \)

27. \( \text{all points on the line } y = -2x + 4 \)

28. \( \text{solution: } (-2, -5) \)

29. \( \text{no solution} \)
30. Solution: \((-1, -3)\)

31. no solution
32. one solution
33. one solution
34. all points on the line \(6y = 8x + 6\)
35. no solution
Section 5.2   The Substitution Method

1. The three different types of systems of linear equations in two variables are consistent, inconsistent, and dependent.
2. When asked to solve a system of linear equations in two variables, the three appropriate types of answers are a specific point of intersection, “no solution”, or “all points on the line (with the specific line specified)”.
3. When asked to solve a system of linear equations in two variables by the Substitution Method, look for the variable with a coefficient of 1 or –1. If no variable have a coefficient of 1 or –1, then choose the easiest variable to isolate.
4. When there is no solution to a system of linear equations in two variables, the last step of the Substitution Method is a contradiction. For example; 2 = 3 or 0 = –4.
5. When there are an infinite number of solutions to a system of linear equations in two variables, the last step of the Substitution Method is an identity. For example; 2 = 2 or –5 = –5.

6. (–3, 5)
7. (–1, 4)
8. (0, –7)
9. no solution
10. (8, 3)
11. (7, –2)
12. all points on the line  – x + 2y = –3
13. (6, 5)
14. all points on the line  3x = 6y + 9
15. (10, –3)
16. (2, –1)
17. no solution
18. (5, 3/2)
19. (50, 60)
20. (20, –30)
21. (–5, –3)
Section 5.3  The Addition/Elimination Method

1. The three different types of systems of linear equations in two variables are consistent, inconsistent, and dependent.
2. When asked to solve a system of linear equations in two variables, the three appropriate types of answers are a specific point of intersection, “no solution”, or “all points on the line (with the specific line specified)”.
3. When asked to solve a system of linear equations in two variables by the Addition/Elimination Method, the first thing to check before you begin is if the equations are “lined-up” (if the like terms and the equal signs are in the same corresponding places in each equation).
4. When asked to solve a system of linear equations in two variables by the Addition/Elimination Method, look to eliminate a variable with a coefficient of 1 or $-1$. If no variable have a coefficient of 1 or $-1$, then choose the easiest to eliminate.
5. When there is no solution to a system of linear equations in two variables, the last step of the Addition/Elimination Method is a contradiction. For example; $0 = -3$ or $0 = 18$.
6. When there are an infinite number of solutions to a system of linear equations in two variables, the last step of the Addition/Elimination Method is an identity. For example; $0 = 0$.

7. $(2, -5)$
8. $(1, 8)$
9. $(2, 3)$
10. $(0, 4)$
11. $(4, 5)$
12. no solution
13. all points on the line $4x = 26 - 5y$
14. $(7, 4)$
15. $(2, -1)$
16. all points on the line $2x - 8y = 6$
17. $\left(\frac{1}{10}, -\frac{7}{15}\right)$
18. no solution
19. $(8, -24)$
20. $(0, 20)$
21. $(10, 15)$
22. $(3, 2)$
Section 5.4  Application Problems of Systems of Linear Equations

1. This answer should be in your own words.
2. If you plan to use a chart to organize the information in a mixture-type of “system of linear equations application problem”, will need a chart with three rows and three columns (no including labels/titles). The rows will be labeled: item or person #1, item or person #2, and “mix”. The columns will be labeled: rate/percent, individual amount, and total price/concentration.
3. 2 pounds of strawberries; 1 pound of watermelon
4. 4 gallons of B-99; 8 gallons of B-80
5. 1.05 liters of solution containing 15% of hydrogen chloride; 0.45 liters of solution containing 25% hydrogen chloride
6. $2500 at 4% simple interest; $1500 at 3% simple interest
7. 87.5 mL of household bleach that is 2% sodium hypochlorite; 37.5 mL of chlorination bleach that is 12% sodium hypochlorite
8. 4 drinks; 6 raffle tickets
9. 24 pounds of amaretto cordials; 36 pounds of mixture
10. 40% of pure juice in the can of concentrate
Chapter 6 — Answers

Section 6.1
Section 6.2
Section 6.3
Section 6.4
Section 6.5
Section 6.6
Section 6.1 Exponents

1. When multiplying like-bases, you can add the exponents to simplify the expression.
2. When dividing like-bases, you can subtract the exponents to simplify the expression.
3. In a simplification of a quotient of like-bases, if there are more factors of a like-base in the numerator, the simplified answer will have that difference of factors in the numerator, and if there are more factors of a like-base in the denominator, the simplified answer will have that difference of factors in the denominator.
4. When raising a power to a power, you can multiply the exponents to simplify the expression.
5. When simplifying a quotient that is raised to a power, it is often easiest to reduce/simplify the quotient/fraction before applying the “outside” power.
6. If an expression is raised to the 0 power, it’s value is 1.
7. a) $9^9$
b) $9^9$
c) $-9$
d) $-27$
e) $-27$
8. a) $-16$
b) $-64$
c) $-64$
d) $16$
e) $16$
9. $x^9$
10. $y^2$
11. $\frac{1}{z^3}$
12. $x^{20}$
13. $1$
14. $7^6$
15. $1$
16. $x^9$
17. $4^7$
18. $\frac{1}{z^{11}}$
19. $(-2)^{40}$ or $2^{40}$
20. $y^5$
21. $x^{347}$
22. $y^{690}$
23. $\frac{1}{c^{343}}$
24. $d^{343}$
25. $x^{10}y^3$
26. $\frac{y^2}{x^4}$
27. $y^6x^{24}$
28. $w^{11}$
29. $-3\alpha^3$\[\frac{2}{2}$
30. $x^{19}y^8$
31. $1$
32. $8\alpha^{12}$
33. $x^3y^8$
34. $9x^2y^{10}$
35. $\frac{3d^6}{e^1}$
36. $\frac{q^7}{2}$
37. $\frac{1}{4x^4}$
38. $-3y^9$
39. $\frac{1}{4n^{12}}$
40. $4\rho^7$
41. $\frac{16x^{12}}{y^8}$
42. $\frac{m^2}{4n^{10}}$
43. $-8x^6$
44. $\frac{100y^2}{81}$
45. $\frac{1}{32q^{20}}$
46. $1$
47. $\frac{1}{125m^{24}}$
48. $\frac{121}{16a^2c^{14}}$
49. $20x^5$
50. $-24y^{14}$
51. $1250m^{12}$
52. $243x^{19}y^9$
53. $9x^2y^{16}$
54. $-72n^{42}$
55. $256y^8z^{11}$
Section 6.2  Negative Exponents

1. This answer should be in your own words.
2. This answer should be in your own words.
3. a) \( \frac{1}{25} \)
   b) \( \frac{-1}{125} \)
   c) \( \frac{-1}{125} \)
   d) \( \frac{1}{25} \)
   e) \( \frac{-1}{25} \)
4. a) \( \frac{-1}{64} \)
   b) \( \frac{-1}{16} \)
   c) \( \frac{1}{16} \)
   d) \( \frac{1}{16} \)
   e) \( \frac{-1}{64} \)
5. \( \frac{1}{x^4} \)
6. \( \frac{1}{32} \)
7. \( x^3 \)
8. \( -8 \)
9. \( xy^3 \)
10. \( \frac{n^5}{m^2} \)
11. \( \frac{1}{x} \)
12. \( y^8 \)
13. \( \frac{1}{x^{19}} \)
14. \( \frac{1}{x^{12}} \)
15. \( 6^5 \)
16. \( 1 \)
17. \( \frac{1}{x^{15}} \)
18. \( 4^6 \)
19. \( 2^{11} \)
20. \( 5^{40} \)
21. \( \frac{1}{y^9} \)
22. \( \frac{x^{15}}{y^3} \)
23. \( \frac{x^8}{y^{12}} \)
24. \( \frac{1}{y^6 x^{15}} \)
25. \( w^8 y^{10} \)
26. \( -\frac{7x^{16}}{2} \)
27. \( \frac{x^{16}}{y^3} \)
28. 1
29. \( -\frac{1}{8x^{12}} \)
30. \( -\frac{x^3}{y^8} \)
31. \( \frac{25x^2}{y^6} \)
32. \( -4e^{15} d^{15} \)
33. \( -\frac{2}{a^9} \)
34. \( -\frac{1}{5x^{12}} \)
35. \( -2y^{10} \)
36. \( \frac{n^8}{3} \)
37. \( \frac{4}{p^5 q^{16}} \)
38. \( \frac{16}{x^{20} y^{12}} \)
39. \( \frac{9}{m^2 n^{14}} \)
40. \( \frac{1}{8x^{18}} \)
41. \( \frac{9y^{10}}{25} \)
42. \( \frac{p^{140}}{32 q^{25}} \)
43. 1
44. \( \frac{125}{m^{33}} \)
45. \( \frac{121}{36 a^{30} c^{24}} \)
46. \( \frac{15}{x^5} \)
47. \[-\frac{5}{8y^{10}}\]
48. \[-\frac{80}{m^{9}}\]
49. \[-\frac{x^{22}y}{27}\]
50. \[-\frac{y^{18}}{9x^{2}}\]
51. \[-\frac{4m^{42}}{27}\]
Section 6.3  Polynomials

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. This answer should be in your own words.
5. This answer should be in your own words.
6. \(8x^4 + x^2\)

<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
</tr>
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<td>(8x^4)</td>
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<td>8</td>
<td>8</td>
<td>4</td>
<td>binomial</td>
</tr>
<tr>
<td>(x^2)</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. \(4x^3 - x^{11} + 2x - 17\)

<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x^3)</td>
<td>3</td>
<td>4</td>
<td>-1</td>
<td>11</td>
<td>none</td>
</tr>
<tr>
<td>(-x^{11})</td>
<td>11</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2x)</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-17)</td>
<td>0</td>
<td>-17</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. \(12x^4y + x^2y^2 - 7y^3\)

<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(12x^4y)</td>
<td>5</td>
<td>12</td>
<td></td>
<td>5</td>
<td>trinomial</td>
</tr>
<tr>
<td>(x^2y^2)</td>
<td>4</td>
<td>1</td>
<td>12</td>
<td>5</td>
<td>trinomial</td>
</tr>
<tr>
<td>(-7y^3)</td>
<td>3</td>
<td>-7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. \(y - 4y^2 + 15\)

<table>
<thead>
<tr>
<th>Individual Terms</th>
<th>The Degree of Each Individual Term</th>
<th>The Coefficient of Each Individual Term</th>
<th>The Leading Coefficient of the Polynomial</th>
<th>The Degree of the Polynomial</th>
<th>Specific Name of the Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>1</td>
<td>1</td>
<td>-4</td>
<td>2</td>
<td>trinomial, quadratic</td>
</tr>
<tr>
<td>(-4y^2)</td>
<td>2</td>
<td>-4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(15)</td>
<td>0</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. \(-5y^6 + y^3 - 14y^2\); degree: 6; leading coefficient: -5

11. \(x^{12} + x^{10} - 7x^2 + 12x\); degree: 12; leading coefficient: 1
Section 6.4  Addition and Subtraction of Polynomials

1. This answer should be in your own words.
2. This answer should be in your own words.
3. \(9x\)
4. \(12y\)
5. \(5x-7\)
6. \(10y+8\)
7. \(11x+19\)
8. \(24x^2 + 3x\)
9. \(8x-45xy + y\)
10. \(6x^2 + 17x^2 y - 8xy\)
11. \(14x-6\)
12. \(m^2 + 57m - 60\)
13. \(3m^2 + 8m - 2\)
14. \(10a^2 - 5a - 16\)
15. \(4y^2 - y - 15\)
16. \(4x^2 + 13xy - 23\)
17. \(x^2 - 25x - 14\)
18. \(-2x - 15\)
19. \(-m^2 + 19m - 18\)
20. \(2n^2 + 10n - 20\)
21. \(13a^2 - 8a + 8\)
22. \(-5y^2 + y - 36\)
23. \(x^2 + 36xy - 28\)
24. \(-x^2 - 29x - 53\)
25. \(m^2 - \frac{1}{6} m - \frac{1}{2}\)
26. \(x^2 - \frac{1}{4} x - 18\)
27. \(\frac{1}{2} a^2 - \frac{1}{2} a - \frac{5}{7}\)
28. \(5x^2 + 4x - 11\)
29. \(y^2 + 9y - 5\)
30. \(-6x + 8\)
31. \(3y^2 - 8y\)
32. \(3m^2 + 9m + 9\)
33. \(6x^2 + 30x - 23\)
Section 6.5 Multiplication of Polynomials

1. The Distribute Property is often used when multiplying polynomials.
2. When computing the square of a binomial you must remember to “FOIL”.
3. a) \(x^2 + 6x + 9\)
b) \(9x^2\)
c) \(x^4 - 8x^2 + 16\)
d) \(16x^4\)
e) This answer should be in your own words.

4. \(8x + 28\)
5. \(-5x - 15\)
6. \(7y^2 + 42y + 21\)
7. \(10x - 60\)
8. \(-2x^2 + 10x - 24\)
9. \(4x^2 + 32x\)
10. \(2y^2 - 6y\)
11. \(9x^2 + \frac{1}{2} x\)
12. \(6x^4 + 3x^3 - 18x^2\)
13. \(2y^3 - y^4 + 5y^3 - 11y^2\)
14. \(4x^3 y - 7x^2 y^2 + xy^3\)
15. \(5m^2 n^2 + 2m^2 n^3\)
16. \(y^2 + 14y + 45\)
17. \(d^2 - 15d + 36\)
18. \(x^2 + 4x - 21\)
19. \(k^2 - \frac{5}{4} k + \frac{3}{8}\)
20. \(p^2 - p - 20\)
21. \(x^2 + 7x + 6\)
22. \(3y^2 + 22y + 7\)
23. \(x^2 + 14x + 49\)
24. \(2x^2 + x - 36\)
25. \(4k^4 - 11k^2 + 7\)
26. \(5y^3 - y^2 - 4\)
27. \(x^2 - 6x + 9\)
28. \(p^4 + 9p^2 + 8\)
29. \(3x^4 - 34x^2 - 24\)
30. \(6k^2 - 53k + 110\)
31. \(xy^2 - xy + 3y - 3\)
32. \(m^2 p^2 + 24mp + 140\)
33. \(x^2 + 6xy - 7y^2\)
34. \(10a^2 - 7ac + c^2\)
35. \(12x^2 - 7xy + y^2\)
36. \(4x^2 + 36x + 81\)
37. \(2x^3 - x^2 y + 6xy - 3y^2\)
38. \(4x^2 - 12xy + 9y^2\)
39. \(x^3 + 8x^2 + 25x + 50\)
40. \( y^3 + 3y^2 + 3y - 7 \)
41. \( 2x^3 - 20x^2 + 59x - 66 \)
42. \( p^4 + 6p^3 - 18p^2 + 12p - 40 \)
43. \( 2m^4 + m^3 - 7m^2 - 3m + 3 \)
44. \( 8x^3 + \frac{5}{2}x^2 + \frac{121}{8}x + 6 \)
45. \( x^3 + y^3 \)
46. \( a^3 - b^3 \)
47. \( 2x^3 + 5x^2 y - 11xy^2 + 4y^3 \)
48. a) \( x^2 - 4 \)
   b) \( y^2 - 25 \)
   c) \( p^2 - \frac{1}{9} \)
   d) \( 4y^2 - 25 \)
   e) \( 9a^2 - 1 \)
   f) This answer should be in your own words.
49. a) \( x^2 + 2x + 1 \)
   b) \( y^2 + 6y + 9 \)
   c) \( k^2 + \frac{1}{2}k + \frac{1}{16} \)
   d) \( 4a^2 + 28a + 49 \)
   e) \( 16x^2 + 8x + 1 \)
   f) This answer should be in your own words.
50. a) \( y^2 - 4y + 4 \)
   b) \( x^2 - 10x + 25 \)
   c) \( a^2 - a + \frac{1}{4} \)
   d) \( 4a^2 - 4a + 1 \)
   e) \( 9x^2 - 12x + 4 \)
   f) This answer should be in your own words.
Section 6.6 Division of Polynomials

1. This answer should be in your own words.
2. When you divide a polynomial with \( n \) terms by a monomial, you will have \( n \) terms in your quotient (answer).
3. This answer should be in your own words.
4. \( 2p + 6 \)
5. \(-3y - \frac{12}{7}\)
6. \(n + 7\)
7. \(2x - \frac{11}{4}\)
8. \(-\frac{2}{3}k - 6\)
9. \(-\frac{5}{3}x + 2\)
10. \(6x + 2\)
11. \(4y^2 + 8y - 2\)
12. \(-6p^2 - 3p + 2\)
13. \(-3x^4 + 15x^2 - 5\)
14. \(4k^2 + 5k - \frac{3}{4}\)
15. \(y^4 + 5y^2 - 3y^3 + \frac{5}{2}y\)
16. \(-2m^7 + 7m^6 + 11m^2 - 1\)
17. \(-\frac{1}{2}x^{10} + 5x^8 - 3x^4 - 2x^2\)
18. \(-2p^2 - 3p + 1 - \frac{5}{p}\)
19. \(-3 - \frac{3}{5}x + \frac{1}{3}y\)
20. \(y + 3 + \frac{19}{y+2}\)
21. \(k + 5 + \frac{1}{k+3}\)
22. \(x - 9 + \frac{42}{x+6}\)
23. \(m + 13 + \frac{46}{m-3}\)
24. \(a - 5 + \frac{6}{a+1}\)
25. \(p - 3\)
26. \(3x + 4 + \frac{4}{x+4}\)
27. \(4y + 11 + \frac{2}{y-2}\)
28. \(3k + 7\)
29. \(2x - 5 + \frac{15}{x+1}\)
30. \(4m - 3\)
31. \(k - 1 + \frac{-6}{5k+2}\)
32. \(p^2 + 4p + 3 + \frac{-16}{p+2}\)
33. \(2y^2 + 2y + 12 + \frac{-47}{y+4}\)
34. \(x - 2 + \frac{12}{x+2}\)
35. \(x + 2\)
36. \(5x - 15 + \frac{52}{x+3}\)
37. \(x^2 + 2x + 4\)
Chapter 7 — Answers

Section 7.1
Section 7.2
Section 7.3
Section 7.4
Section 7.5
Section 7.6
Section 7.7
Section 7.8
Section 7.1  Greatest Common Factor

1. In general, to factor an expression means to rewrite the expression as a product (multiplication).
2. This answer should be in your own words.
3. If an expression has no greatest common factor, it may still be factorable by another factoring method.
4. An expression that isn’t factorable is called prime.
5. This answer should be in your own words.
6. $4(x + 3)$
7. $5(2p + 3)$
8. $3(2y + 1)$
9. $6(3x - 1)$
10. $7(3k - 2)$
11. $12(3y - 5)$
12. $44(2n + 1)$
13. prime
14. $27(x + 2)$
15. $2y(y^2 + 2)$
16. $8p^3(2p - 1)$
17. $5x(5x + 9)$
18. $3a^2(5a - 39)$
19. $3y^4(10y^2 - 3)$
20. $11x^2(3x^3 + x - 6)$
21. $7p^2(5p^2 + 4p - 1)$
22. $30xy(2x^2y + 3)$
23. $8yz^2(10y^2 - 9z)$
24. $7a^3c^2\left(10a^3 - 8ac + 13c^2\right)$
25. $13p^2\left(5p^3 + q^2 + 3q^4\right)$
26. $8mn\left(6m^3 - 8m^2n + 1\right)$
27. $x^2y^2\left(9x^2 - 22xy - 18y^2\right)$
28. $13w\left(4w^3z^2 - 2wz^3 + 3\right)$
29. $15xy\left(3x^2 + 4xy - 6y^2\right)$
30. $12ac\left(4a^3 + 2a^2c^2 - c^3\right)$
31. $3x^{\frac{3}{2}}\left(x^{\frac{1}{2}} + 2\right)$
32. $2a^{\frac{5}{2}}\left(7 + a^{\frac{3}{2}}\right)$
33. $4x^{-3}(x + 2)$
34. $5y^{-3}(5 + y)$
35. $(a + 5)(x + 4)$
36. $(z + 1)(y - 7)$
37. $(q - 3)(7p - 2)$
38.  \((c - 4)(3d + 1)\)
39.  \((x + 9)(x + 2)\)
40.  \((m - 3)(m - 1)\)
Section 7.2  Factoring by Grouping

1. In general, to factor an expression means to rewrite the expression as a product (multiplication).
2. The two types of factoring methods covered in Chapter 7 so far are GCF and factoring by grouping.
3. When asked to factor an expression, try factoring by grouping if the expression has four terms.
4. This answer should be in your own words.
5. Always look for a GCF first, even if you eventually use another factoring method.
6. This answer should be in your own words.
7. An expression that isn’t factorable is called prime.
8. \((2x^2 + 5)(3x + 2)\)
9. \((3p^3 + 2)(4p + 5)\)
10. \((5y^2 + 7)(y - 3)\)
11. \((6k^2 - 1)(k^2 + 4)\)
12. prime
13. \((2x^2 - 7)(8x + 3)\)
14. \((a + 1)(c + 2)\)
15. \(2(9w^2 + 5)(w - 4)\)
16. \((9y^2 - 2)(4y + 7)\)
17. \((2x^2 + 1)(11x + 3)\)
18. \((10p - 3)(3p - 8)\)
19. \((9m^2 + 1)(m + 9)\)
20. prime
21. \((3pq - 4)(pq + 2)\)
22. \((2m^2 - 1)(13m + 1)\)
23. \(3y(2x - 1)(x + 10)\)
24. \(3a(a + 8)(a + 7)\)
25. \(x(xy^2 + xy + 2x + 3)\)
26. prime
27. \((2y^2 - 3)(4y - 1)\)
28. \(x^2(3x^2 - 1)(x + 1)\)
Section 7.3  Factoring Trinomials with a Leading Coefficient of 1

1. In general, to factor an expression means to rewrite the expression as a product (multiplication).
2. The three types of factoring methods covered in Chapter 7 so far are GCF, factoring by grouping, and “un-foiling”.
3. When asked to factor an expression, try un-foiling if the expression has three terms.
4. Always look for a GCF first, even if you eventually use another factoring method.
5. This answer should be in your own words.
6. An expression that isn’t factorable is called prime.
7. This answer should be in your own words.
8. 
   a) \((x + 4)(x + 5)\)
   b) \((x - 4)(x - 5)\)
   c) \((x - 5)(x + 4)\)
   d) \((x + 5)(x - 4)\)
   e) This answer should be in your own words.
   f) This answer should be in your own words.
9. \((x + 5)(x + 10)\)
10. \((p + 1)(p + 11)\)
11. \((y - 3)(y - 7)\)
12. \((a - 2)(a - 4)\)
13. \((m - 8)(m + 5)\)
14. \((p - 9)(p + 7)\)
15. \((x + 14)(x - 2)\)
16. \((a + 6)(a - 3)\)
17. prime
18. \((n + 8)(n - 4)\)
19. \((x - 6)(x + 1)\)
20. \((p + 4)(p + 13)\)
21. \((y - 10)(y + 3)\)
22. prime
23. \((cd - 6)(cd - 7)\)
24. \((xy + 6)(xy - 4)\)
25. \((p - 3q)(p - 4q)\)
26. \((mn - 6)(mn + 5)\)
27. \((x + 5y)(x + 12y)\)
28. prime
29. \((x + y)(x + y)\) or \((x + y)^2\)
30. \(2(q + 4)(q + 9)\)
31. \(5(a^2 - 3a - 12)\)
32. \(3(n - 2)(n - 5)\)
33. \(4(xy - 2)(xy - 6)\)
34. \(q(p + 8)(p - 3)\)
35. \(2c(c - 9d)(c + 2d)\)
36. \(xy(xy - 15)(xy - 2)\)
37. \((a^3 - 3)(a^3 - 11)\)
38. \((z^4 - 7)(z^4 + 2)\)

39.

a) \((w + 5)^2\)

b) \((y + 7)^2\)

c) \((m + 3)^2\)

d) This answer should be in your own words.

e) \((x + \frac{1}{4})^2\)

40.

a) \((p - 4)^2\)

b) \((n - 6)^2\)

c) \((a - 8)^2\)

d) This answer should be in your own words.

e) \((x - \frac{1}{2})^2\)
Section 7.4  Factoring a Difference of Squares

1. In general, to factor an expression means to rewrite the expression as a product (multiplication).
2. The three types of factoring methods covered in Chapter 7 so far are GCF, factoring by grouping, and “un-foiling”.
3. When asked to factor an expression, look for a difference of squares if the expression has two terms.
4. Always look for a GCF first, even if you eventually use another factoring method.
5. This answer should be in your own words.
6. An expression that isn’t factorable is called prime.
7. This answer should be in your own words.
8. The expression $x^2 + 9$ is prime. The explanation should be in your own words.
9. The expression $4x^2 + 16$ is factorable to $4(x^2 + 4)$. The explanation should be in your own words.
10. If an expression has two terms and it is not a difference of squares, it may be factored. The explanation should be in your own words.

11. $(x + 5)(x - 5)$
12. $(a + 11)(a - 11)$
13. $(p + 9)(p - 9)$
14. $(y + 1)(y - 1)$
15. prime
16. $(k + 14)(k - 14)$
17. $(2c + 3)(2c - 3)$
18. $(x + y)(x - y)$
19. $(9w + 1)(9w - 1)$
20. $(7d + 4)(7d - 4)$
21. $(6 + n)(6 - n)$
22. $(st + 8)(st - 8)$
23. $(10w + 13z)(10w - 13z)$
24. $(y^2 + 12)(y^2 - 12)$
25. $(15 + x^3)(15 - x^3)$
26. $(p + \frac{1}{2})(p - \frac{1}{2})$
27. $5(a + 4)(a - 4)$
28. $3(m + 11)(m - 11)$
29. $x(x + 14)(x - 14)$
30. $(v + \frac{3}{4})(v - \frac{3}{4})$
31. $2(z^2 + 49)$
32. $25(q + 3)(q - 3)$
33. $4(2c + 1)(2c - 1)$
34. $9a^2(4a + 1)(4a - 1)$
35. $8(y + 1)(y - 1)$
36. $(x^2 + 4)(x + 2)(x - 2)$
37. $12(k^2 + 1)(k + 1)(k - 1)$
Section 7.5  Factoring Trinomials with a non-1 Leading Coefficient

1. In general, to factor an expression means to rewrite the expression as a product (multiplication).
2. The three types of factoring methods covered in Chapter 7 so far are GCF, factoring by grouping, and “un-foiling” for a trinomial or a binomial difference of squares.
3. When asked to factor an expression, try un-foiling or a-c grouping if the expression has three terms.
4. Always look for a GCF first, even if you eventually use another factoring method.
5. This answer should be in your own words.
6. An expression that isn’t factorable is called prime.
7. This answer should be in your own words.
8. The expression \(2x^2 + 3x + 1\) is factorable into \((2x+1)(x+1)\). The explanation should be in your own words.
9.
   a) \((2x+1)(x+7)\)
   b) \((2x-1)(x-7)\)
   c) \((2x+7)(x+1)\)
   d) \((2x-7)(x-1)\)
   e) \((2x-1)(x+7)\)
   f) \((2x+1)(x-7)\)
   g) \((2x+7)(x-1)\)
   h) \((2x-7)(x+1)\)
10. \((2y+3)(y+1)\)
11. \((3m+1)(m+5)\)
12. \((5x-7)(x-2)\)
13. \((2a+11)(3a+1)\)
14. \((6p-11)(p-1)\)
15. \((4d+3)(d-4)\)
16. \((2x+3)(2x-5)\)
17. prime
18. \((2q-3)(4q-3)\)
19. \((3c+8)^2\)
20. \((2x-7)(x+3)\)
21. \((10y-1)(y+16)\)
22. \((9n+2)(n-4)\)
23. \((6a-5)^2\)
24. \((3m-2)(7m+8)\)
25. \((2x+3y)(x+5y)\)
26. \((3a-d)^2\)
27. \((12pq+5)(pq+1)\)
28. \((3cd+2)(5cd-6)\)
29. \(5(10n+3)^2\)
30. \(q(4q-3)(5q+6)\)
31. \(2k(2k-3)(7k-3)\)
32. \(6(2x+5y)(3x+8y)\)
Section 7.6 General Factoring

1. In general, to factor an expression means to rewrite the expression as a product (multiplication).
2. The factoring methods covered in Chapter 7 so far are GCF, factoring by grouping, and “un-foiling” or a-c grouping for a trinomial or a binomial difference of squares.
3. Always look for a GCF first, even if you eventually use another factoring method.
4. This answer should be in your own words.
5. An expression that isn’t factorable is called prime.
6. \((t + 10)(t - 10)\)
7. \(4x(x + 3)\)
8. \((y - 6)(y + 5)\)
9. \((p^2 - 7)(p + 3)\)
10. \((6m + 5)(3m - 2)\)
11. \(3(a - 6)(a - 9)\)
12. prime
13. \(5(x+5)(x-5)\)
14. \(2y(2y-9)(3y+1)\)
15. \(n(5n^2 + 3)(n + 1)\)
16. \(3(15+q)(15-q)\)
17. \(t(t - 144)\)
18. \((a + 5)(a + 2)(a - 2)\)
19. \(4(c - 3d)(c - 5d)\)
20. \(8(x^2 + 1)(x + 1)(x - 1)\)
21. \((z^2 + 13)(z + 1)(z - 1)\)
22. \(m(8m + 11)^2\)
23. \((9p + 4q)(9p - 4q)\)
24. \(a \cdot c(7ac - 4)(2ac - 3)\)
25. \(6y^2(3y^2 - 8)(y + 5)\)
26. prime
27. \(10k^5(k - 8)(k - 1)\)
28. \((2w + 1)(z + 6)(z - 6)\)
29. \((x^2 + y^2)(x + y)(x - y)\)
30. \(3m(10m + 9)(m - 5)\)
31. \(2(p^2 + 1)(p + 7)(p - 7)\)
32. \(5(8a + 3)(2a + 1)(2a - 1)\)
33. \(2xy(9x - 2y)(x - 6y)\)
Section 7.7  Solving Quadratic Equations by Factoring

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. \( x = 6 \) or \( x = -10 \)
5. \( p = 7 \) or \( p = -4 \)
6. \( y = 3 \) or \( y = 12 \)
7. \( q = 7 \) or \( q = 8 \)
8. \( n = -\frac{5}{2} \) or \( n = -1 \)
9. \( a = 5 \) or \( a = -4 \)
10. \( x = 0 \) or \( x = 2 \)
11. \( m = -8 \) or \( m = -9 \)
12. \( k = -\frac{8}{3} \) or \( k = 5 \)
13. \( y = 5 \)
14. \( a = \pm 3 \)
15. \( p = 0 \) or \( p = 5 \)
16. \( n = -7 \) or \( n = -5 \)
17. \( x = 6 \) or \( x = 13 \)
18. \( q = 0 \) or \( q = 3 \)
19. \( k = \frac{10}{3} \) or \( k = 6 \)
20. \( m = -1 \) or \( m = -9 \)
21. \( x = -7 \) or \( x = 2 \)
22. \( y = -\frac{5}{4} \) or \( y = 8 \)
23. \( p = -12 \) or \( p = -4 \)
24. \( a = 11 \)
25. \( z = 1 \) or \( z = -7 \)
26. \( k = 6 \) or \( k = -8 \)
27. \( x = \frac{3}{10} \) or \( x = -5 \)
28. \( n = -\frac{7}{4} \) or \( n = 1 \)
29. \( y = 9 \) or \( y = -3 \)
30. \( q = 4 \) or \( q = 5 \)
Section 7.8  General Application Problems Involving Quadratic Equations

1. \(x = 13\) cm
2. \(x = 4\) feet
3. \(x = 10\) inches
4. 8 inches; 6 inches
5. 13 feet
6. 16 and 18
7. 5 and 7
8. –8 and –12; 8 and 12
9. –3 and –4; 2 and 6
10. width = 8 inches; length = 11 inches
Chapter 8 — Answers

Section 8.1
Section 8.2
Section 8.3
Section 8.4
Section 8.5
Section 8.6
Section 8.1 Fraction Review

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. This answer should be in your own words.
5. \( \frac{5}{11} \)
6. \( \frac{4x}{3} \)
7. \( \frac{6}{7} \)
8. \( \frac{4}{7} \)
9. \( \frac{11}{8} \)
10. \( \frac{9}{2} \)
11. \( \frac{2}{15} \)
12. 1
13. \( \frac{19}{30} \)
14. 18
15. \( \frac{30}{17} \)
16. \( \frac{-61}{14} \) or \( -4 \frac{5}{14} \)
17. \( \frac{43}{56} \)
18. \( \frac{1}{7} \)
19. \( \frac{37}{35} \)
20. \( \frac{25}{39} \)
Section 8.2  Simplifying Rational Expressions

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. This answer should be in your own words.
5. This answer should be in your own words.
6. This answer should be in your own words.
7. The factoring “methods” are GCF, grouping, and un-foiling/a-c grouping for a trinomial or difference of squares.
8. a) \( \frac{1}{3} \)
   b) \( \frac{x+2}{x+6} \) does not simplify because there are no common FACTORS and all like-terms are combined.
   c) \( \frac{2x+1}{x+3} \)
   d) \( x - 2 \)
   e) \( \frac{-x}{x+3} \)
9. a) \(-1\)
   b) \(-1\)
   c) \( \frac{x+3}{x-3} \) does not simplify because there are no common FACTORS and all like-terms are combined
   d) \(-1\)
   e) \(-1\)
10. \( \frac{5}{11x} \)
11. \( 12p^4 \)
12. \( \frac{1}{2m} \)
13. \( \frac{x+6}{3} \)
14. \( 2y - 1 \)
15. \( \frac{x+2}{2} \)
16. \( \frac{a-14}{a+5} \)
17. \( w - 2 \)
18. \( x + 8 \)
19. \( \frac{z+6}{z-2} \)
20. \( \frac{2}{m+3} \)
21. \( \frac{5(p+5)}{4(p+10)} \)
22. \( -\frac{a+2}{2} \)
23. \( \frac{2d+4}{d+8} \)
24. \( \frac{1}{2(n-3)} \)
25. \( \frac{y-2}{3-6} \)
26. \( \frac{2x-7}{x-12} \)
27. \( \frac{q-5}{q+8} \)
28. \( \frac{2m-3}{2} \)
29. \( \frac{1}{2(k-5)} \)
30. \( \frac{2z+7}{k+7} \)
31. \( \frac{-(2a+3d)}{8} \)
32. \( \frac{2p^2-3}{p-5} \)
33. \( \frac{2mn+5}{2n(7m-1)} \)
Section 8.3 Multiplication and Division of Rational Expressions

1. This answer should be in your own words.
2. This answer should be in your own words.
3. When dividing two rational expressions you may NOT cancel “right away”.
4. This answer should be in your own words.
5. The factoring “methods” are GCF, grouping, and un-foiling/a-c grouping for a trinomial or difference of squares.
6. This answer should be in your own words.
7. When all the factors cancel in a multiplication or division problem, the answer is 1.
8. \(4xy^4\)
9. \(\frac{1}{20}\)
10. \(\frac{1}{x}\)
11. \(\frac{a-4}{a-7}\)
12. 1
13. \(\frac{2z+1}{3}\)
14. \(\frac{3y^2}{2}\)
15. \(\frac{1}{3c^2d^2}\)
16. \(\frac{2a}{3}\)
17. \(\frac{a-2}{2}\)
18. 4
19. 1
20. \(\frac{1}{2}\)
21. \(\frac{x}{3(x+5)}\)
22. \(9(3y-5)\)
23. 1
24. \(\frac{a^2(a-c)}{c^2(a+c)}\)
25. 2
26. \(\frac{4z+3}{2(z-12)}\)
27. \(\frac{-(6n+1)}{w+3}\)
28. \(\frac{2m+8}{6n}\)
29. \(-1\)
30. \(\frac{-(a+9)}{2(a-9)}\)
31. \(\frac{-2p^3}{3p+5q}\)
Section 8.4  Addition and Subtraction of Rational Expressions

1. When adding or subtracting two rational expressions, one CANNOT cancel a numerator of one expression with the denominator of the other.
2. When adding or subtracting two rational expressions, the expressions must have a least common denominator.
3. When multiplying or dividing two rational expressions, the expressions do NOT need to have a least common denominator.
4. This answer should be in your own words.
5. This answer should be in your own words.
6. When all terms cancel in the numerator of an addition or subtraction problem, the answer is 0.
7. \( \frac{2}{7} \)
8. \( \frac{x+3}{x+2} \)
9. \( \frac{a-4}{a+5} \)
10. \( \frac{2}{x} \)
11. \( \frac{-p+9}{3p} \)
12. \( \frac{1}{a+2} \)
13. \( \frac{5}{c^2-4} \)
14. \( \frac{1}{x+8} \)
15. \( \frac{x^2+18x+1}{(y+6)(y-2)} \)
16. \( \frac{a-3}{a-1} \)
17. \( \frac{q+2}{q-7} \)
18. \( \frac{k-1}{2k-3} \)
19. \( \frac{-c-2}{(3c+1)(c+5)} \)
20. \( \frac{2(d+1)}{4d+3} \)
21. \( \frac{m-1}{2m} \)
22. \( \frac{-1}{x-5} \)
23. \( \frac{y+4}{10-y} \)
24. \( \frac{-7}{q+2} \)
25. \( \frac{7}{3c} \)
26. \( \frac{5x+3}{c^2} \)
27. \( \frac{21p-5}{30p^2} \)
28. \( \frac{t^2+3k+3}{4(k+1)} \)
29. \( \frac{3a^2-2a-2}{a(a-2)} \)
30. \( \frac{7n+23}{(n+1)(n+5)} \)
31. \( \frac{11d-2}{2d(d+2)} \)
32. \( \frac{-2m-1}{m(m+3)} \)
33. \( \frac{c^2+c+3}{(c+1)(c-1)} \)
34. \( \frac{4q+15}{q(q+5)} \)
35. \(\frac{-11z+47}{4z(z-3)}\)
36. \(\frac{y^2-10y+10}{6y(y-1)}\)
37. \(\frac{13x-31}{(x+2)(x-2)(x-3)}\)
38. \(\frac{-7k-4}{(2k+3)(k-6)(k+3)}\)
39. \(\frac{m^2+m-16}{(m+3)(m-3)(m+8)}\)
40. \(\frac{3d-1}{(d+1)(d-1)}\)
41. \(\frac{3a+2}{3a(a-2)}\)
42. \(\frac{n+5}{(n+6)(n+3)}\)
43. \(\frac{3p-4}{(p+2)(p-2)}\)
44. \(\frac{c+3}{(2c-1)(c-4)}\)
45. \(\frac{1}{(z+4)(z+1)}\)
46. \(\frac{2(5z^2+12z-15)}{(z+3)(z-3)(z+5)}\)
Section 8.5  Rational Equations

1. This answer should be in your own words.
2. This answer should be in your own words.
3. The denominator of a rational expression CANNOT be 0.
4. This answer should be in your own words.
5. This answer should be in your own words.
6. Yes, it is important to check your answers after solving an equation with rational expressions. The explanation should be in your own words.

Find the value(s) of the variable for which the expression is undefined, and then write a phrase that gives all values for which the expression is defined.

7. \( x \neq -2 \); the expression is defined for all real numbers except \(-2\).
8. \( p \neq 4 \); the expression is defined for all real numbers except \(4\).
9. \( y \neq 3 \); the expression is defined for all real numbers except \(3\).
10. \( m \neq 0 \); the expression is defined for all real numbers except \(0\).
11. \( a \neq -1 \text{ and } a \neq -9 \); the expression is defined for all real numbers except \(-1\) and \(-9\).
12. \( n \neq -3 \text{ and } n \neq -8 \); the expression is defined for all real numbers except \(-3\) and \(-8\).
13. \( d \neq \pm 2 \); the expression is defined for all real numbers except \(-2\) and \(2\).
14. \( z \neq 0 \text{ and } z \neq 9 \); the expression is defined for all real numbers except \(0\) and \(9\).
15. \( c \neq -7 \text{ and } c \neq 2 \); the expression is defined for all real numbers except \(-7\) and \(2\).
16. The expression is defined for all real numbers.
17. \( x = -\frac{1}{9} \)
18. \( y = \frac{1}{3} \)
19. \( z = \frac{22}{5} \)
20. \( m = 2 \)
21. no solution
22. \( p = \frac{10}{3} \)
23. \( a = \frac{1}{3} \)
24. \( n = 5 \text{ or } n = 7 \)
25. \( c = -2 \)
26. no solution; \(-2\) is an extraneous solution
27. \( k = -4 \)
28. \( y = -2 \text{ or } y = 12 \)
29. no solution; \(-3\) is an extraneous solution
30. \( m = -5 \text{ or } m = 0 \)
31. \( z = -12 \)
32. \( n = 4 \); \(-3\) is an extraneous solution
33. \( a = -\frac{1}{2} \text{ or } a = -3 \)
34. \( k = -\frac{1}{3}; -5 \) is an extraneous solution
Section 8.6 General Application Problems Involving Rational Equations

1. This answer should be in your own words.
2. This answer should be in your own words.
3. This answer should be in your own words.
4. To find a distance, multiply rate and time.
5. To find rate, divide distance by time.
6. To find time, divide distance by rate.
7. This answer should be in your own words.
8. 8 and 4
9. The number is either 6 or \(-\frac{1}{3}\).
10. width = 32 inches; length = 20 inches
11. 3.2 miles
12. jet’s rate: 120 mph; biplane’s rate: 60 mph
13. 6 mph
14. ran: 2 miles; biked: 6 miles
15. 3 mph
16. full rate: 8 mph; empty rate: 10 mph
17. 0.6 hour or 36 minutes
18. 36 minutes
19. 6.25 hours or 6 hours 15 minutes
Chapter 9 — Answers

Section 9.1
Section 9.2
Section 9.3
Section 9.4
Section 9.5
Section 9.6
Section 9.1 Introduction to Square Roots

1. This answer should be in your own words.
2. This answer should be in your own words.
3. No, the square root of a negative number does **NOT** have a real number value.
4. The expression under the square root sign is called the radicand.
5. 6
6. 1
7. 12
8. $\frac{1}{2}$
9. −10
10. 0
11. not a real number
12. 14
13. $\frac{1}{7}$
14. −9
15. $-\frac{3}{8}$
16. 15
17. not a real number
18. −20
19. $\frac{1}{100}$
20. 6
21. 2
22. a) 4
   b) 16
   c) $x$
   d) 4
   e) 9
   f) $-x$ (The negative sign in front of the $x$ makes the negative $x$-value positive.)
   g) 4
   h) 3
   i) $x$
Section 9.2  Simplifying Radical Expressions — Part I

1. This answer should be in your own words.
2. When factors are “pulled out from a square root”, **multiplication** is between those factors and the factors that remain under the square root.
3. If the radicand in a square root expression has a variable raised to an exponent, the short cut rule for simplifying the square root for that variable is to divide the exponent by 2.
4. This answer should be in your own words.
5. a) True  b) True  c) False  d) False  e) True  f) True  g) False  h) True  i) False
6. When multiplying two single-term square root expressions, it’s “easiest” to write the expression under one radical sign first.
7. Assuming all variables are nonnegative, the simplified answer for a problem that “squares a square root” or “square roots a square” is the radicand. In other words, “squaring” and “square rooting” are inverses.
8. \(3\sqrt{2}\)
9. \(2\sqrt{5}\)
10. \(3\sqrt{5}\)
11. \(10\sqrt{2}\)
12. not a real number
13. \(9\sqrt{2}\)
14. \(12\sqrt{5}\)
15. \(4\sqrt{7}\)
16. \(15\sqrt{7}\)
17. \(11\sqrt{5}\)
18. \(12\sqrt{3}\)
19. \(6\sqrt{11}\)
20. \(-10\sqrt{5}\)
21. \(20\sqrt{7}\)
22. \(21\sqrt{6}\)
23. \(x\sqrt{x}\)
24. \(z^5\sqrt{z}\)
25. \(k^4\sqrt{k}\)
26. \(n^{200}\)
27. \(p^{200}\sqrt{p}\)
28. \(y^2\sqrt{5}\)
29. \(2a^4\sqrt{2}\)
30. \(4m^6\)
31. \(7c^6d^4\sqrt{2d}\)
32. \(6y^{12}z^5\sqrt{3y}\)
33. \(8p^3q^4\sqrt{3q}\)
34. \(8d^{10}c^8d\sqrt{5c}\)
35. \(18v^{10}u^{16}\sqrt{2v}\)
36. \(2n^3\sqrt{2m}\)
37. \(7x^{25}y^{21}\sqrt{10z}\)
38. \(5a^{75}\sqrt{6}\)
39. \(6e^{18}d^{12}\sqrt{15d}\)
40. 5
41. 16
42. 8
43. 11
44. \(2\sqrt{5}\)
45. \(6\sqrt{5}\)
46. \(6\sqrt{21}\)
47. \(\sqrt{22}\)
48. 30
49. \(7\sqrt{10}\)
50. 42
51. \(3x^2\)
52. \(5y\sqrt{2y}\)
53. \(3a^4\sqrt{10}\)
54. \(9z^4\sqrt{2z}\)
55. \(10m^2n^2\sqrt{6}\)
56. \(4xy^3\sqrt{yz}\)
57. \(5a^2c^3\sqrt{14}\)
58. \(24p^6q^4\sqrt{2}\)
59. \(25w^6\)
60. \(11k^5\)
61. \(3a^9b\)
62. \(36x^{18}\)
63. \(14y^3z\)
64. \(15m^2\)
65. \(16c^2d^2\)
Section 9.3  Addition, Subtraction, and Multiplication of Radical Expressions

1. Only add square root expressions that have like-radicands. The explanation should be in your own words.
2. This answer should be in your own words.
3. $9\sqrt{2}$
4. $5\sqrt{11}$
5. $5\sqrt{10}$
6. $12\sqrt{2}$
7. $8\sqrt{14}$
8. $13\sqrt{5} - 3\sqrt{6}$
9. $-6\sqrt{3} + 7\sqrt{2}$
10. $7\sqrt{3}$
11. $5\sqrt{5}$
12. $5\sqrt{3}$
13. $10 + 2\sqrt{11}$
14. $\sqrt{7}$
15. $40\sqrt{3} - 12\sqrt{5}$
16. $9\sqrt{6}$
17. $-11 + 3\sqrt{2}$
18. $15 - 12\sqrt{3}$
19. $12 + 5\sqrt{11}$
20. $39\sqrt{2} + \sqrt{34}$
21. $5\sqrt{2} + 2$
22. $13\sqrt{7} + 13\sqrt{5}$
23. $2\sqrt{10} - 12$
24. $9\sqrt{11} - \sqrt{22}$
25. $18\sqrt{5} + 12$
26. $32\sqrt{3} - 48$
27. $x^2 + x\sqrt{6}$
28. $y^2 + y\sqrt{y}$
29. $6a\sqrt{a} + 2a^2$
30. $15 + 4\sqrt{5}$
31. $6\sqrt{14} - 42$
32. $40\sqrt{11} + 165$
33. $12\sqrt{7} + 20\sqrt{42}$
34. $6\sqrt{5} + 20\sqrt{2}$
35. $72\sqrt{2} - 12\sqrt{14}$
36. $8x + 13\sqrt{x}$
37. $9a\sqrt{a} + 9a$
38. $16z\sqrt{z} + 16z^2$
39. $5c\sqrt{5c} + 5c\sqrt{5}$
40. $14n + 14n\sqrt{n}$
41. $18 + 6\sqrt{5} + 3\sqrt{2} + \sqrt{10}$
42. $8 + 8\sqrt{7} - \sqrt{3} - \sqrt{27}$
43. $40 - 4\sqrt{5} - 10\sqrt{6} + 3\sqrt{2}$
44. $57 + 16\sqrt{2}$
45. $21 - 8\sqrt{4}$
46. $40 + 20\sqrt{5} + 12\sqrt{2} + 6\sqrt{10}$
47. $36 + 91\sqrt{3}$
48. $62 + 114\sqrt{3}$
49. $82 - 21\sqrt{14}$
50. $26 + 8\sqrt{10}$
51. $4 - 2\sqrt{3}$
52. $3x^2 + 7x\sqrt{x} + 2x$
53. $y^2 - 2y\sqrt{z} + z$
54. $2n^2 - 9n\sqrt{3n} + 12n$
55. $-11$
56. $-22$
57. $w^2 - 64w$
58. $8a - 16a^2$
59. $-z^2$
60. This answer should be in your own words.
Section 9.4 Division of Radical Expressions

1. When simplifying a square root whose radicand is a quotient, usually it’s “easiest” to simplify/reduce the fraction first, before taking the square root.

2. When simplifying an expression that consists of a quotient of square roots (without an addition or subtraction), simplify the fraction first.

3. The three conditions that must be met in order for a square root expression to be considered “simplified” are:
   a) No radicand can contain a factor that is a perfect square.
   b) The radicand is not a quotient.
   c) There are no radicals in the denominator.

4. This answer should be in your own words.

5. Expressions that require rationalizing the denominator are expressions with a single radical term in the denominator and expressions with two terms (at least one of which has a radical term). The next part of the answer should be in your own words.

6. $\frac{2}{3}$

7. $\sqrt{2}$

8. $2$

9. $-3$

10. $\sqrt{2}$

11. $2\sqrt{2}$

12. $\sqrt{2}$

13. $\sqrt[3]{5}$

14. $\sqrt[3]{3}$

15. $\sqrt[3]{5}$

16. $\sqrt[3]{3}$

17. $\sqrt[3]{5}$

18. $2\sqrt{2}$

19. $x^2$

20. $\frac{2m\sqrt{2}}{5}$

21. $\frac{2n}{3}$

22. $\frac{1}{9y^4}$

23. $\frac{\sqrt[6]{15}}{2w^3}$

24. $\frac{\sqrt[3]{q}}{3p}$

25. $\frac{m^{1/2}n^{3/2}}{\sqrt[n]{5n}}$

26. $\frac{1}{8xy^2}$

27. $\frac{1}{4}d^3\sqrt{c}$

28. $\frac{\sqrt[3]{12}}{3}$

29. $\frac{\sqrt[3]{5}}{2}$

30. $\frac{\sqrt[3]{5}}{3}$

31. $\frac{\sqrt{5}}{5}$

32. $\sqrt{2}$

33. $\frac{\sqrt[3]{2}}{2}$

34. $\frac{\sqrt[3]{5}}{y}$

35. $\frac{\sqrt{5}}{3\pi}$
36. \( \frac{2 \sqrt{p}}{p} \)
37. \( \frac{\sqrt{x}}{x} \)
38. \( \frac{m \sqrt{m}}{6} \)
39. \( z \sqrt{2} \)
40. \( \frac{2c \sqrt{c}}{8d^2} \)
41. \( \frac{x^2y}{xy^2} \)
42. \( \frac{\sqrt{p}}{3pq} \)
43. \( \frac{m^3 \sqrt{m}}{4n^2} \)
44. \( \frac{3a^4 \sqrt{a}}{5} \)
45. \( \frac{4a^3y}{3x^2y^2} \)
46. \( 5(2 - \sqrt{3}) \)
47. \( \frac{3 \times \sqrt{3}}{2} \)
48. \( \frac{\sqrt{6} + \sqrt{5}}{-3} \)
49. \( 2(3 - \sqrt{5}) \)
50. \( \frac{4 \sqrt{10}(3 - \sqrt{2})}{7} \)
51. \( 2 \sqrt{3} (\sqrt{8} + \sqrt{5}) \)
52. \( \frac{\sqrt{8} \sqrt{5}}{64 - x} \)
53. \( \frac{\sqrt{11}(\sqrt{11} + \sqrt{5})}{11 - y} \)
54. \( \frac{2 \sqrt{1} (\sqrt{1} - \sqrt{5})}{a - e} \)
55. \( \frac{3 \sqrt{1 + \sqrt{5}}}{x^2 - y} \)
Section 9.5  Simplifying Radical Expressions — Part II

1. This answer should be in your own words.
2. This answer should be in your own words.
3. \(-\frac{3+\sqrt{5}}{2}\)
4. \(\frac{1}{4}\)
5. \(3 + \sqrt{10}\)
6. \(-\frac{5-\sqrt{11}}{2}\)
7. \(\frac{7+5\sqrt{2}}{2}\)
8. \(-\frac{4+\sqrt{11}}{5}\)
9. \(-\frac{2-\sqrt{10}}{2}\)
10. \(-1\)
Section 9.6  Radical Equations

1. a) \( 4x^2 \)
   b) \( 4x \)
   c) \( x^2 + 4x + 4 \)
   d) \( x + 4\sqrt{x} + 4 \)

2. The first part of this answer should be in your own words. The inverse of square rooting is squaring.

3. It is necessary to check your answers when solving a square root equation. The rest of this answer should be in your own words.

4. \( x = 64 \)
5. \( y = 16 \)
6. \( a = 81 \)
7. \( m = 100 \)
8. \( w = 8 \)
9. \( p = 9 \)
10. no solution; 12 is an extraneous solution.
11. \( n = 22 \)
12. \( q = 106 \)
13. \( k = 41 \)
14. \( w = \frac{1}{2} \)
15. \( d = -3 \)
16. \( y = 11 \)
17. \( x = 4 \)
18. \( k = 4 \)
19. no solution
20. \( a = 2 \)
21. \( p = 1 \)
22. \( n = -3 \)
23. no solution; \( -\frac{3}{2} \) is an extraneous solution.
24. \( x = 16; 9 \) is an extraneous solution.
25. \( k = -4; -7 \) is an extraneous solution.
26. \( w = 5; -4 \) is an extraneous solution.
27. \( y = -4; -9 \) is an extraneous solution.
28. \( z = 3; 0 \) is an extraneous solution.
29. \( a = 9; 1 \) is an extraneous solution.
30. \( c = 4; -\frac{8}{9} \) is an extraneous solution.
31. \( m = \frac{1}{2} \) and \( m = 7 \)
32. no solution; \( \pm 1 \) are extraneous solutions.
33. \( x = 3; 11 \) is an extraneous solution.
Chapter 10 — Answers

Section 10.1
Section 10.2
Section 10.3
Section 10.4
Section 10.5
Section 10.1 Factoring

1. Factoring methods covered in Chapter 7 are GCF, grouping, and “un-foiling” or a-c grouping a trinomial or binomial difference of squares.

2. This answer should be in your own words.

3. \( x = -6 \) or \( x = 1 \)

4. \( y = 0 \) or \( y = 4 \)

5. \( a = -\frac{1}{3} \) or \( a = 5 \)

6. \( w = -1 \) or \( w = 12 \)

7. \( p = \pm 4 \)

8. \( n = -8 \) or \( n = 3 \)

9. \( z = -2 \) or \( z = 9 \)

10. \( x = -1 \) or \( x = -\frac{1}{2} \)

11. \( x = -8 \) or \( x = -2 \)
Section 10.2  The Square Root Property

1. This answer should be in your own words.
2. This answer should be in your own words.
3. \( x = \pm 7 \)
4. \( a = \pm 5 \)
5. \( y = \pm 3\sqrt{2} \)
6. \( z = \pm \frac{3}{2} \)
7. \( n = -11 \) or \( n = 1 \)
8. no real solution
9. \( c = -9 \) or \( c = 11 \)
10. \( q = -2 \pm 3\sqrt{7} \)
11. \( w = 3 \pm 2\sqrt{5} \)
12. \( p = 7 \) or \( p = 1 \)
13. \( x = 6 \pm 2\sqrt{3} \)
14. no real solution
15. \( y = -10 \pm \sqrt{11} \)
16. \( z = 1 \) or \( z = -2 \)
17. \( m = \frac{2}{3} \) or \( m = -2 \)
18. \( w = -\frac{7 \pm \sqrt{3}}{4} \)
19. \( d = -\frac{5 \pm 2\sqrt{5}}{2} \)
20. \( x = -5 \) or \( x = 1 \)
21. \( y = 4 \) or \( y = 1 \)
22. \((a + 4)^2\)
23. \((n - 5)^2\)
24. 
25. \((z - \frac{1}{2})^2\)
26. \(x = 14\)
27. \(y = 0 \) or \( y = -4 \)
28. \(d = -\frac{11 \pm \sqrt{11}}{2} \)
29. \(x = 22\)
30. no solution; \(-12\) is an extraneous solution.
Section 10.3 Completing the Square

1. This answer should be in your own words.
2. This answer should be in your own words.
3. \( x = -12 \) or \( x = 2 \)
4. \( p = 3 \) or \( p = -5 \)
5. \( y = -3 \pm 2 \sqrt{3} \)
6. \( d = -6 \pm \sqrt{6} \)
7. no real solution
8. \( w = 1 \pm \sqrt{10} \)
9. \( a = -2 \pm 2 \sqrt{5} \)
10. \( z = 7 \pm 2 \sqrt{15} \)
11. \( n = 3 \pm \sqrt{3} \)
12. \( c^2 + 5c + 4 = 0 \)
13. no real solution
14. \( q = \frac{-2 \pm \sqrt{3}}{2} \)
15. \( y = 4 \) or \( y = -1 \)
16. \( a = -13 \) or \( a = 2 \)
17. \( z = -6 \pm \sqrt{62} \)
18. \( m = \pm \sqrt{6} \)
19. \( w = 7 \) or \( w = -11 \)
20. \( d = 7 \pm 2 \sqrt{10} \)
Section 10.4  The Quadratic Formula

1. The Quadratic Formula is  \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).
2. The Quadratic Formula is derived by solving  \( ax^2 + bx + c = 0 \) for  \( x \) by completing the square.
3. This answer should be in your own words.
4. This answer should be in your own words.
5.  \( x = -3 \) or  \( x = -\frac{1}{2} \)
6.  \( a = 3 \) or  \( a = 6 \)
7.  \( z = \frac{5 \pm \sqrt{33}}{2} \)
8.  \( n = \frac{-3 \pm \sqrt{43}}{4} \)
9.  \( y = \pm \frac{\sqrt{11}}{2} \)
10.  \( c = -\frac{41}{2} \) or  \( c = 0 \)
11.  \( m = \frac{5 \pm \sqrt{11}}{3} \)
12. no real solution
13.  \( x = -1 \pm 2\sqrt{3} \)
14.  \( d = \frac{13 \pm \sqrt{114}}{2} \)
15.  \( z = -4 \pm \sqrt{114} \)
16.  \( a = -2 \) or  \( a = 6 \)
17.  \( y = \frac{12 \pm 7\sqrt{5}}{2} \)
18.  \( q = \frac{8 \pm 3\sqrt{5}}{2} \)
19.  \( w \approx 0.17 \) or  \( w \approx -3.84 \)
20.  \( n \approx 0.29 \) or  \( n \approx -1.54 \)
21.  \( p \approx 1.39 \) or  \( p \approx -0.73 \)
Section 10.5  Application Problems Involving Quadratic Equations

For each problem below, define variables, write an equation, solve the equation, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate.

1. \( x \approx 10.44 \text{ cm} \)
2. \( x \approx 8 \text{ feet} \)
3. \( x \approx 12.65 \text{ inches} \)
4. width \( \approx 18.35 \text{ cm} \); height \( \approx 16.35 \text{ cm} \)
5. \( \approx 20.40 \text{ feet} \)
6. \( \approx 9.70 \text{ feet} \)
7. a) \( 1 \text{ foot} \)
   b) \( 21 \text{ feet} \)
   c) \( \approx 0.47 \text{ second on the way up}; \approx 2.53 \text{ seconds on the way down} \)
   d) \( \approx 3.02 \text{ seconds} \)
8. a) \( 0 \text{ feet} \)
   b) \( 63 \text{ feet} \)
   c) \( \approx 0.74 \text{ second on the way up}; \approx 3.96 \text{ seconds on the way down} \)
   d) \( \frac{80}{17} \text{ or } \approx 4.71 \text{ seconds} \)