Math 4 -- Precalculus Algebra & Trig Test 3 -- Chapters 6, 7
M. Eastman -- Spring 2008

1. Fill in the table below with radian measure of each angle, and the exact values and decimal approximations correct to three decimal places (for non-exact values) of the trigonometric functions for each angle. If a value is not defined, write ND.

   Note: Table asks for values for angles with degree measure 0°, 30°, 45°, 60°, and 90°.

2. Use the reference angle principle and your table above to get the ***exact*** value of the following expressions:

   a. $sin\,(-60°)$ b. $cos\left(\frac{5\pi}{4}\right)$ c. $tan\,(480°)$ d. $cos\left(-\frac{13\pi}{6}\right)$

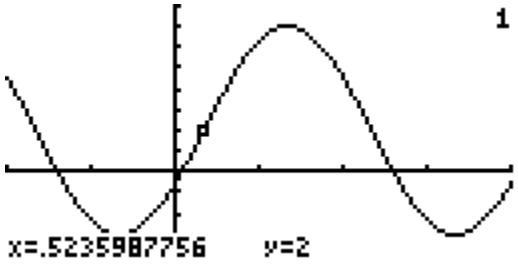
3. Let α be and angle between 0 and 2π such that $cos\,\alpha = \frac{5}{6}$ and $sin\,\alpha < 0$.

   a. In what quadrant is α ?

   b. Find the ***exact*** value of the following values: $sin\,\alpha$, $tan\,\alpha$, $sec\,\alpha$, $csc\,\alpha$, $cot\,\alpha$, $cos\,(-\alpha)$, $sin(-\alpha)$, $tan(-\alpha)$. You may rationalize denominators, but it is not necessary.

4. Sketch one period the graph of the function $y = 4\,sin(2x - \pi) - 1$; ***label*** the high and low points and all the points on the 'midline' with ***ordered pairs***.

5. Find a sine function and a cosine function that would produce the following graph [the adjacent points $\left(\frac{\pi}{6}, 2\right)$ midline and $\left(\frac{2\pi}{3}, 7\right)$ high point were labeled on the graph]:

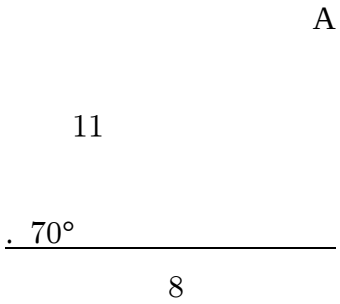


6. Prove that the following equations are identities:

   a. $sin\,A + cos\,A = \frac{sin\,A}{1 - cot\,A} - \frac{cos\,A}{tan\,A - 1}$ b. $csc^2\theta + cot^2\theta = csc^4\theta - cot^4\theta$

7. The figure shows two ships at points P and Q, which are in the same vertical plane as an airplane at the point R. When the the height of the airplane is 3500 feet, the angle of depression to P is 48 °, and that to Q is 25°. Find the distance between the two ships. Round your answer to the nearest 10 feet.

8. Find the area of the given triangle. (Draw the two sides from the ends of the line to the point A).



9. Find all values of x in the interval $[\,0, 2\pi\,]$ that satisfy the given equations:

a. $cos\,(2x) = \frac{\sqrt{3}}{2}$

b. $2\,(sin\,x)^2 = 1$

c. $2\,cos^{\,2}(x) + cos\,(x) = 1$

Math 4 - Precalculus **Midterm 3 - ANSWERS**
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1.

θ	θ	$cos\,\theta$		$sin\,\theta$		$tan\,\theta$	
(degrees)	(radians)	exact	approx	exact	approx	exact	approx
0°	0	1	1	0	0	0	0
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	.866	$\frac{1}{2}$	.5	$\frac{\sqrt{3}}{3}$	.577
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	.707	$\frac{\sqrt{2}}{2}$	.707	1	1
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	.5	$\frac{\sqrt{3}}{2}$	.866	$\sqrt{3}$	1.732
90°	$\frac{\pi}{2}$	0	0	1	1	undef.	---

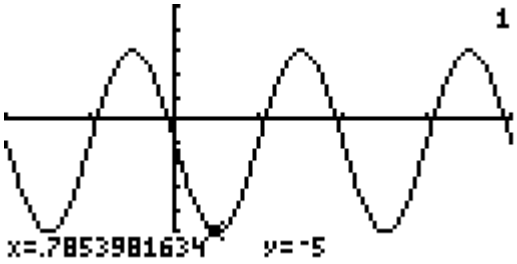
2. a. $-\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{2}}{2}$ c. $-\sqrt{3}$ d. $\frac{\sqrt{3}}{2}$

3. The angle α is in quadrant IV.

$sin\,\alpha = -\frac{\sqrt{11}}{6}$ $tan\,\alpha = -\frac{\sqrt{11}}{5}$ $sec\,\alpha = \frac{6}{5}$ $csc\,\alpha = -\frac{6\sqrt{11}}{11}$

$cot\,\alpha = -\frac{5\sqrt{11}}{11}$ $cos\,(-\alpha) = \frac{5}{6}$ $sin\,(-\alpha) = \frac{\sqrt{11}}{6}$ $tan(-\alpha) = \frac{\sqrt{11}}{5}$

4.



5. a. $f(x) = 5\,sin\left(x - \frac{\pi}{6}\right) + 2$ b. $f(x) = 5\,cos\left(x - \frac{2\pi}{3}\right) + 2$

6. identities

7. $d = \frac{3500}{tan\,48°} + \frac{3500}{tan\,25°} \approx 10657.18 \approx 10,660$ feet

8. $A = \frac{1}{2}bh = \frac{1}{2}(8)(11\,sin\,70°) = 44\,sin\,70° \approx 41.35$ square units

9. a. $\{\,\frac{\pi}{12},\ \frac{11\pi}{12},\ \frac{13\pi}{12},\ \frac{23\pi}{12}\,\}$ b. $\{\,\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}\,\}$ c. $\{\,\frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}\,\}$