Please show your work on these pages. You may use calculators on this test, but answer with exact values: for example π is exact, while 3.14 is not. You may use one page of notes. Put your answers in the space provided—answers without supporting work may not receive full credit.

1. Find the values of $x$ that satisfy each inequality. Use interval notation.
   a. $x^3 - 3x^2 - 16x + 48 < 0$
   b. $\frac{x-3}{x+7} \geq 6$

2. Given: $f(x) = \sqrt{16 - x^2}$
   $g(x) = \sqrt{x^2 - 9}$
   Find, or state a reason why the quantity can't be found:
   a. the domain of $f$.
   b. the domain of $g$.
   c. $g \circ f (-2)$
   d. $f \circ g (4)$
   e. $f \circ g (x)$
   f. the domain of $f \circ g$.

3. Given the function $f(x) = x^3 + 6x^2 - 107x + 70$
   a. List the possible rational zeroes of $f$.
   c. Find the exact values of all the zeroes of $f$.

4. Solve for $x$: in part b. state both the exact value of $x$ and an approximation correct to two decimal places.
   a. $\log_4(x) + \log_4(x + 6) = 2$
   b. $3^{x-1} = 21$

5. The population of a certain town was 14,000 in 1975. In 1980 the population of the town had grown to 17,100. Assume that the population grows exponentially at a continuous rate, and that the growth rate remains constant.
   a. Find the formula for the population $t$ years after 1975.
   b. Use the formula from part a to estimate the population in 2005 (to the nearest hundred).
   c. Assume that the town continues to grow at the same rate. Estimate the year when the population reaches 75,000.
6. Sketch the graph of the following functions. **Label the scales on the axes.** Find the key points (intercepts, any maximum and minimum points, points where the graph crosses an asymptote), and draw and label any asymptotes of the function (horizontal, vertical and/or 'slant').

a. \( f(x) = \frac{2x^2 - x - 10}{x^2 - 7x + 10} \)

b. \( g(x) = 3 \sin \left(2x + \frac{\pi}{3}\right) - 1 \)

Extra credit (5 points): Find all the \( x \)-intercepts for this function.

7. Determine the polynomial of degree 6 whose graph is shown below; each mark on the \( x \)-axis represents one unit. The intercepts have integer coordinates. Note the point on the graph that is listed with the 'TRACE' feature (The polynomial can be left in factored form).

8. Find the exact value of:

a. \( \cos \left(\frac{7\pi}{6}\right) \)

b. \( \tan (75^\circ) \)

c. \( \sin \left(-\frac{\pi}{8}\right) \)

d. \( \tan^{-1} \left(-\sqrt{3}\right) \)

e. \( \sin^{-1} \left(\sin \left(\frac{4\pi}{3}\right)\right) \)

f. \( \cos \left(\sin^{-1} \left(\frac{5}{8}\right)\right) \)

9. Verify the identities algebraically:

a. \( \cos \left(\frac{\pi}{2} + t\right) = -\sin t \)

b. \( \tan^2 x - \sin^2 x = \tan^2 x \cdot \sin^2 x \)

c. \( \cos (4\theta) = 8 \cos^4(\theta) - 8\cos^2(\theta) + 1 \)
10. Find all values of $x$ in the interval $[0, 2\pi]$:
   a. $\sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{2}$
   b. $\cos(2x) + \cos(x) = 0$

11. Find the unknown side (to the nearest tenth of a unit) and the unknown angles (to the nearest degree) of the triangle:

   Connect the three points $A, B$, and $C$ to view the triangle.

   \[
   \begin{array}{c}
   B \\
   \beta \\
   6 \\
   40^\circ \\
   A \\
   11 \\
   C \\
   \end{array}
   \]

12. A flagpole is on top of a building that is 24 feet high. From a point on level ground that is 100 feet away from a point that is directly below the flagpole, the flagpole subtends an angle of $10^\circ$ (see figure: the point $A$ is at the top of the flagpole, the point $B$ is at the bottom of the flagpole and the top of the building, the point $D$ is at the base of the building, and the point $C$ is the point of view. Connect all the points). Estimate the length of the flagpole to the nearest tenth of a foot.

   \[
   \begin{array}{c}
   A \\
   \\
   B \\
   10^\circ \\
   24' \\
   C \\
   \\
   D \\
   100'
   \end{array}
   \]
1. a. $(-\infty, -4) \cup (3,4)$ b. $[-9, -7)$

2. a. $[-4, 4]$ b. $(-\infty, -3] \cup [3, \infty)$ c. $\sqrt{3}$ d. 3 e. $\sqrt{25 - x^2}$ f. $[-5, -3] \cup [3, 5]$

3. a. $\pm 1, \pm 2, \pm 5, \pm 7, \pm 10, \pm 14, \pm 35, \pm 70$ b. $-14, 4 \pm \sqrt{11}$

4. a. $x = 2 \ (x \neq -8)$ b. $x = \frac{\ln 21}{\ln 3} + 1 \approx 3.77$

5. a. $P(t) = 14000e^{0.4t}$ b. $P(30) \approx 46,500$ c. $2017 \ (P = 75000 \text{ when } t = 42)$

6. a. Intercepts: $(0, -1), (-2, 0), \left(\frac{5}{2}, 0\right)$ Vertical Asymptotes: $x = 2 \ x = 5$

Horizontal Asymptote: $y = 2$

b. $y$-intercept: $\left(0, \frac{3\sqrt{3}}{2}\right)$ High Points: $\left(\frac{\pi}{12} + n \pi, 2\right)$ Low Points: $\left(\frac{7\pi}{12} + n \pi, -4\right)$

$x$-intercepts: $\left(\frac{1}{2} \sin^{-1}\left(\frac{1}{3}\right) - \frac{\pi}{6} + n \pi, 0\right), \left(\frac{1}{2} \sin^{-1}\left(\frac{1}{3}\right) + \frac{5\pi}{6} + n \pi, 0\right)$

7. $P(x) = -\frac{1}{25}(x + 4)^2(x - 2)^3(x - 5)^3$

8. a. $-\frac{\sqrt{3}}{2}$ b. $\frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$ c. $-\frac{\sqrt{2} - \sqrt{2}}{2}$ d. $-\frac{\pi}{3}$ e. $-\frac{2\pi}{3}$ f. $\frac{\sqrt{39}}{8}$
9. a. \[ \cos\left(\frac{\pi}{2} + t\right) = \cos\left(\frac{\pi}{2}\right)\cos(t) - \sin\left(\frac{\pi}{2}\right)\sin(t) = 0 \cdot \cos(t) - 1 \cdot \sin(t) = -\sin(t) \]
   
b. both sides simplify to: \[ \frac{\sin^4 x}{\cos^2 x} \]
   
c. \[ \cos(4\theta) = 2\cos^2(2\theta) - 1 = 2(2\cos^2\theta - 1)^2 - 1 = 8\cos^4(\theta) - 8\cos^2(\theta) + 1 \]

10. a. \[ \frac{5\pi}{24}, \quad \frac{13\pi}{24}, \quad \frac{29\pi}{24}, \quad \frac{37\pi}{24} \]
    
b. \[ \frac{\pi}{3}, \quad \pi, \quad \frac{5\pi}{3} \]

11. \[ a \approx 7.5 \quad \beta \approx 109^\circ \quad \gamma \approx 31^\circ \]

12. \[ \angle BCD = \tan^{-1}\left( \frac{24}{100} \right) \approx 13.5^\circ \]
   
   \[ \tan(10^\circ + 13.5^\circ) = \frac{AB + 24}{100} \]
   
   so \[ AB = 100 \tan(23.5^\circ) - 24 \approx 19.5 \text{ feet} \]