Math 5B -- Calculus II **Name** ______________________________

M. Eastman -- Fall 2008 **Test 2 -- Chapters 7, 8**

1. Evaluate:

a. $\int \frac{5x+2}{x^3-4x}\,dx$ b. $\int sin^5 x\,dx$ c. $\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$

2. Determine whether the following integrals converge or diverge. Evaluate any integral that converges. Justify your answers by showing the necessary work.

a. $\int_0^\infty x\,e^{-x^2}\,dx$ b. $\int_1^4 \frac{x}{x^2-4}\,dx$

3. The region in the first quadrant under the first 'hump' of the curve $y = sin\,x$ is rotated around the y-axis. Find the volume of the solid that is obtained.

4. Given the function $y = x^3$ between the points $(1,1)$ and $(2,8)$:

a. Set up and simplify the integrand, *but do not evaluate*, the integral that will find the length of this portion of the curve.

b. If this portion of the curve is rotated around the x-axis, find the area of the surface that is generated.

1. a. $\int \left(\frac{\left(-\frac{1}{2}\right)}{x} + \frac{\left(\frac{3}{2}\right)}{x-2} + \frac{(-1)}{x+2} \right) dx = -\frac{1}{2} ln\,|x| + \frac{3}{2}\, ln\,|x-2| - ln\,|x+2| + C$

b. $\int sin^5 x\, dx = \int (1 - cos^2 x)^2\, [sinx\, dx]$ Set $u = cos\, x \ :\ du = -\, sinx\, dx$

$= \int (1-u^2)^2\, [-\, du] = \ldots = -\, cos\, x + \frac{2}{3} cos^3 x - \frac{1}{5} cos^5 x + C$

c. $\int \frac{1}{x^2\sqrt{x^2-9}}\, dx = \frac{1}{3}\int \frac{1}{x^2\sqrt{\frac{x^2}{9}-1}}\, dx$ Set $\frac{x}{3} = sec\,\theta \ :\ dx = 3\, sec\,\theta\, tan\,\theta\, d\theta$

$= \frac{1}{3}\int \frac{3\, sec\,\theta\, tan\,\theta}{(3\, sec\,\theta)^2\sqrt{sec^2\theta - 1}}\, d\theta = \ldots = \frac{1}{9}\int \frac{1}{sec\,\theta}\, d\theta = \frac{1}{9}\int (cos\,\theta)\, d\theta$

$= \frac{1}{9}\, sin\,\theta + C = \frac{\sqrt{x^2-9}}{9x} + C$

2. a. $\int_0^\infty x\, e^{-x^2}\, dx = \lim_{T\to\infty}\int_0^T x\, e^{-x^2}\, dx = \ldots$(sub $u = -\, x^2$)$\ldots = \lim_{T\to\infty}\left(-\frac{1}{2}e^{-x^2}\right)\Big|_0^T$

$= \lim_{T\to\infty}\left(-\frac{1}{2}e^{-T^2} - \frac{1}{2}e^0\right) = \lim_{T\to\infty}\frac{1}{2}\left(1 - \frac{1}{e^{T^2}}\right) = \frac{1}{2}$ Converges to $\frac{1}{2}$.

b. $\int_1^4 \frac{x}{x^2-4}\, dx = \int_1^2 \frac{x}{x^2-4}\, dx + \int_2^4 \frac{x}{x^2-4}\, dx$

$= \lim_{b\to 2^-}\int_1^b \frac{x}{x^2-4}\, dx + \lim_{a\to 2^+}\int_a^4 \frac{x}{x^2-4}\, dx$ Set $u = x^2 - 4 \ :\ du = 2x\, dx$

$= \lim_{b\to 2^-}\left(\frac{1}{2}\, ln\,|x^2-4|\right)\Big|_1^b + \lim_{a\to 2^+}\left(\frac{1}{2}\, ln\,|x^2-4|\right)\Big|_a^4$ Note that both integrals diverge.

3. $V = \int_0^\pi 2\pi(x)(sin\, x)\, dx = \ldots$(by IBP)$\ldots = (-\, x\, cos\, x + sin\, x)\Big|_0^\pi = 2\pi^2$

4. a. $s = \int_1^2 \sqrt{1 + (3x^2)^2}\, dx = \int_1^2 \sqrt{1+9x^4}\, dx$

b. $A = \int_1^2 2\pi(x^3)\sqrt{1+9x^4}\, dx$ Set $u = 1 + 9x^4 \ :\ du = 36\, x^3 dx$

$= \frac{2\pi}{36}\int_{10}^{145}\sqrt{u}\, du = \frac{\pi}{18}\left(\frac{2}{3}u^{3/2}\right)\Big|_{10}^{145} = \frac{\pi}{27}\left(145\sqrt{145} - 10\sqrt{10}\right)$