Math 5B -- Calculus II **Name** ______________________________

M. Eastman -- Spring 2005 **Test 2 -- Chapter 7**

Please show your work on these pages, and put your answers in the places that are provided. If you need more space, use the back of the pages; indicate on the original page where the work is continued. Answers without supporting work will receive little or no credit. Answers should be exact. No graphing calculators are allowed. You may use one 4" × 6" card of notes.

1. Write out the form of the partial fraction decomposition of the function. Do *not* determine the value of the coefficients.

$$\frac{3x^2-5x-10}{(x-3)^2(x^2+4)^3}$$

2. Evaluate:

$$\int sin^3x \cdot cos^4x\, dx$$

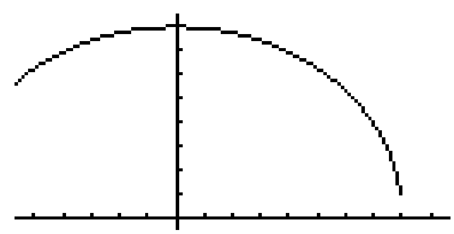
3. Evaluate:

a. $\int \frac{1}{(9-x^2)^{3/2}}\, dx$

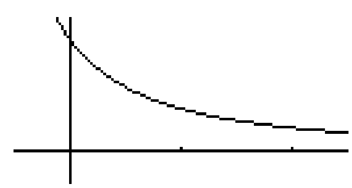
b. $\int \frac{ln(x)}{x^2}\, dx$

c. $\int_1^2 \frac{\sqrt{x^2-1}}{x}\, dx$

4. Find the area under the curve $f(x) = \sqrt{64 - x^2}$ between $x = 0$ and $x = 4$.



5. The region in the first quadrant under the curve $y = \frac{1}{x^2+4x+3}$ between $x = 0$ and $x = 2$ is rotated around the y-axis. Find the volume of the solid that is obtained.



1. $\frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{Cx+D}{(x^2+4)} + \frac{Ex+F}{(x^2+4)^2} + \frac{Gx+H}{(x^2+4)^3}$

2. Let $u = cos\,x$.

   Integral becomes $\int (1-u^2)u^4(-du) = \ldots = \frac{1}{7}cos^7 x - \frac{1}{5}cos^5 x + C$

3. a. Trig substitution: $\frac{x}{3} = sin\,\theta$

   Answer: $\frac{x}{9\sqrt{9-x^2}} + C$

   b. Use IBP: $u = ln(x)\;\; dv = \frac{1}{x^2}dx$

   Answer: $-\frac{1}{x}ln(x) - \frac{1}{x} + C$

   c. Trig Substitution: $x = sec\,\theta$

   Answer: $\frac{3\sqrt{3}-\pi}{3}$

4. $A = \int_0^4 \sqrt{64-x^2}\,dx = 8\int_0^4 \sqrt{1-\frac{x^2}{64}}\,dx$

   Let $\frac{x}{8} = sin\,\theta$

   Integral becomes $\int_0^{\frac{\pi}{6}} \cos^2\theta\,d\theta = \ldots = \frac{16\pi}{3} + 8\sqrt{3}$

5. Use shells: $V = \int_0^2 2\pi(x)\left(\frac{1}{x^2+4x+3}\right)dx = 2\pi\int_0^2 \frac{x}{x^2+4x+3}\,dx$

   use partial fractions: $\frac{x}{x^2+4x+3} = \frac{\left(\frac{3}{2}\right)}{x+3} + \frac{\left(-\frac{1}{2}\right)}{x+1}$

   integral yields $2\pi\left(\frac{3}{2}ln|x+3| - \frac{1}{2}ln|x+1|\right)\Big|_0^2 = \ldots = \pi(3\,ln\,5 - 4\,ln\,3)$