Math 5B -- Calculus II **FINAL EXAM**
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Please show your work on these pages; correct answers without supporting work may not receive full credit. If you need more space use the back of the pages; indicate on the original problem where the work is located. Put your answer to each problem in the space provided. Answers should be exact unless the problem asks for an approximation. No graphing calculators are allowed, scientific calculators are okay.

1. Evaluate: for the definite integrals please give the *exact* answer.

a. $\int sin^3 x\, cos^2 x\, dx$ b. $\int \frac{3x-8}{x^3+x}\, dx$

c. $\int_1^e x^3\, ln\, x\, dx$ d. $\int_0^4 \sqrt{64-x^2}\, dx$

2. Let $\mathbb{R}$ be the region in the first quadrant under the curve $f(x) = \frac{1}{x^2+1}$ and to the right of the line $x = 1$.

a. Find the area of $\mathbb{R}$, or explain why such an area cannot be found.

b. The region $\mathbb{R}$ is rotated around the x-axis. Find the volume that is generated, or explain why such a volume cannot be found.

3. Find the centroid of the region in the first quadrant bounded by the curve $y = e^x$ and the line $x = 2$.

4. A circular drum is on its side. The drum is four feet in diameter, and is five feet long. It is filled with water to a depth of three feet (or one foot from the top of the drum). Find the total force exerted on one (circular) end of the drum. [Note: the density of water is 62.5 pounds per cubic foot].

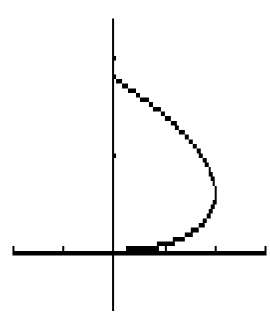
5. Given the parametric equations: $x = 3t - t^3$

$y = 3t^2 \qquad t \geq 0$

a. Find the length of this curve from $(0, 0)$ to $(0, 9)$.

b. Find the area bounded by the curve and the y-axis.

[You should use horizontal strips]



6. Determine whether the following series converge or diverge. Be specific with your answers when appropriate (absolute vs. conditional convergence). State which test(s) you use.

a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \cdot (ln(n))^2}$

b. $\displaystyle\sum_{n=0}^{\infty} \frac{(2n)!}{3^n (n!)^2}$

7. Find the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{7(x-2)^n}{3^n \cdot \sqrt{n}}$

8. Determine whether the following series are convergent or divergent. If it is convergent, find its sum. Justify your answer.

a. $\displaystyle\sum_{n=1}^{\infty} ln\left(\frac{n+1}{n}\right)$

b. $48 - 36 + 27 - \frac{81}{4} + \frac{243}{16} - \frac{729}{64} + \ldots$

9. Use Taylor series to find the sum of the following series:

a. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n \cdot n!}$

b. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(\pi)^{2n}}{9^n \cdot (2n)!}$

10. Find the first five non-zero terms of the Taylor series for $f(x) = \sqrt[3]{x}$ at $a = 1$.

11. a. Find the Maclaurin series for $f(x) = cos\sqrt{x}$ by using the (known) Taylor series for $cos\, x$ at $x = 0$. Write your answer using $\sum$ notation.

b. Use the series to estimate $\displaystyle\int_0^1 cos\sqrt{x}\, dx$ with a ***fraction*** (of integers) to within 0.001, using the fewest number of terms necessary.

c. Find another approximation of $\displaystyle\int_0^1 cos\sqrt{x}\, dx$ using Simpson's Rule, with $n = 4$ subdivisions. Express your answer correct to four decimal places.

1. a. $\frac{cos^5 x}{5} - \frac{cos^3 x}{3} + C$

   b. $4\ln\left(\frac{x^2+1}{x^2}\right) + 3\,tan^{-1}x + C$

   c. $\frac{2}{9}e^3 + \frac{1}{9}$

   d. $\frac{16\pi}{3} - 8\sqrt{3}$

2. a. $\int_1^\infty \frac{1}{x^2+1}\,dx$ converges to $\frac{\pi}{4}$

   b. $\int_1^\infty \pi\left(\frac{1}{x^2+1}\right)^2 dx$ converges to $\frac{\pi^2-2\pi}{8}$

3. $\left(\frac{e^2+1}{e^2-1}, \frac{e^2+1}{4}\right)$

4. $F = 62.5\int_{-2}^{1}(2-y)\left(2\sqrt{4-y^2}\,dy\right) = 62.5\left(\frac{16\pi}{3} + 4\sqrt{3}\right)$

5. a. $6\sqrt{3}$ b. $\frac{36}{5}\sqrt{3}$

6. a. Converges--absolutely (integral test) b. Diverges (ratio test)

7. $[-1, 5)$

8. a. diverges (telescoping series) b. converges to $\frac{192}{7}$

9. a. $e^{-2/3}$ b. $cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

10. $1 + \frac{1}{3}(x-1) - \frac{2}{9}(x-1)^2 + \frac{5}{81}(x-1)^3 - \frac{10}{81}(x-1)^4$

11. a. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(2n)!}$ b. $1 - \frac{1}{4} + \frac{1}{72} = \frac{55}{72}$ c. 0.7635