Math 5B -- Calculus II **Test 3 -- Chapter 11**
M. Eastman -- Spring 2009

1. Determine whether the sequence converges or diverges. Justify your answers. If it converges, find the limit.

a. $\left\{ tan\left(\frac{2n\pi}{1+6n}\right) \right\}_{n=1}^{\infty}$ b. $a_k = \sqrt[k]{5^{(1+2k)}}$

2. Given the series $\sum_{k=1}^{\infty} \frac{10}{k^2+4k+3}$

a. Use the technique of partial fractions to write the general term a_k of this series as a sum or difference of two terms.

b. Find a formula for the nth partial sum S_n of this series (simplify by canceling out terms).

c. Evaluate $\lim_{n\to\infty} S_n$ to determine the sum of this convergent series.

3. Consider the series $\sum_{k=1}^{\infty} \frac{1}{k^4}$. This is a convergent p-series, since $p = 4 > 1$. According to the integral test, the remainder R_n of a convergent series satisfies the inequalities $\int_{n+1}^{\infty} f(x)\,dx \leq R_n \leq \int_{n}^{\infty} f(x)\,dx$, where $f(x)$ is a continuous function such that $f(k) = a_k$. Use one or both of these inequalities to find the <u>*smallest*</u> value of n such that the partial sum S_n is within 0.00001 of the actual sum of the series.

4. Determine whether the following series converge or diverge. State which test(s) you use, and justify your answer.

a. $\sum_{k=1}^{\infty} k \cdot e^{-k^2}$4. b. $\sum_{n=1}^{\infty} n \cdot sin\left(\frac{\pi}{n}\right)$ c. $\sum_{n=1}^{\infty} (-1)^n \frac{(3n^2+5)^n}{(2n-1)^{2n}}$

5. Determine whether each series is convergent or divergent. If it converges, find the sum. Justify your answers.

a. $\sum_{n=0}^{\infty} \frac{3^n+4^n}{6^n}$ b. $\sum_{n=0}^{\infty} (-1)^n \frac{(5)^{2n}}{(2n)!}$

6. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+3)^n}{7^n \cdot n}$

7. Find the 3rd degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $a = 1$.

8. a. Find the Maclaurin series for $f(x) = cos\sqrt{x}$ by using the (known) Maclaurin series for $cos\,x$. Write your answer using $\sum$ notation.

b. Use the series to estimate $\int_0^2 cos\sqrt{x}\,dx$ with a ***fraction*** to within 0.001, using the fewest number of terms necessary.

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1. converges to $tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ b. converges to 25

2. a. $a_k = \frac{5}{k+1} - \frac{5}{k+3}$ b. $S_n = \frac{5}{2} + \frac{5}{3} - \frac{5}{n+2} - \frac{5}{n+3}$ c. $\frac{25}{6}$

3. $\int_n^\infty \frac{1}{k^4}\,dx < .001 \quad \rightarrow \quad n > \sqrt[3]{\frac{100000}{3}} \approx 32.18... \quad \rightarrow \quad n_{min} = 33$

4. a. converges (integral test)
   b. diverges (test for divergence)
   c. converges - absolutely (root test)

5. a. converges to 5
   b. converges to $cos\,5$

6. $[\,-10, 4\,)$

7. $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

8. a. $cos(\sqrt{x}) = \sum_{n=0}^{\infty} (-1)^n \frac{(\sqrt{x})^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{(2n)!}$
   b. $\int_0^2 cos\sqrt{x}\,dx \approx \frac{199}{180}$