

1. Determine whether the sequence converges or diverges. Justify your answers. If it converges, find the limit.

a. $\left\{ \cos\left(\frac{1}{n}\right) \right\}_{n=1}^{\infty}$

b. $a_k = \ln(3k - 1) - \ln(k + 1)$

2. Given the series $\sum_{k=1}^{\infty} \frac{6}{k^2+6k+8}$

- a. Use the technique of partial fractions to write the general term a_k of this series as a sum or difference of two terms.
b. Find a formula for the n th partial sum S_n of this series.
c. Evaluate $\lim_{n \rightarrow \infty} S_n$ to determine the sum of this convergent series.

3. Determine whether each series is convergent or divergent. If it converges, find the sum. Justify your answers.

a. $\sum_{n=1}^{\infty} \frac{(2n-1)(2n+1)}{16n^2}$

b. $\sum_{n=1}^{\infty} (-1)^n \frac{2^{2n+1}}{5^{n-1}}$

4. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer. State any tests you use in your solution.

a. $\frac{2}{7} - \frac{2}{11} + \frac{2}{15} - \frac{2}{19} + \frac{2}{23} - \frac{2}{27} + \dots$

b. $\sum_{k=0}^{\infty} \frac{(-3)^k}{k!}$

5. Determine whether the following series converge or diverge. State which test(s) you use, and justify your answer.

a. $\sum_{n=2}^{\infty} \frac{1}{n \cdot \sqrt{\ln(n)}}$

b. $\sum_{n=1}^{\infty} (-1)^n \frac{(3n+1)^n}{(4n-3)^n}$

c. $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+9}}{n^3-n^2+1}$

6. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{8(x-4)^n}{5^n \cdot \sqrt{n}}$

7. Find the first four non-zero terms of the Taylor series for $f(x) = \frac{1}{\sqrt[3]{x}}$ centered at $a = 1$.

1. a. converges to 1. b. converges to $\ln 3$.
2. a. $a_k = \frac{3}{k+2} - \frac{3}{k+4}$ b. $S_n = 1 + \frac{3}{4} - \frac{3}{n+2} - \frac{3}{n+4}$ c. $\frac{7}{4}$
3. a. diverges (test for divergence). b. converges to $-\frac{40}{9}$ (geometric series).
4. a. conditionally convergent -- alternating series test, integral test, or comparison test.
b. absolutely convergent -- ratio test.
5. a. diverges -- integral test.
b. converges (absolutely) -- root test.
c. converges -- (limit) comparison test.
6. $[-1, 9)$
7. $1 - \frac{1}{3}(x-1) + \frac{2}{9}(x-1)^2 - \frac{14}{81}(x-1)^3$