Please show all your work on these pages. Graphing calculators are not allowed; you may use a scientific calculator. All answers should be exact (e.g. $\sqrt{2}$ is exact: 1.414... is not). Answers obtained without supporting work may not receive full credit.

1. Find the area of the region inside the cardioid $r = 2 + 2 \sin \theta$ and outside of the circle $r = 1$.

2. Given the points $A = (2, 3, 1)$, $B = (3, 2, 5)$, and $C = (6, 3, -3)$, find:
   a. parametric equations for the line that passes through $A$ and $B$.
   b. the angle $\angle BAC$.
   c. the equation of the plane (in the form $ax + by + cz = d$) through these three points.

3. If $r(t) = <4t, 2 \cos t, 2 \sin t>$, find: (20 points)
   a. the length of the curve between $(0, 2, 0)$ and $(2\pi, 0, 2)$.
   b. $T\left(\frac{\pi}{3}\right)$
   c. the curvature $\kappa$ (as a function of $t$).

4. Given the equation $z^2 = 36 + 9x^2 - 4y^2$:
   a. Classify this quadric surface.
   b. Find the equation of the tangent plane to this surface at the point $(2, 3, -6)$.
   c. Suppose that the positive $y$-axis points north and the $x$-axis east. If one travels on this surface in a northeast direction from the point $(2, 3, -6)$, does one's elevation (as measured by the $z$-value) increase or decrease? Explain.

5. A package in the shape of a rectangular box can be mailed by parcel post if the sum of its length and girth is at most 84 inches (girth is the perimeter of a cross-section perpendicular to the length). Find the dimensions of the package with largest volume that can be mailed by parcel post.
6. Evaluate $\int_0^1 \int_{2x}^{2} e^{y^2} \, dy \, dx$  [Note: $\int e^{y^2} \, dy$ cannot be done]

7. Use Green's Theorem to evaluate $\oint_C (e^x + 5y) \, dx + (7x - \cos y) \, dy$, where $C$ is the simple closed curve given by the $x$-axis, the line $x + y = 6$, and the parabola $y = x^2$, traversed counterclockwise (see figure).

8. Given: $\mathbf{F}(x, y) = <3x^2 - 2xy, 10y - x^2>$
   a. Evaluate $\int_C \mathbf{F} \cdot \, d\mathbf{r}$ directly, where $C$ is the line segment from $(1, 0)$ to $(2, 3)$.
   b. Evaluate $\int_C \mathbf{F} \cdot \, d\mathbf{r}$ directly, where $C$ is the part of the parabola $y = x^2 - 1$ from $(1, 0)$ to $(2, 3)$.
   c. Use the Fundamental Theorem of line integrals to evaluate $\int_C \mathbf{F} \cdot \, d\mathbf{r}$ along any path $C$ that starts at $(1, 0)$ and ends at $(2, 3)$.

9. Given $\mathbf{F}(x, y, z) = z \sin y \, \mathbf{i} + x \cos z \, \mathbf{j} + (1 + z^2) \, \mathbf{k}$
   a. Find curl $\mathbf{F}$.
   b. Find div $\mathbf{F}$.
   c. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \, d\mathbf{S}$, where $S$ is the surface of the solid bounded by the first-octant portion of the paraboloid $f(x, y) = 1 - x^2 - y^2$ and the coordinate planes.
1. Area = \( 2 \int_{-\pi/6}^{\pi/6} \frac{1}{2}((2 + 2\sin \theta)^2 - 1^2) \, d\theta \)

2. a. \( x = 2 + t; \ y = 3 - t; \ z = 1 + 4t \)
   b. \( \cos^{-1} \left( -\frac{1}{2} \right) = \frac{2\pi}{3} \)
   c. \( x + 5y + z = 18 \)

3. a. \( \int_0^{\pi/2} \sqrt{(4)^2 + (-2\sin t)^2 + (2\cos t)^2} \, dt = \pi \sqrt{5} \)
   b. \( < \frac{2}{\sqrt{5}}, \frac{-\sqrt{3}}{2\sqrt{5}}, \frac{1}{2\sqrt{5}} > \)
   c. \( \frac{1}{10} \)

4. a. hyperboloid of one sheet
   b. \( 3x - 2y + z = -6 \)
   c. decreases; directional derivative is negative
   \[ z = f(x, y) = -\sqrt{36 + 9x^2 - 4y^2} \]
   \[ D_u(f) \bigg|_{(2,3)} = \nabla f \cdot \left< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right> = -\frac{1}{\sqrt{2}} \]

5. Minimize \( A = lwh \) subject to \( 2l + 2w + h = 84 \)
   dimensions are \( 14'' \times 14'' \times 28'' \)

6. switch order of integration: \( \int_0^2 \int_0^{\pi/2} e^{y^2} \, dx \, dy = \ldots = \frac{1}{4} (e^4 - 1) \)
7. \[ \int_0^4 \int_{\sqrt{y}}^{6-y} (2) \, dx \, dy = \ldots = \frac{64}{3} \]

8. a. \[ \int_0^1 (84t - 6t^2) \, dt = 40 \]
   b. \[ \int_1^2 (16x^3 + 3x^2 - 18x) \, dx = 40 \]
   c. \[ (x^3 - x^2y + 5y^2) \bigg|_{(2,3)}^{(1,0)} = 40 \]

9. a. \[ \langle x \sin z, \sin y, \cos z - z \cos y \rangle \]
   b. \[ 2z \]
   c. \[ \int \int \int_{x^2 + y^2 \leq 1} \int_0^{1-x^2-y^2} (2z) \, dz \, dA = \ldots = \frac{\pi}{12} \]