Math 11A - Calculus with Applications I **Final Exam**
M. Eastman - Fall 2006

1. a. Find $\frac{dy}{dx}$, given $y = x^3 \cdot e^{arctan(x)}$

   b. Find $f''(x)$ [the second derivative] of $f(x) = ln\,(x^2 - 3x + 5)$:

2. Find the critical points for the function $f(x) = \sqrt[5]{x^2 - 6x - 40}$.

3. Use the technique of logarithmic differentiation to find $\frac{dy}{dx}$, if $y = (ln\,x)^{5x}$. Write your answer explicitly in terms of x.

4. The graph of the equation $x^2 + 2y^2 = 4x + 8y + 5$ is an ellipse (graph is shown below):

   a. Find the point on the graph of this equation that crosses the ***positive*** x-axis.

   b. Find the general formula for $\frac{dy}{dx}$ (in terms of both x and y).

   c. Find the equation, in ***slope-intercept form***, of the line tangent to the graph of this equation at the point from part a.

5. Evaluate the following limits, if they exist (if a limit does not exist, write DNE): justify your answers using algebraic techniques and limit laws. Numerical calculations will recieve only partial credit.

   a. $\lim_{x\to 0} \frac{\sqrt{x^2+16} - 4}{x^2}$ b. $\lim_{x\to 0} \frac{sin(x)-x}{x^3}$ c. $\lim_{x\to \pi^-} \frac{sin\,x}{1-cos\,x}$

6. A motorist leaves a town at noon traveling north at 60 mph. At 1 pm, another motorist leaves the same town traveling east at 50 mph. How fast is the distance between the two motorists changing at 2 pm?

7. Find the value of the constants a and b that makes g a continuous function for all real numbers:

$$g(x) = \begin{cases} 4x - 3 & x \le 2 \\ ax + b & 2 < x < 6 \\ x^2 - 9 & x \ge 6 \end{cases}$$

8. Use the formula $f(x) \approx f(a) + f'(a)(x - a)$ to find a fraction (of integers) that approximates $\sqrt{35}$.

9. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 9x^2 + 15x + 20$ on the interval [0, 8]. Justify your answers using calculus.

10. Given the function $f(x) = x^4 - 18x^2 + 32$, list the coordinates of all intercepts and inflection points.

11. Given the function $f(x) = \frac{2x^3 - 6x^2}{x^2 - x - 6}$, identify all asymptotes (vertical, horizontal and/or oblique). Justify your answers with appropriate algebra and calculus techniques.

12. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 9 - x^2$. What are the dimensions of the rectangle with the largest area? What is the largest area?

1. a. $3x^2 \cdot e^{arctan(x)} + \frac{x^3 e^{arctan(x)}}{1+x^2}$ b. $\frac{[2](x^2-3x+5)-(2x-3)[2x-3]}{(x^2-3x+5)^2} = \frac{-2x^2+6x+1}{(x^2-3x+5)^2}$

2. $x = -4,\ 3,\ 10$

3. $\frac{dy}{dx} = (ln\, x)^{5x}\left(5\, ln\,(ln\, x) + \frac{5}{ln\, x}\right)$

4. a. $(5, 0)$ b. $\frac{dy}{dx} = \frac{4-2x}{4y-8} = -\frac{(x-2)}{2(y-2)}$ c. $y = \frac{3}{4}x - \frac{15}{4}$

5. a. $\frac{1}{8}$ b. $-\frac{1}{6}$ c. 0

6. $\frac{970}{13} \approx 74.6$ mph

7. $a = \frac{11}{2}$ $b = -6$

8. $\sqrt{35} \approx 6 + \frac{1}{12}(-1) = \frac{71}{12}$

9. $f_{max} = 76$ @ $x = 8$, $f_{min} = -5$ @ $x = 5$

10. Intercepts: $(0, 32),\ (4, 0),\ (-4, 0),\ \left(\sqrt{2}, 0\right),\ \left(-\sqrt{2}, 0\right)$

    Inflection points: $\left(\sqrt{3},\ -13\right),\ \left(-\sqrt{3},\ -13\right)$

11. Vertical asymptote: $x = -2$ Oblique asymptote: $y = 2x - 4$

12. Maximize $A = (2x)y = 2x(9 - x^2)$ $A' = 18 - 6x^2 = 0$ at $x = \pm\sqrt{3}$

    Dimensions: $\left(2\sqrt{3}\right) \times (6)$ Area: $12\sqrt{3}$ square units