

1. Find  $f'(x)$  [do not simplify your results]:
  - a.  $f(x) = \tan(4x - \pi) \cdot \sec(x^3)$
  - b.  $f(x) = \left(\frac{x^2}{3x+2}\right)^4$
2. Find  $\frac{dy}{dx}$ ; simplify your answer :
  - a.  $y = \frac{\sin x}{1 - \cos x}$
  - b.  $y = \ln(x^2 + 1) - 2x \tan^{-1}(x)$
3. Find the critical points for the function  $\sqrt[3]{x^2 - 7x - 30}$ :
4. Find the equation, in *slope-intercept form*, of the line tangent to the graph of  $f(x) = (x - 7)^{\sqrt{x+1}}$  at the point where  $x = 8$ .
5. Given the equation  $x^3 - x^2y + y^2 = 20$ 
  - a. Find the point on the graph of this equation in the *first quadrant* whose  $x$ -coordinate is 2.
  - b. Find the formula for  $\frac{dy}{dx}$  (in terms of both  $x$  and  $y$ ).
  - c. Find the equation, in *slope-intercept form*, of the line tangent to the graph of this equation at the point from part a.
6. Assume that  $x$  and  $y$  are differentiable functions of  $t$ . Find  $\frac{dy}{dt}$  when  $x^2 - xy + y^2 = 19$ ,  $\frac{dx}{dt} = 3$  for  $x = 2$ , and  $y > 0$ .
7. Use the formula  $f(x) \approx L(x) = f(a) + f'(a)(x - a)$  to find a fraction (of integers) that approximates  $\sqrt[3]{122}$ . Be sure to identify the function  $f(x)$  and the value of  $a$  that is used.
8. Find the absolute maximum and minimum values of the function  $f(x) = 2x^3 - 9x^2 - 24x + 35$  on the interval  $[-2, 7]$ . Justify your answers using calculus.
9. Sketch the graph of  $f(x) = (x - 5)^2 \cdot (x + 2) = x^3 - 8x^2 + 5x + 50$ . Identify all intercepts, asymptotes, critical numbers, local extrema, and inflection points.
10. A rectangle has its base on the  $x$ -axis, its lower left corner at  $(0, 0)$ , and its upper right corner on the curve  $y = 6e^{-x/2}$ . Find the dimensions of the rectangle that has the largest area. What is the largest area?
11. Evaluate the following limits, if they exist (if not, write DNE). Justify your answers.
  - a.  $\lim_{x \rightarrow 4} \frac{\sqrt{6x+1}-5}{2x-8}$
  - b.  $\lim_{x \rightarrow 1} \frac{\cos(\pi x)+1}{\ln(x)-x+1}$
  - c.  $\lim_{x \rightarrow \infty} x \cdot \sin\left(\frac{\pi}{2x}\right)$

1. a.  $4[\sec^2(4x - \pi)] \cdot \sec(x^3) + \tan(4x - \pi)[3x^2 \cdot \sec(x^3) \tan(x^3)]$   
b.  $4\left(\frac{x^2}{3x+2}\right)^3 \frac{[2x](3x+2) - x^2[3]}{(3x+2)^2}$
2. a.  $\frac{1}{\cos x - 1}$  b.  $-2 \tan^{-1}(x)$
3.  $-3, \frac{7}{2}, 10$
4.  $y = 3x - 23$
5. a.  $(2, 6)$  b.  $\frac{dy}{dx} = \frac{3x^2 - 2xy}{x^2 - 2y}$  c.  $y = \frac{3}{2}x + 3$
6. When  $x = 2, y = 5$ . Implicitly differentiating yields  $\frac{dy}{dt} = \frac{2x-y}{x-2y} \frac{dx}{dt}$   
Substituting values:  $\left. \frac{dy}{dt} \right|_{(x,y)=(2,5), \frac{dx}{dt}=3} = \frac{3}{8}$
7. Let  $f(x) = \sqrt[3]{x}$ . Note that  $f(125) = 5$   $\sqrt[3]{122} \approx f(125) + f'(125)([122] - 125) = \frac{124}{25}$
8.  $f_{min} = -77$  when  $x = 4$   $f_{max} = 112$  when  $x = 7$
9. Intercepts:  $(0, 50); (5, 0); (-2, 0)$  Asymptotes: none  
Critical numbers:  $x = \frac{1}{3}, 5$  Min:  $f(5) = 0$  Max:  $f(\frac{1}{3}) = \frac{1372}{27} \approx 50.8$   
Inflection points:  $(\frac{8}{3}, \frac{686}{27})$
10. Maximize area =  $A = b \cdot h = x \cdot (y) = x \cdot (6e^{-x/2})$   
 $\frac{dA}{dt} = 0$  when  $x = 2$  dimensions:  $b = 2; h = \frac{6}{e}$  maximum area =  $\frac{12}{e}$
11. a.  $\frac{3}{10}$  b.  $-\pi^2$  c.  $\frac{\pi}{2}$