

1. Find $f'(x)$:
 - a. $f(x) = \tan^{-1}(4x) \cdot \tan\left(\frac{1}{4x}\right)$
 - b. $f(x) = \left(\frac{\csc x}{5 + \sin x}\right)^3$

2. Given the equation $x^y = y^x$, find $\frac{dy}{dx}$ by using the technique of logarithmic differentiation (don't try to solve for y in terms of x).

3. Find the equation(s), in *slope-intercept form*, of the line(s) tangent to the parabola $y = x^2$ that also go through the point $(1, -8)$.

4. The graph of the first derivative f' of a function f is shown (graph from problem 4.3.6 p.295)
 - a. On what interval(s) is f increasing?
 - b. At what value(s) of x does f have a local maximum?
 - c. At what value(s) of x does f have a local minimum?
 - d. What are the x -coordinates of the inflection point(s) of f ?

5. Evaluate the following limits, if they exist. Justify your answers.
 - a. $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{81 - x^2}$
 - b. $\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{x^2}$
 - c. $\lim_{x \rightarrow \infty} \left(1 + \frac{4}{x}\right)^x$

6. Two carts, A and B , are connected by a rope 33 feet long that passes over a pulley P (see the figure). The point Q is on the floor 12 feet directly beneath P and between the carts. Cart A is being pulled away from Q at a speed of 2 feet per second. How fast is cart B moving toward Q when cart A is 5 feet from Q ? Label the units of your answer.

7. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 9x^2 + 15x + 40$ on the interval $[0, 8]$. Justify your answers using calculus:

8. Use differentials (or a linear approximation) to estimate $(2.98)^5$.

9. A rectangular beam will be cut from a cylindrical log of radius 10 inches. The strength of the beam is proportional to the product of its width and the square of its depth. Find the dimensions of the strongest beam that can be cut from the cylindrical log.

10. Sketch the graph of $f(x) = \frac{x}{x^2+4}$. Identify all intercepts, asymptotes, critical numbers, local extrema, and inflection points. [Note: "none" is a possible answer].

1. a. $\left[\frac{4}{1+16x^2}\right] \tan\left(\frac{1}{4x}\right) + \tan^{-1}(4x) \left[\frac{-1}{4x^2} \sec^2\left(\frac{1}{4x}\right)\right]$
 b. $3\left(\frac{\csc x}{5+\sin x}\right)^2 \left(\frac{(5+\sin x)[- \csc x \cot x] - \csc x [\cos x]}{(5+\sin x)^2}\right)$

2. $\frac{dy}{dx} = \frac{xy \ln y - y^2}{xy \ln x - x^2}$

3. $y = 8x - 16$; $y = -4x - 4$

4. a. $(0, 1)$; $(3, 5)$ b. $x = 1, 5$ c. $x = 3$ (and endpoints 0, 6) d. $x = 2, 4$

5. a. $-\frac{1}{108}$ b. $\frac{9}{2}$ c. e^4

6. Find $\frac{dy}{dt}$ when $x = 5$ ($y = 16$) given $\frac{dx}{dt} = 25$ ft/sec

Equation is : $\sqrt{x^2 + 144} + \sqrt{y^2 + 144} = 33$

$\frac{dy}{dt} = -\frac{25}{26}$ ft/sec

7. $f_{min} = 15$ at $x = 5$ $f_{max} = 96$ at $x = 8$

8. $(2.98)^5 \approx 243 + 5(3)^4[-.02] = 234.9$

9. Maximize: $S = k w (d^2) = k w (476 - w^2)$; width = $8\sqrt{3}$ inches depth = $8\sqrt{6}$ inches

10. Intercept: $(0, 0)$ Asymptote: $y = 0$ Critical numbers: $x = \pm 2$

Max: $\frac{1}{4}$ @ $x = 2$ Min: $-\frac{1}{4}$ @ $x = -2$ Inf. points: $(0, 0)$, $\left(2\sqrt{3}, \frac{\sqrt{3}}{8}\right)$, $\left(-2\sqrt{3}, -\frac{\sqrt{3}}{8}\right)$

