

1. Find $f'(x)$ [do not simplify your results]:

a. $f(x) = e^{\tan x} \cdot \sec\left(2x - \frac{\pi}{3}\right)$

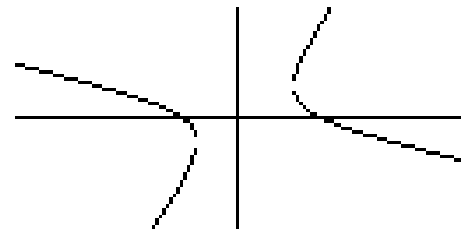
b. $f(x) = \left(\frac{x^2}{\cosh(5x)-4}\right)^3$

2. Find $\frac{dy}{dx}$; simplify your answer :

a. $y = \ln(\csc(3x) - \cot(3x))$

b. $y = (4x^2 + 1)\tan^{-1}(2x) - 2x$

3. The equation $x^2 + 2xy - y^2 - x = 6$ represents a "rotated" hyperbola, that is, a hyperbola whose axes are not parallel to the coordinate axes.



a. Find the formula for $\frac{dy}{dx}$ (in terms of both x and y).

b. Find the coordinates of the point(s) on the graph where the tangent line is vertical.

c. Find the equation, in *slope-intercept form*, of the line tangent to the graph of this equation at the point where the hyperbola crosses the positive x -axis.

4. Water is being pumped at a rate of 0.3 cubic meters per minute into a tank that has the shape of an inverted cone. The tank is 5 meters tall and has a diameter at the top of 2 meters. How fast is the water level rising when the depth of the water is 3 meters? Recall that the volume of a cone is one-third the area of the base times the height ($V = \frac{1}{3}Bh$).

5. Find the value(s) of a and b for which $f(x)$ is continuous for all x : justify your answers by using the definition of continuity.

$$f(x) = \begin{cases} ax + 6 & x < 3 \\ b & x = 3 \\ x^2 - a & x > 3 \end{cases}$$

6. Find the absolute maximum and minimum values of the function $f(x) = 2\cos x - \sin(2x)$ on the interval $[-\frac{\pi}{2}, \pi]$. Justify your answers.

7. Evaluate the following limits, if they exist (if not, write DNE). Justify your answers.

a. $\lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{(x-4)^2}$

b. $\lim_{x \rightarrow 1} \frac{\cos(\pi x) + 1}{\ln(x) - x + 1}$

c. $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^x$

8. A rectangular storage container with an open top is to have a volume of 15 cubic meters. The length of its base is twice the width. Material for the base costs \$100 per square meter; material for the sides costs \$30 per square meter. Find the dimensions of the box that can be constructed for the least cost. What is the minimum cost?

9. Sketch the graph of $f(x) = x^{2/3} \cdot (x - 10) = x^{5/3} - 10x^{2/3}$. Identify all intercepts, asymptotes, critical numbers, local extrema, and inflection points. [Note: "none" is a possible answer].

10. On the graph of the function $y = f(x) = x^3$, when the point is moved from the point where $x = 4$ to the point where $x = 4.1$, find:

a. Δy (the actual change in y : don't round off)

b. dy (the differential of y for these values)

11. Find the most general antiderivative of the given functions $f(x)$:

a. $f(x) = \frac{1}{x^4} + \frac{5}{\sqrt{1-x^2}}$

b. $f(x) = \frac{8x^4 - 7x^2 + 3}{x}$

1. a. $e^{\tan x} [\sec^2 x] \cdot \sec\left(2x - \frac{\pi}{3}\right) + e^{\tan x} \left[2 \sec\left(2x - \frac{\pi}{3}\right) \tan\left(2x - \frac{\pi}{3}\right)\right]$
b. $3\left(\frac{x^2}{\cosh(5x)-4}\right)^2 \left[\frac{[2x](\cosh(5x)-4) - x^2[5 \sinh(5x)]}{(\cosh(5x)-4)^2}\right]$
2. a. $3 \csc(3x)$ b. $8x \tan^{-1}(2x)$
3. a. $\frac{dy}{dx} = \frac{1-2x-2y}{2x-2y}$ b. $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ and $(2, 2)$ c. $y = -\frac{5}{6}x + \frac{5}{2}$
4. Given $\frac{dV}{dt} = 0.3$, find $\frac{dh}{dt}$ when $h = 3$: $V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\left(\pi\left(\frac{1}{5}h\right)^2\right)h = \frac{\pi}{75}h^3$
 $\frac{dx}{dt} = \left(\frac{25}{\pi h^2}\right) \frac{dV}{dt} \Bigg|_{h=3, \frac{dV}{dt}=0.3} = \frac{25(0.3)}{9\pi} = \dots = \frac{5}{6\pi}$ meters/minute
5. $a = \frac{3}{4}$ or $b = \frac{33}{4}$
6. $f_{min} = -2$ at $x = \pi$ $f_{max} = \frac{3\sqrt{3}}{2}$ at $x = -\frac{\pi}{6}$
7. a. *DNE* b. $-\pi^2$ c. $\frac{1}{e^2}$
8. Let $x =$ width of base; Given volume $V = (x)(2x)h = 15 \rightarrow h = \frac{15}{2x^2}$
Minimize cost: $C = 100(x \cdot 2x) + 30(2xh + 2(2xh)) = 200x^2 + 180xh$
 $C = 200x^2 + 180x\left(\frac{15}{2x^2}\right) = 200x^2 + \frac{1350}{x} \rightarrow \frac{dC}{dx} = 0$ when $x = \frac{3}{2}$
Dimensions: $\frac{3}{2} \times 3 \times \frac{10}{3}$ Minimum cost = \$1350
9. Intercepts: $(0, 0)$; $(10, 0)$ Asymptotes: none
Critical numbers: $x = 0, 4$ Min: $f(4) = -6\sqrt[3]{16} = -12\sqrt[3]{2}$ Max: $f(0) = 0$
Inflection point: $\left(-2, -12\sqrt[3]{4}\right)$;
10. a. 4.921 b. 4.8
11. a. $-\frac{1}{3x^3} + 5 \sin^{-1}x + C$ b. $2x^4 - \frac{7}{2}x^2 + 3 \ln|x| + C$