Math 19B - Calculus II **Midterm 2 - Chapters 7, 8, 11**

M. Eastman - Spring 2006

1. Write out the form of the partial fraction decomposition of the function. Do *<u>not</u>* determine the value of the coefficients.

$\frac{4x^2-7x+8}{(x-5)^3(x^2+9)^2}$

2. Evaluate: $\int \frac{x-15}{x^2+3x-28}\,dx$

3. Evaluate: $\int tan^3x\, sec^5x\, dx$

4. Evaluate: $\int \frac{1}{x^2+2x+10}\,dx$

5. Determine whether the following integrals converge or diverge. Evaluate those that are convergent.

a. $\int_2^{\infty} x\, e^{-x} dx$ b. $\int_{\sqrt{2}}^{2} \frac{1}{\sqrt{4-x^2}}\,dx$

6. Given the equation $y = \frac{x^3}{6}$ between the points $\left(2, \frac{4}{3}\right)$ and $\left(3, \frac{9}{2}\right)$:

a. Set up, but do not evaluate, an integral that finds the length of this segment of the curve.

b. If this segment of the curve is rotated around the x-axis, find the area of the surface that is generated.

7. Determine whether the following sequences converge or diverge. Justify your answers. If a sequence converges, find its limit.

a. $\left\{\frac{ln\, n}{\sqrt{n}}\right\}_{n=1}^{\infty}$ b. $a_n = tan^{-1}\left(\frac{\sqrt{3n^2+1}}{n+4}\right)$

8. Determine whether the following series converge or diverge. If a series converges, find its sum. Justify your answers.

a. $\sum_{n=1}^{\infty} \frac{n(n+1)}{16n^2-9}$ b. $\sum_{n=1}^{\infty} \frac{4}{n^2+2n}$

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1. $\frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx+E}{x^2+9} + \frac{Fx+G}{(x^2+9)^2}$

2. $\int \frac{x-15}{x^2+3x-28}\,dx = \int \left(\frac{2}{x+7} - \frac{1}{x-4}\right) dx = 2\,ln|\,x+7\,| - ln|\,x-4\,| + C$

3. (Substitute $u = sec\,x$) Integral is $\frac{1}{7}sec^7 x - \frac{1}{5}sec^5 x + C$

4. (Complete the square: set $u = \frac{x+1}{3}$) Integral is $\frac{1}{3}\,tan^{-1}\left(\frac{x+1}{3}\right) + C$

5. a. Converges to $\frac{3}{e^2}$

   b. Converges to $\frac{\pi}{4}$

6. a. $\int_2^3 \sqrt{1 + \frac{x^4}{4}}\,dx$

   b. $SA = \int_2^3 2\pi\left(\frac{x^3}{6}\right)\sqrt{1 + \frac{x^4}{4}}\,dx = \left(\text{let } u = \frac{x^4}{4}\right) = \frac{\pi\left(85\sqrt{85} - 40\sqrt{5}\right)}{36}$

7. a. Converges to 0.

   b. Converges to $\frac{\pi}{3}$.

8. a. Diverges (test for divergence: $\lim_{n\to\infty}\left(\frac{n(n+1)}{16n^2-9}\right) = \frac{1}{16} \neq 0$).

   b. Converges to 3.