Math 19B -- Calculus II **FINAL EXAM**
M. Eastman -- Spring 2006

1. Evaluate:

   a. $\int tan^4 x \, sec^4 x \, dx$ b. $\int sin^2 x \, dx$

2. Evaluate: $\int \frac{x^2+6}{x^3-6x^2+9x} \, dx$

3. Evaluate: show your work and give the *exact* answer. $\int_1^e x^4 \, ln \, x \, dx$

4. Find the area of the region inside the circle $x^2+y^2=36$ to the right of the line $x=3$.

5. Let $\mathbb{R}$ be the region in the first quadrant under the curve $f(x)=e^{-x}$.

   a. Find the area of $\mathbb{R}$, or explain why such an area cannot be found.

   b. The region $\mathbb{R}$ is rotated around the y-axis. Find the volume that is generated, or explain why such a volume cannot be found.

6. Find the length of the curve given by $y=\frac{1}{3}\sqrt{x}\,(x-3)$ from $\left(1,\ -\frac{2}{3}\right)$ to $(9,6)$.

7. Determine whether the following series converge or diverge. Be specific with your answers when appropriate (absolute vs. conditional convergence). State which test(s) you use.

   a. $\sum_{n=2}^{\infty} \frac{1}{n\cdot(ln(n))^2}$ b. $\sum_{n=0}^{\infty} \frac{(2n)!}{3^n(n!)^2}$

8. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{4^n\cdot\sqrt{n}}$

9. Use Taylor series to find the sum of the following series:

a. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{5^n}{n!}$

b. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{(\pi)^{2n+1}}{3\cdot 9^n\cdot(2n+1)!}$

10. Find the first four non-zero terms of the Taylor series for $f(x) = \sqrt[3]{x}$ at $a = 1$.

11. a. Find the Maclaurin series for $f(x) = cos\sqrt{x}$ by using the (known) Taylor series for $cos\, x$ at $x = 0$. Write your answer using $\sum$ notation.

b. Use the series to estimate $\displaystyle\int_0^1 cos\sqrt{x}\, dx$ with a ***fraction*** (of integers) to within 0.001, using the fewest number of terms necessary. Justify your answer.

Math 19B -- Calculus II **FINAL EXAM - Form A**
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1. a. $\frac{tan^7 x}{7} + \frac{tan^5 x}{5} + C$ b. $\frac{1}{2}x - \frac{1}{4}sin\,x + C$

2. $\frac{1}{3}\,ln(x^2(x-3)) - \frac{5}{(x-3)} + C$

3. $\frac{4}{25}e^5 + \frac{1}{25}$

4. $A = \int_3^6 2\sqrt{36 - x^2}\,dx = 12\pi - 9\sqrt{3}$

5. a. $\int_0^\infty e^{-x}\,dx$ converges to 1

   b. $\int_0^\infty 2\pi(x)\,e^{-x}\,dx$ converges to 2π

6. $L = \int_1^9 \sqrt{\left(\frac{1}{2}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\right)^2 + 1}\,dx = \frac{32}{3}$

7. a. Converges--absolutely (integral test)

   b. Diverges (ratio test)

8. $[-1, 7)$

9. a. e^{-5}

   b. $sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

10. $1 + \frac{1}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{5}{81}(x-1)^3$

11. a. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(2n)!}$

    b. $1 - \frac{1}{4} + \frac{1}{72} = \frac{55}{72}$