Math 19B -- Calculus II

M. Eastman -- Winter 2003 **FINAL EXAM**

1. Evaluate:

a. $\int tan^3 x\, sec^3 x\, dx$ b. $\int \frac{x+6}{x^3-2x^2}\, dx$ c. $\int \frac{1}{x^2\sqrt{x^2-9}}\, dx$

2. The region in the first quadrant under the curve $y = e^{-x^2}$ and to the right of the line $x = 1$ is rotated around the y-axis. Find the volume of the region that is generated, or explain why such a volume cannot be found.

3. Use the arc length formula and a definite integral to find the length of the curve $y = \sqrt{16 - x^2}$ between the points $(0, 4)$ and $\left(2\sqrt{3}\,, 2\right)$.

4. Determine whether the following series are convergent or divergent. If a series is convergent, find its sum. Justify your answer.

a. $8 - 6 + \frac{9}{2} - \frac{27}{8} + \frac{81}{32} - \frac{243}{128} + \ldots$

b. $\sum_{n=0}^{\infty} \frac{3^n}{(n)!}$ [Think of a known Maclaurin series]

5. Determine whether each series is absolutely convergent, conditionally convergent or divergent. Justify your answer. State which test(s) you use.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$

b. $\sum_{n=0}^{\infty} \frac{\left(8-3n^2\right)^n}{(2n+9)^{2n}}$

6. Find the interval of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x-5)^n}{n \cdot 3^n}$

7. Find the first four non-zero terms of the Taylor series for $f(x) = \sqrt{x}$ centered at $a = 1$.

8. a. Find the Maclaurin series for $f(x) = x \cdot cos\, x$ by using the (known) Maclaurin series for $cos\, x$. Use $\sum$ notation.

b. Use the series to estimate $\int_0^1 x \cdot cos\, x\, dx$ to within 0.001, using the fewest number of terms necessary.

c. Evaluate $\int_0^1 x \cdot cos\, x\, dx$ using integration by parts. Write the exact value of the integral.

FINAL EXAM - ANSWERS

1. a. $\frac{sec^5 x}{5} - \frac{sec^3 x}{3} + C$ b. $2\ln\left(\frac{x-2}{x}\right) + \frac{3}{x} + C$ c. $\frac{\sqrt{x^2-9}}{9x} + C$

2. $\int_1^\infty 2\,\pi\,x \cdot e^{-x^2}\,dx = \lim_{M\to\infty}\int_1^M 2\,\pi\,x\cdot e^{-x^2}\,dx = \lim_{M\to\infty}\pi\left(-e^{x^2}\right)\Big|_1^M = \ldots = \frac{\pi}{2e}$

3. $L = \int_0^{2\sqrt{3}}\sqrt{1+\left(\frac{-x}{\sqrt{16-x^2}}\right)^2}dx = \int_0^{2\sqrt{3}}\frac{4}{\sqrt{16-x^2}}\,dx$

   Substitute $sin\theta = \frac{x}{4}$. Integral becomes $\int_0^{\pi/3} 4\,d\theta = \frac{4\pi}{3}$

4. a. converges to $\frac{32}{7}$ (geometric series) b. converges to e^3

5. a. Conditionally converges (alt. series test, p-series) b. Absolutely converges (root test)

6. (2, 8]

7. $1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{8} + \frac{(x-1)^3}{16}$

8. a. $\sum_{n=1}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n)!}$ b. $\frac{1}{2} - \frac{1}{8} + \frac{1}{144} = \frac{55}{144}$ c. $sin(1) - cos(1) + 1$