Math 19B -- Calculus II **Test 1 -- Form A**
M. Eastman -- Fall 2009 **ANSWERS**

1. $g'(x) = -\frac{sec^2 x}{\sqrt{tan^4 x + 3}}$

2. a. $F'(x) = \frac{1}{x+\sqrt{9+x^2}} \cdot [1 + \frac{1}{2}\frac{2x}{\sqrt{9+x^2}}] = \frac{1}{x+\sqrt{9+x^2}} \cdot [\frac{\sqrt{9+x^2}}{\sqrt{9+x^2}} + \frac{x}{\sqrt{9+x^2}}] = \frac{1}{\sqrt{9+x^2}}$

   b. $\int_0^4 \left(\frac{1}{\sqrt{9+x^2}}\right) dx = ln\,|\,x + \sqrt{9+x^2}\,|\,\Big|_0^4 = ln\left(4+\sqrt{25}\right) - ln\left(0+\sqrt{9}\right) = ln\,3$

3. a. $\Delta x = \frac{4}{n}$ b. $x_i^* = 1 + \frac{4i}{n}$

   c. $\int_1^5 (x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{4i}{n}\right)^2 \cdot \frac{4}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{4}{n} + \frac{32i}{n^2} + \frac{64i^2}{n^3}\right)$

   $= = \lim_{n\to\infty}\left(\frac{4}{n}(n) + \frac{32}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right) = 4 + 16 + \frac{64}{3} = \frac{124}{3}$

4. a. $\frac{3}{2}x^{10/3} - 3x^{7/3} + C$ b. $\frac{20}{3}$

5. $A = \int_{-6}^{4}((16-x^2)-(2x-8))\,dx = \frac{500}{3}$ square units

6. shells: $\int_0^2 2\pi(y+1)[(6-y)-(y^2)]\,dx$

   washers: $\int_0^4 \pi\left(\left(\sqrt{x}+1\right)^2 - 1^2\right)dx + \int_4^6 \pi\left(((6-x)+1)^2 - 1^2\right)dx$

   Both integrals yield the volume : $V = \frac{76\pi}{3}$

7. a. $W = \int_0^{50}(200-4x)\,dx$ [here x is distance from bottom of chain's original position]

   -or- $W = \int_0^{50}(x)\,[4dx]$ [x is distance that a 'chunk' of chain (which 'weighs' $4\,dx$ pounds) is lifted]

   $W = 5000$ foot pounds

   b. $W = \int_0^{30}(200-4x)\,dx =$ -or- $= \int_{20}^{50}(x)\,[4dx] = ... =$ 4200 foot pounds

8. a. $-x\,e^{-x} - e^{-x} + C$ b. $2\,e^3 - 2\,ln\,(2) + 2$