Math 19B -- Calculus II **Test 1 -- Form B**
M. Eastman -- Fall 2009 **ANSWERS**

1. $g'(x) = -\frac{sec\,x\,tan\,x}{\sqrt{sec^3x+1}}$

2. a. $F'(x) = \frac{1}{\sqrt{x^2+16}+x} \cdot [\frac{1}{2}\frac{2x}{\sqrt{x^2+16}} + 1] = \frac{1}{\sqrt{x^2+16}+x} \cdot [\frac{x}{\sqrt{9+x^2}} + \frac{\sqrt{x^2+16}}{\sqrt{x^2+16}}] = \frac{1}{\sqrt{x^2+16}}$

   b. $\int_0^3 \left(\frac{1}{\sqrt{x^2+16}}\right) dx = ln\,|\sqrt{x^2+16}+x\,|\Big|_0^3 = ln\Big(\sqrt{25}+3\Big) - ln\left(\sqrt{16}+0\right) = ln\,2$

3. a. $\Delta x = \frac{4}{n}$ b. $x_i^* = 2 + \frac{4i}{n}$

   c. $\int_2^6 (x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n}\Big(2+\frac{4i}{n}\Big)^2 \cdot \frac{4}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{16}{n} + \frac{64i}{n^2} + \frac{64i^2}{n^3}\right)$

   $= = \lim_{n\to\infty}\left(\frac{16}{n}(n) + \frac{64}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right) = 16 + 32 + \frac{64}{3} = \frac{208}{3}$

4. a. $\frac{3}{2}x^{8/3} - 3x^{5/3} + C$ b. $\frac{506}{15}$

5. $A = \int_{-2}^{6}((16-x^2) - (-4x+4))\,dx = \frac{256}{3}$ square units

6. washers: $\int_0^2 \pi[((6-y)+1)^2 - (y^2+1)^2]\,dx$

   shells: $\int_0^4 2\pi(x+1)\big(\sqrt{x}\big)dx + \int_4^6 2\pi(x+1)(6-x)dx$

   Both integrals yield the volume : $V = \frac{884\pi}{15}$

7. a. $W = \int_0^{60}(180-3x)\,dx$ [here x is distance from bottom of chain's original position]

   -or- $W = \int_0^{60}(x)\,[3dx]$ [x is distance that a 'chunk' of chain (which 'weighs' $3\,dx$ pounds) is lifted]

   $W =$ 5400 foot pounds

   b. $W = \int_0^{40}(180-3x)\,dx =$ -or- $= \int_{20}^{60}(x)\,[3\,dx] = ... =$ 4800 foot pounds

8. a. $\frac{1}{2}x^2 ln\,x - \frac{1}{4}x^2 + C$ b. $1 - \frac{4}{e^3} = \frac{e^3-4}{e^3}$