Math 5B -- Calculus II **Exam 2 -- Form B**

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1. Use substitution: $u = sec\,x$ Answer: $\frac{1}{7}sec^7x - \frac{2}{5}sec^5x + \frac{1}{3}sec^3x + C$

2. Use trig substitution: $x = 3\,sec\,\theta$ Answer: $\frac{\sqrt{x^2-9}}{9x} + C$

3. $\frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx+E}{(x^2+4)} + \frac{Fx+G}{(x^2+4)^2}$

3. Note : $\frac{x+13}{3x^2+x-15} = \frac{4}{3x-5} + \frac{(-1)}{x+2}$ Answer : $\frac{4}{3}\,ln\,|\,3x-5\,| - ln\,|\,x+2\,| + C$

4. a. $\int_0^\infty x\,e^{-x^2}dx = \lim_{T\to\infty}\int_0^T x\,e^{-x^2}dx$ Let $u = -x^2$

$= \ldots = \lim_{T\to\infty}\frac{1}{2}\,e^{-x^2}\Big|_0^T = \lim_{T\to\infty}\left(\frac{1}{2}e^{-T^2} - \frac{1}{2}\right) = \frac{1}{2}$ converges to $\frac{1}{2}$

b. $\int_1^{10}(x-2)^{-4/3}\,dx = \int_1^2 (x-2)^{-4/3}\,dx + \int_2^{10}(x-2)^{-4/3}\,dx$

$= \lim_{b\to 2^-}\left(\int_1^b (x-2)^{-4/3}\,dx\right) + \lim_{a\to 2^+}\left(\int_a^{10}(x-2)^{-4/3}\,dx\right)$

Both integrals diverge. Hence the original integral diverges.

5. a. $s = \int_1^4 \sqrt{1+[2x]^2}\,dx = \int_1^4 \sqrt{1+4x^2}\,dx$ --or--

$s = \int_1^{16}\sqrt{\left[\frac{1}{2\sqrt{y}}\right]^2+1}\;dx = \int_1^{16}\sqrt{\frac{1}{4y}+1}\;dx$

b. $A = \int_1^4 2\pi(x)\sqrt{1+[2x]^2}\,dx = \int_1^4 2\pi(x)\sqrt{1+4x^2}\,dx =$ --or--

$A = \int_1^{16} 2\pi\left(\sqrt{y}\right)\sqrt{\frac{1}{4y}+1}\;dy = \int_1^{16} 2\pi(y)\sqrt{\frac{1}{4}+y}\;dy$

Both integrals yield $A = \frac{\pi}{6}\left(65\sqrt{65} - 5\sqrt{5}\right)$

6. a. $a_k = \frac{3}{k+1} - \frac{3}{k+3}$ b. $s_n = \frac{3}{2} + 1 - \frac{3}{n+2} - \frac{3}{n+3}$ c. $\frac{5}{2}$

7. a. diverges -- integral test b. converges -- ratio test