Math 22 - Calculus of Several Variables **Final Exam - Form A**
M. Eastman - Fall 2007

1. Given the points $\mathbb{A} = (5,\ 3,\ -1)$, $\mathbb{B} = (4,\ 5,\ -2)$, and $\mathbb{C} = (0,\ 8,\ -1)$, find:
   a. parametric equations for the line through $\mathbb{A}$ and and is parallel to the line through $\mathbb{B}$ and $\mathbb{C}$.
   b. the angle $\angle\mathbb{BAC}$.
   c. the equation of the plane (in the form $ax + by + cz = d$) through these three points.

2. Given the curve defined by $\mathbf{r}(t) = \ < t^2,\ 2t,\ ln\, t >$, find:
   a. $\mathbf{T}(2)$ (the unit tangent to this curve when $t = 2$).
   b. the length of this curve between the points $(1,\ 2,\ 0)$ and $(16,\ 8,\ ln\, 4)$
   c. the vector equation for the line tangent to this curve when $t = 4$.

3. Given $f(x, y, z) = 2x^3y\ + 7xy^2z - 4x^3z^2$, find:
   a. $f_{xz}(x, y, z)$ b. $\nabla f(-1,\ 1,\ 2)$
   c. the directional derivative of f at $(-1,\ 1,\ 2)$ in the direction of the point$(2,\ -5,\ 4)$.
   d. The equation of the tangent plane to the level surface $f(x, y, z) = 0$ at the point $(-1,\ 1,\ 2)$.

4. Find the critical points for $f(x, y) = x^4 + y^2 - 8x^2 - 2y + 24$. Find the value of f at each critical point, the value of D (for the second derivative test), and classify the function value at each critical point as either a (local) maximum, (local) minimum, or a saddle point. The number of lines in the table do not necessarily indicate the number of points. Justify your answers.

5. Find three positive numbers x, y, and z whose sum is 12 such that $f(x, y, z) = xy^2z^3$ is a maximum. What is the maximum value of f?

6. Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = [0,\ 2] \times [-1, 3]$.

7. Evaluate: $\displaystyle\int\!\!\int_{\mathbb{D}} e^{(x/y)}\, dA$, where $\mathbb{D} = \{ (x, y) \mid 1 \le y \le 2,\ y \le x \le y^3 \}$

8. Evaluate by reversing the order of integration: $\displaystyle\int_0^2 \int_{3x}^{6} \frac{1}{\sqrt{y^2+4}}\, dy\, dx$

9. Evaluate by converting to polar coordinates: $\displaystyle\int_0^2 \int_x^{\sqrt{8-x^2}} \left(\frac{1}{x^2+y^2+1}\right) dy\, dx$

10. Use a triple integral to find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 36$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

11. Use a change of variables to evaluate $\displaystyle\int\!\!\int_{\mathbb{R}} \left(\frac{2x+y}{x-3y}\right) dA$, where $\mathbb{R}$ is the parallelogram enclosed by the lines $2x + y = 2$, $2x + y = 5$, $x - 3y = 1$, and $x - 3y = 7$.

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1. a. $x = 5 - 4t$
$y = 3 + 3t$
$z = -1 + t$

b. $cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

c. $x + y + z = 7$

2. a. $< \frac{8}{9}, \frac{4}{9}, \frac{1}{9} >$

b. $15 + ln\, 4$

c. $< 16 + 8t, 8 + 2t, ln\, 4 + \frac{1}{4}t >$

3. a. $7y^2 - 24x^2z$
b. $< -28, -30, 9 >$
c. $\frac{114}{7}$
d. $28x + 30y9 - z + 16 = 0$

4.

critical point	function value	$D(x,y)$	classification
$(0,1)$	23	-23	saddle
$(2,1)$	7	64	minimum
$(-2,1)$	7	64	minimum

5. $x = 2\,,\ y = 4\,,\ z = 6$; $f_{max} = 6912$

6. $V = \int_0^2 \int_{-1}^3 (12 - 3x - 2y)\, dy\, dx = \ldots = 56$ cubic units

7. $\int_1^2 \int_y^{y^3} e^{\left(\frac{x}{y}\right)}\, dx\, dy = \ldots = \frac{e^4 - 4e}{2}$

8. $\int_0^2 \int_{3x}^6 \frac{1}{\sqrt{y^2+4}}\, dy\, dx = \int_0^6 \int_0^{\frac{1}{3}y} \frac{1}{\sqrt{y^2+4}}\, dx\, dy = \ldots = \frac{2}{3}\left(\sqrt{7} - 1\right)$

9. $\int_0^2 \int_x^{\sqrt{8-x^2}} \left(\frac{1}{x^2+y^2+1}\right) dy\, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\sqrt{8}} \left(\frac{1}{r^2+1}\right) r\, dr\, d\theta = \ldots = \frac{\pi}{8}\, ln\, 9$

10. $\int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^6 \rho^2 sin\,\phi\, d\rho\, d\phi\, d\,\theta = \ldots = 72\,\pi\sqrt{2}$

11. $\int_1^7 \int_2^5 \left(\frac{u}{v}\right) [\frac{1}{25}]\, du\, dv = \ldots = \frac{21}{50}\, ln\, 7$