Math 22 - Calculus of Several Variables **FINAL EXAM**
M. Eastman - Winter 2007

1. Given $\mathbb{L}_1 :$ $x = 4 - t$ $\mathbb{L}_2 :$ $x - 1 = \frac{y+1}{2} = z - 3$

$y = 1 + t$

$z = 2 + 2t$

a. Determine whether the lines $\mathbb{L}_1$ and $\mathbb{L}_2$ are parallel, skew, or intersecting

b. If the lines are parallel, find the equation of the plane containing both lines. If they are skew, find the distance between the two lines. If they intersect, find the point of intersection.

2. Given the points $\mathbb{A} = (3,\, 0,\, 2)$, $\mathbb{B} = (4,\, 3,\, 2)$, and $\mathbb{C} = (1,\, -3,\, 0)$, find:

a. the area of triangle $\Delta\mathbb{ABC}$

b. the equation of the plane (in the form $ax + by + cz = d$) through these three points.

c. the parametric equations of the line passing through the point $\mathbb{A}$ that is parallel to the line that passes through the points $\mathbb{B}$ and $\mathbb{C}$.

3. If $\boldsymbol{r}(t) = \, < 4\,sin\,t,\, 2t,\, 4\,cos\,t >$, find:

a. the length of the curve between $(0,\, 0,\, 4)$ and $(0,\, 2\pi,\, -4)$

b. $\boldsymbol{T}\left(\frac{\pi}{4}\right)$

c. the vector equation for the line $\boldsymbol{L}(t)$ tangent to this curve when $t = \frac{\pi}{2}$.

4. Given the equation $z = 16x^2 + 4y^2 - 64$:

a. Classify this quadric surface.

b. Find the equation of the tangent plane to this surface at the point $(\,2,\ 1,\ 4)$.

c. Suppose that the positive y-axis points north and the x-axis east. If one travels on this surface in a *northwest* direction from the point $(\,2,\ 1,\ 4)$, does one's elevation (as measured by the z-value) increase or decrease? Explain your reasoning and state any quantities that you calculate for justification.

5. Given $f(x,y) = x^3 + y^2 - 3x - 6y + 15$

a. Find all the critical points of f: enter your points in the first column of the table.

b. Find the function value of f at each critical point: enter the values in the second column.

c. Classify each function value as either a local maximum, local minimum, or a saddle point. Justify your answers with the appropriate calculations.

6. Find the dimensions of the rectangular box in the first octant with three faces in the coordinate planes and one vertex on the plane $4x + 2y + z = 8$ that has the largest volume. What is the largest volume?

7. Evalute: $\int_0^2 \int_{x^2}^4 x\,\sqrt[3]{y^2+1}\;dy\,dx$

8. Evalute: $\int\int_{\mathbb{D}} (2y - 4x)\,dA$, where $\mathbb{D}$ is the region in the xy-plane bounded by the parabola $y = x^2$ and the line $y = x + 2$.

9. Use a triple integral to find the volume inside the sphere $x^2 + y^2 + z^2 = 4$, and above the cone $z = \sqrt{3x^2 + 3y^2}$. [Note: this triple integral is very difficult when calculated using rectangular coordinates] .

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1. a. the lines intersect b. $\left(\frac{8}{3}, \frac{7}{3}, \frac{14}{3}\right)$

2. a. 3.5 square units b. $6x - 2y - 3z = 12$ c. $x = 3 + 3t$, $y = 6t$, $z = 2 + 2t$

3. a. $2\pi\sqrt{5}$ b. $< \frac{\sqrt{10}}{5}, \frac{\sqrt{5}}{5}, -\frac{\sqrt{10}}{5} >$ c. $< 4, \pi + 2t, -4t >$

4. a. Elliptic paraboloid

   b. $64x + 8y - z = 132$

   c. decreases; directional derivative is negative $\left(D_{\boldsymbol{u}} f(x,y) = \nabla f \cdot \boldsymbol{u} = -28\sqrt{2}\right)$

5.

critical point	function value	type of point
$(1,3)$	$f(1,3) = 4$	local minimum $(D = 12)$
$(-1,3)$	$f(-1,3) = 8$	saddle point $(D = -12)$

6. $\frac{2}{3} \times \frac{4}{3} \times \frac{8}{3}$ Volume $= \frac{64}{27}$

7. switch the order of integration: $\int_0^4 \int_0^{\sqrt{x}} x\sqrt[3]{y^2+1}\, dx\, dy = \ldots = \frac{3}{16}\left(17\sqrt[3]{17} - 1\right)$

8. $\int_{-1}^{2} \int_{x^2}^{x+2} (2y - 4x)\, dy\, dx = \ldots = \frac{27}{5}$

9. Using cylindrical coordinates, $V = \int_0^{2\pi} \int_0^1 \int_{\sqrt{3}r}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$

   Using spherical coordinates, $V = \int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^2 \rho^2 sin\, \phi\, d\rho\, d\phi\, d\theta$

   With either coordinate system, the volume is $\frac{8\pi\left(2-\sqrt{3}\right)}{3}$.